CDC01-INV2601

An Assessment of the Current Status of Algorithmic Approaches to the Verification of Hybrid Systems

B. Izaias Silva[1], Olaf Stursberg[2], Bruce H. Krogh[1], and Sebastian Engell[2]

[1] Dept. of Electrical and Computer Eng., Carnegie Mellon University, Pittsburgh, PA 15213 (USA), {izaias,krogh}@cmu.edu

[2] Process Control Lab (CT-AST), University of Dortmund, D-44221 Dortmund (Germany), {o.stursberg, s.engell}@ct.uni-dortmund.de

Abstract. As an introductory paper for the invited session *Recent Developments in Tools for Verification of Hybrid Systems*, this paper reviews the current status of implemented verification techniques for hybrid systems. We focus on tools that perform *model checking* for hybrid systems with varying levels of complexity. Features of the tools are described using a batch reactor example to illustrate what is required to develop an appropriate model for each tool. The concluding section suggests directions for future research and tool development based on the needs of industry for tools to perform verification and validation of designs for embedded control systems.

Key Words. hybrid systems, model checking, verification, computer-aided control system design.

I. Introduction

Formal verification refers to methods for determining whether or not given properties (specifications) are true for a given model of a dynamic system. During the past decade, tools for computer-based modeling and verification of discrete-state, or logical, systems have gained industrial importance, especially in the areas of hardware design and communication protocols [13]. Extending formal verification techniques to systems with continuous as well as discrete dynamics has been one of the main research issues in the field of hybrid systems. Several research groups have introduced tools based on this research that offer the possibility to perform formal verification for various classes of hybrid system models using a variety of computational methods. The objective of this paper is to provide a brief survey of the current status of these tools and to offer some perspective on what needs to be done to develop tools that will reach the same level of acceptance in industry as achieved by the tools for purely discrete systems.

In general, there are two approaches to formal verification: *theorem proving* and *model checking*. Methods for dealing with hybrid systems are being explored in both approaches. Theorem proving aims at inferring (or contradicting) a specification for a system model using the methods of logical proof systems, whereas in the model checking approach the

state-transition relation is used in iterative computations to arrive at the set of states (a fixed point) for which the given specification is true. The attractive feature of the theorem proving approach is that it is not restricted to finite-state systems, making it particularly appealing for hybrid systems. Theorem proving is not algorithmic, however, meaning that the proof procedure needs human direction and intervention for all but the most trivial verification problems. In contrast, algorithms exist and have been implemented for model checking using symbolic representations of sets of system states that, in many cases, solve verification problems of considerable complexity automatically [12]. The caveat is that these algorithms handle only finite-state systems.

In this paper and invited session we focus exclusively on the model checking approach because this has been the approach used in the implementation of most of the tools for computer-based verification of hybrid systems. To perform model checking, a finite-state approximation (often called an *abstraction*) of the continuous dynamics needs to be constructed. Representing and constructing these finite-state abstractions is the main function and contribution of these tools. It is well known from the hybrid systems literature that only hybrid systems with very trivial continuous dynamics (simple integrators and some types of simple linear dynamics [1], [24]) have equivalent finite-state models (called *bisimulations*) that can be used for verification. Consequently, verification of properties for the finite state approximation may be inconclusive. For example, discovering a state is reachable in the finite-state approximation may not imply it is reachable in the underlying hybrid system. When the verification results are inconclusive, the tools may provide mechanisms for making the finite-state approximation less conservative, but there can be no guarantee this process of refinement will ever terminate.

Several methods and tools for performing verification of hybrid system have been reported in the literature (see, e.g., recent proceedings of the annual workshop *Hybrid Systems: Computation and Control* [22], [31], [26]). We have the following objectives for the invited session *Recent Developments in Tools for Verification of Hybrid Systems.* First, the session provides a forum for reporting on the latest developments and experiences for tools that have been recognized as representing the state of the research in the hybrid systems community. Second, by presenting the tools together in one session, it makes it possible to more clearly identify and discuss the differences and similarities among the approaches being pursued. Finally, the session provides an opportunity for the participants to discuss perspectives on what might be the most fruitful directions for future research and development, and perhaps even collaboration. The tools presented in the session use various representations, computational methods, and basic formalisms, ranging from timed automata to hybrid systems with nonlinear continuous dynamics.

This survey paper provides a comparative overview of several tools from the following perspectives: the basic modeling formalism, specifications, the user interface, the computational representations, and computational routines. We use an example described in the following section to illustrate how the user develops models amenable to the formalism used by each tool described in Sec. III. Verification results for the example from two tools developed by the authors are also included in this section. Based on experiences with the example application and on other case studies in literature, we assess the status of hybrid system verification tools in Sec. IV. The concluding section discusses directions for further development towards the goal of making these tools accessible and useful to industrial users.

II. A Batch Reactor System

To illustrate the type of problem which is typically considered in hybrid system verification, we consider the discretely-controlled chemical batch reactor shown in Fig. 1 (taken from [30]). In this system a reactor is filled by two liquid streams F_A and F_B with temperatures T_A, T_B and concentrations c_A, c_B of two dissolved substances A and B. The streams can be controlled through the valves v_A and v_B in the inlet pipes. The stirred content of the reactor is cooled by a cooling jacket if the supply of cooling water is switched on by opening valve v_C. Cooling is necessary since an exothermic chemical reaction $2A + B \rightarrow D$ leads to an increase of the reactor temperature T_R. The reaction product can be discharged through the valve v_O which is (as all other valves) controlled by a discrete controller. Measurements of the temperature T_R, the liquid volume V_R, and the concentration c_A indicate whether these variables exceed specific thresholds or not.

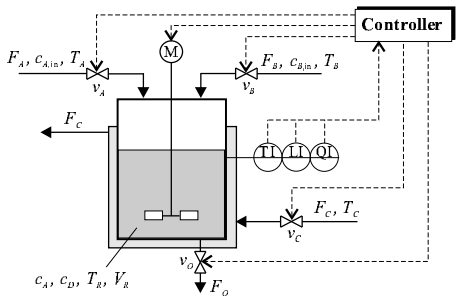

Fig. 1. Batch reactor schematic.

We consider the production procedure shown as a transition model in Fig. 2. Initially, one half of the reactor volume is already filled with solution B (and v_B is closed). In the first step (denoted by z_1), valve v_A is opened to supply the solution A (with concentration c_A^{in}) until the volume V_R reaches an upper limit V_{High}. The chemical reaction leads to an increase of the temperature (state z_2) such that T_R eventually reaches a threshold $T_R = T_{High}$. At this point the controller has to open the valve v_C in order to prevent an overheating of the reactor. From state z_3 (reaction with cooling) three different states can be reached: the 'normal' operation is that the concentration c_A has dropped down to $c_{A,des}$ corresponding to a sufficiently high concentration of the product D, and the reactor is emptied through valve v_O. If, alternatively, the temperature increases further to an upper threshold T_{Alarm}, the state z_5 is reached. (Note that T_R can show an over-shooting behavior when v_C is opened.) As a third possibility, a specified reaction time t_{final} can elapse before the desired concentrations are reached and the procedure terminates in state z_6. The reaction time is measured by a clock t_R, which is reset when valve v_A is opened.

Obviously, the states z_5 and z_6 should be excluded from the course of operation. A discrete controller has to be designed such that it switches the valves v_A, v_C, and v_0 in order to ensure that the operation always ends in state z_4. The objective of formal verification for this system is to determine if the forbidden states are reachable. Specifically, such an analysis will show if the temperature threshold $T_R = T_{High}$ is chosen appropriately to guarantee T_{Alarm} is never exceeded **and** to ensure that the desired product concentration is reached (for which T_{High} must not be chosen too low).

The verification task requires a timed or hybrid model of the process behavior (i. e. of the continuous variables V, c_A and T_R). As shown in the next section, different types and complexities of the process model must be chosen for

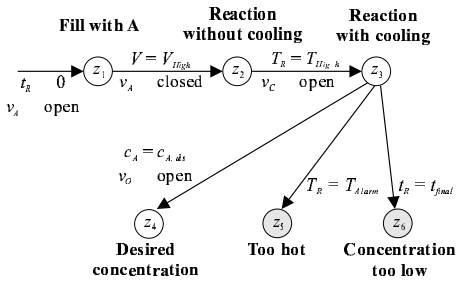

Fig. 2. Operation procedure for the batch reactor.

TABLE I

HYBRID MODEL OF THE REACTOR SYSTEM.

Equations
$\frac{dc_A}{dt} = \frac{s_1 \cdot F_A}{V_R} \cdot (c_A^{in} - c_A) - 2 \cdot r$
$\frac{dT_R}{dt} = \frac{s_1 \cdot F_A}{V_R} \cdot (T_A - T_R) - \frac{s_2 \cdot k_C \cdot A_C}{\rho \cdot c_P \cdot V_R} \cdot (T_R - T_C) - \frac{H_R \cdot r}{\rho \cdot c_P}$
$\frac{dV_R}{dt} = s_1 \cdot F_A$
$\frac{dt_R}{dt} = 1$
$r = c_A^2 \cdot k_0 \cdot \exp\left(\frac{-E_A}{R_m \cdot T_R}\right)$, $A_C = \frac{\pi}{4} \cdot D_R^2 + \frac{4}{D_R} \cdot V_R$
Variables
c_A - concentration of substance A (kmole/m^3)
T_R - temperature of the reactor content (K)
V_R - liquid volume within the reactor (m^3)
t_R - reaction time (sec)
$s_1 \in \{0,1\}$ - switch the valve v_A to close or open
$s_2 \in \{0,1\}$ - switch the valve v_C to close or open

different tools. For some approaches, it is necessary to split up the behavior into the filling part and the procedure continuing from state z_2. While the analysis for the first part determines the admissible values for c_A and T_R under the condition that $V = V_{High}$, the verification for the second part evaluates the reachable set starting from the final values of the first part, but for a constant volume V_{High}. For those tools that can handle switched nonlinear dynamics, the hybrid model given in Table I is used. For approaches that can handle hybrid systems with linear dynamics only, the ODEs for T_R are linearized for suitable regions of the state space.

III. REPRESENTATIVE MODEL CHECKING TOOLS

This section briefly surveys the representative selection of model checkers for hybrid systems and describes how they could be applied to the batch reactor system. These tools are selected from those that have attracted the most attention in recent years and cover a large range of different classes of hybrid systems. We have included the application of one tool for timed automata, one for linear hybrid automata, another for hybrid systems with linear continuous dynamics, and two for systems with switched nonlinear dynamics. Three other tools are briefly described. More comprehensive description

of the tools Uppaal, HyTech, Taxys/Kronos, d/dt and the MLD-verifier can be found in the other contributions to the session [6], [20], [14], [5], [8].

A. UPPAAL

The tool UPPAAL is an environment to model, simulate, and verify systems represented as networks of timed automata (TA) [9]. These are defined according to [2] with an extension by data variables and synchronization mechanisms to model the communication of separate automata. UPPAAL can analyze simple liveness properties as well as reachability properties. By using *clock difference diagrams* [25], the timed automata are internally represented in a compact format (similar to that known as binary decision diagrams for discrete systems) that allows a relatively efficient analysis. Further information about UPPAAL can be found online at `www.docs.uu.se/docs/rtmv/uppaal/`.

In order to investigate the reactor system with UPPAAL, the relevant part of the process behavior has to be translated manually by the user into a set of concurrent TA. The model can then be entered using the UPPAAL graphical interface, illustrated in Fig. 3. In this figure, the controller corresponds to the operation procedure in Fig. 2. The variable t is a globally defined clock that measures the reaction time (limited to 3000 sec). The synchronization labels establish the communication with the automaton representing the reactor, and the labels are marked by '!' or '?' to denote the sender, or the receiver respectively. The upper part of the figure shows a TA that approximates the reactor behavior. The state 'Fill' denotes the filling procedure, the states 'S1' to 'S11' represent distinct regions for the temperature T_R and the concentration c_A, i. e. T_R decreases in direction from 'S1' to 'S9' and c_A increases in direction from 'S5' to 'S8'. The crucial and difficult modelling step is to determine the possible state transitions, including the guards and the state invariants. They can be obtained by simulation of the hybrid model given in Table I, by measurements from the real plant, or by automated approximation techniques (see Sec. III-E). Of course, the first two alternatives do not produce a conservative model in general.

The analysis of the composition of both automata using UPPAAL'S verification algorithm can then reveal if all behaviors terminate in one of the desired states 'S2', 'S5', or 'S9'. For the shown models, we get the result that the terminal state 'S10' is reachable, i. e. it is not ensured that the concentration c_A eventually falls below the desired threshold before the reaction is terminated.

B. HYTECH

HYTECH verifies specifications given as temporal logic expressions for a class of hybrid automata using symbolic model checking in the continuous state space. If the verification fails, it generates a diagnostic error trace. HYTECH can only model flows of the form $A\dot{x} \leq B$, characterizing so-called *linear hybrid automata* (LHA). HYTECH handles parallel composition of processes and parametric analysis. For theory details, see [3]. For information about the tool, see [21]. Information is online at `http://www-cad.eecs.berkeley.edu/~tah/HyTech/`. HYPERTECH, a new version of HYTECH, focuses on performing the reachability operations using interval approximation. For further details, see [19].

The developers of HYTECH suggest the following three approaches to verify systems of higher complexity than that of LHA [18].

1. **Clock transition models.** In this approach, continuous state variables are replaced by *clock variables* (pure

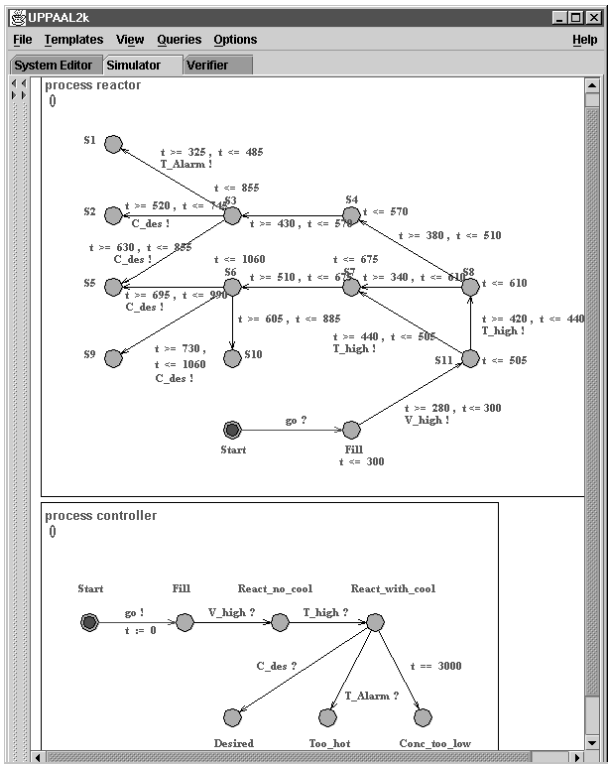

Fig. 3. The reactor modeled in UPPAAL.

integrators with different rates) with constraints identifying the regions for which the given rates are valid. To make this model effective, the state space is partitioned into a number of regions and the original differential equations are represented by the rates of the clock variables. For the batch reactor example, the variables T_R, c and t_R would be replaced by clock variables. The switches between control modes are now made with respect to the clock variables.

2. **Rate translation.** This approach retains the original state variables, but approximates their continuous behavior with piecewise-constant bounds on the first derivatives. For the batch reactor example one possible partition of the state space is illustrated in figure 4.

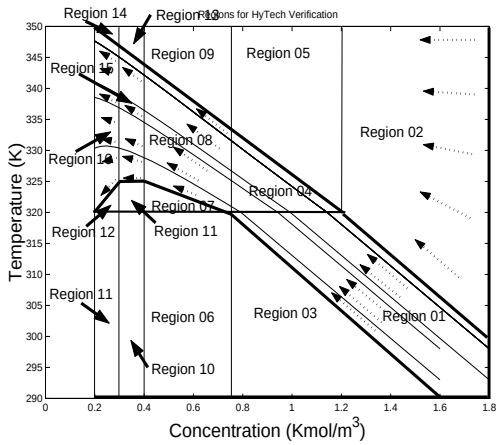

Fig. 4. Partition of the state space of the batch reactor example.

3. **Linear phase-portrait approximation.** In this approach, the derivatives of the state variables can be constrained in linear combinations, e.g., $\dot{x}_1 \leq \dot{x}_2 + c$, giving a better approximation to the original state equations. This method is described in detail in [18].

C. d/dt

The tool *d/dt* performs verification and control synthesis for hybrid systems. It performs a reachability analysis for hybrid systems with linear continuous dynamics. Reachable sets for linear dynamic systems are represented with collections of orthogonal (rectangular) polyhedra computed by performing so-called *face lifting* to create efficient over-approximations [16]. Collections of orthogonal polyhedra are represented with a very efficient data structure supported by a library of routines for performing the set operations required for model checking [11]. *d/dt* allows models with uncertainty in the input in the dynamics equation, i.e., models of the form $\dot{x} = Ax + Bu, u \in U$.

To apply the current version of *d/dt* to the batch reactor example, the user must linearize the system dynamics around the operating point of interest. Figure 5 illustrates the concept of face lifting for the initial condition set in the batch reactor example. To approximate the states reachable from this set (the shadedset in the figure) in some time step ΔT, each face is moved by an amount that bounds all possible trajectories starting on the face. Since the vector field points into the original set at all the points on two of the faces in the figure, the approximation moves the boundary out for only two of the faces. The approximation is then repeated beginning from the new set. Applying this process to partitions of the initial state set makes it possible to obtain arbitrarily close approximations to the actual reachable set. We were not able to obtain the tool *d/dt* to include final verification results in this paper.

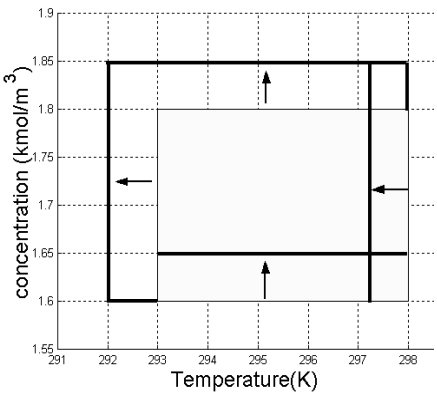

Fig. 5. Using face lifting concept to compute the reachable set.

D. CheckMate

CHECKMATE is a MATLAB-based tool for simulation and verification of hybrid dynamic systems with arbitrary nonlinear continuous dynamics. CHECKMATE allows any Simulink/StateFlow blocks for simulation. For verification it allows logical operators (AND, OR, XOR, etc.), MUX/DEMUX and the following custom Simulink blocks:

- **Switched Continuous System Block (SCSB)**. A SCSB models continuous dynamics (linear or nonlinear);

- **Polyhedral Threshold Block (PTHB)**. The output is 1 if the input x lies within the polyhedron $C\mathbf{x} \leq d$ and 0 otherwise;
- **Finite State Machine Block**. A FSMB is a MATLAB/Stateflow chart where all inputs are logical functions of the outputs of PTHBs or FSMBs, only one output (discrete data type) is allowed and it has no hierarchy.

CHECKMATE performs verification by computing a finite-state approximation using general polyhedral over-approximations to the sets of reachable states for the continuous dynamics called *flowpipes*. The system specification is expressed as a restricted CTL formulas using only universal quantifiers. If the verification is successful, the user is informed and the program terminates. If the verification result is inconclusive, the user can ask CheckMate to refine the current approximation and attempt the verification again. CHECKMATE automatically searches for the states of the approximate transition system that led to failure, splits them, recomputes the reachable states for the new states and, again, evaluates the logic expression representing the specification. Properties of individual trajectories of the system can also be verified. For the theoretical background of CHECKMATE, see [15]. Details about the tool are given in [28] and online information is available at `www.ece.cmu.edu/~ebk/checkmate`. Currently, CHECKMATE is being modified to allow an efficient way to perform verification for sampled-data hybrid systems [27].

Figure 7 shows a CHECKMATE model for the batch reactor example. The SCSB labeled *dynamics* models the switched continuous nonlinear dynamics for the system in Table I. The operation of the controller is represented by the FSMB *status*. This block receives events representing the crossings of the temperature threshold to toggle the heating. Also, it detects if the system overheats. The time for the reaction is exceeded or if a low concentration is measured. The PTHBs *low_concentration, timeout, overheat* and *cross_up* generate discrete outputs 0 or 1 depending on whether or not one of these thresholds is crossed. Figure 6 shows the trajectories starting from the vertices of the set of initial states as simulated using the CHECKMATE model.

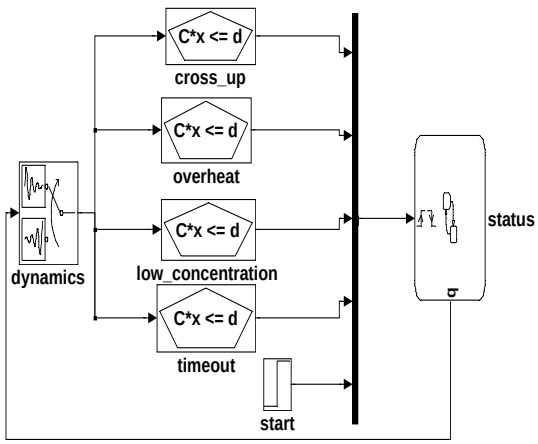

Fig. 6. Batch reactor modeled in CHECKMATE

For the batch reactor example, verification was performed to see if the system will achieve a concentration $c \leq 0.2$ within a time horizon of one hour. For the initial controller design, CheckMate returned a negative result even after two refinements of the finite-state approximation. To analyze the situation, we plotted the 3D flowpipe shown in figure 8. We can see that the flowpipe hits the hyperplane corresponding to $t = 60$ min before hitting the concentration limit.

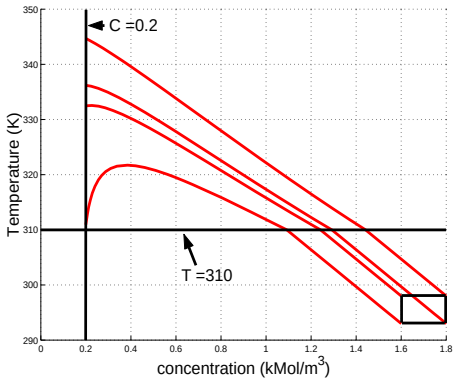

Fig. 7. CheckMate simulation of the trajectories starting from the vertices of the set of initial states (concentration x temperature).

Since the approximation is conservative, the result is inconclusive. We modified the controller threshold so that it would switch the cooler on at $T = 320$K. For this case, CheckMate verified the system will behave correctly on the first pass. The result is illustrated by the flowpipe in Fig. 9. This verification procedure took approximately 40 min. on a 600 MHz Pentium III PC.

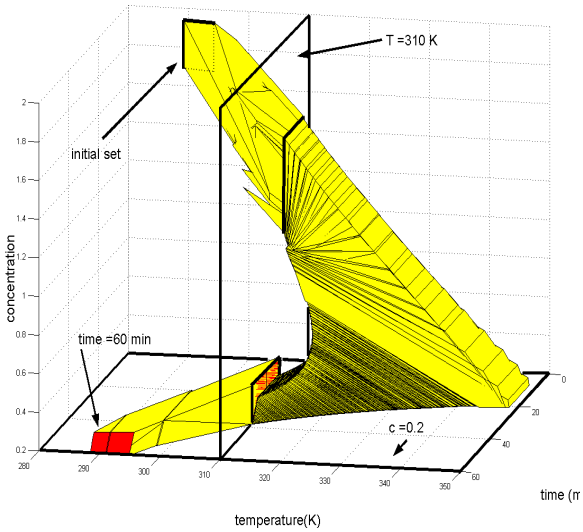

Fig. 8. Flowpipe for verification with threshold T=310K

E. VERDICT

The tool VERDICT provides an environment for modular modelling and verification of timed and hybrid systems [30]. The modelling step is based on hybrid condition/event systems (HCES). In a block-diagram editor the structure of the system is built in a modular manner (usually a module for the controller and set of modules for the other components), and the communication is established by condition and event signals. The behavior of each module is described by a discrete, timed or hybrid transition system, where the latter can contain nonlinear ODEs with autonomous or externally triggered switching of the dynamics. While the hybrid modules are textually specified by a description language for HCES called CELESTE, the modules with discrete and timed behavior can be build by a state-diagram editor also.

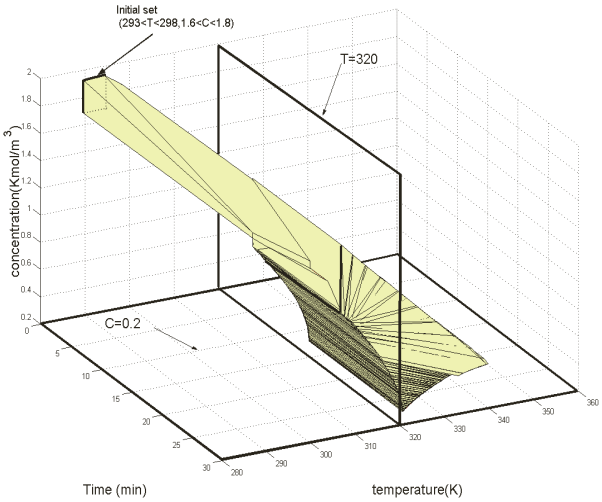

Fig. 9. Flowpipe for verification with threshold T=320K

The controller model can alternatively be imported as code given in the common PLC languages *Instruction List* and *Sequential Function Charts*, and VERDICT comprises transformation algorithms to convert the code into modules given as HCES [23].

The concept of VERDICT is not to provide a special analysis algorithm for HCES but to translate the HCES model into the input languages of a choice of different model checkers for discrete and timed automata. So far VERDICT includes transformation routines into the input models of HYTECH, KRONOS, the tool SMV [13], and UPPAAL (the latter is currently implemented). The conversion routine includes an approximation step for the hybrid components of the HCES in order to get models to which these tools are applicable. Two different approximation algorithms are currently available, which both rely upon an orthogonal partitioning of the continuous state space of the hybrid modules [29]. One algorithm determines the transitions between different partition elements based on computation of representative trajectories. The alternative one uses interval arithmetic, or constrained optimization, respectively, to approximate the original dynamics by differential inclusions, which then make it possible to determine the transitions and time constraints (i.e. guards and invariants). The result is a TA model (specified in the languages of HYTECH, KRONOS or UPPAAL), a rectangular automaton (in *Hytech* language), or an additional transformation generates a discrete automaton specified in SMV language. Online information about *Verdict* is available at `astwww.chemietechnik.uni-dortmund.de/ verdict/`.

The *Verdict* model of the reactor system is shown in Fig. 10. The right upper window contains the block-diagram containing one module each for reactor and controller. The measurements reported from the reactor to the controller are modeled as event signals, and the controller output (settings of the valves v_A and v_C) is given as condition signals. The controller model itself is specified as state-transition system (right lower window), and the hybrid behavior according to Tab. I is formulated in CELESTE (left window).

For the given example, the analysis was carried out with HYTECH. Depending on the chosen partitioning and approximation method, the transformation of HCES lead to TA models of approx. 300 to 900 states and 4500 to 18000 transitions, and required between 7 and 54 min for computation (Pentium PC, 200 MHz). The analysis correctly shows

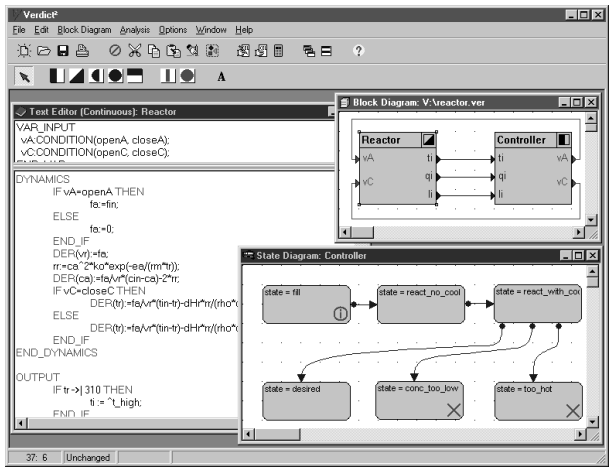

Fig. 10. Reactor system in Verdict

that the critical state z_6 (respectively z_5 with $T_{Alarm} = 345\text{K}$) is reachable for a temperature threshold $T_{High} = 310\text{K}$ (respectively $T_{High} = 330\text{K}$). It shall be noted that for the largest TA models the analysis could not be completed with HyTech due to a huge memory consumption of more that 120MB. For the completed runs, the analysis took less than 10 minutes in all cases.

F. Other Tools

In addition to the tools described above, the following three hybrid system model checkers have attracted some attention recently.

- veriSHIFT: In contrast to the tools described above, which represent reachable regions by polyhedra, this tool uses ellipsoids to conservatively approximate the reachable sets for hybrid automata with linear differential inclusions [10]. The advantages of the ellipsoidal approximation are a reduced memory requirement and a polynomial complexity with respect to time (while the worst-case complexity of polyhedral operations is exponential). Online information is available at `robotics.eecs.berkeley.edu/ olegb/VeriSHIFT/`.
- Verifier for MLD systems: This tool uses mathematical programming to perform reachability analysis for Mixed Logical Dynamical (MLD) systems, which combine discrete-time linear dynamics with logical propositions [7]. In contrast to the other tools discussed in this paper, MLDs model *discrete-time* hybrid systems. The algorithm determines the reachable set by solving a mixed-integer optimization problems. Recent developments in this approach will be presented in this invited session [8]. Online information is available at `www.aut.ethz.ch/ hybrid/verification.msql`.
- Kronos: This is a well-established model checker for TA. Based on a symbolic representation of the clock space by linear constraints for the clock values, forward or backward analysis reveals whether properties specified in the temporal logic TCTL are satisfied [17]. Recently, Kronos has been extended by a compiler for the synchronous language *Esterel*, resulting in a tool named Taxys. Information about *Kronos* and related tools can be found online at `www-verimag.imag.fr/TEMPORISE/tools.shtml.en`.

TABLE II

TOOL FEATURES.

Features	UPPAAL	HYTECH	*d/dt*	CHECKMATE	VERDICT
Graphical user interface	✓	-	-	✓	✓
Simulation	✓	-	-	✓	-
Reachability specifications	✓	✓	✓	✓	✓
temporal logic specs.	✓	✓	-	✓	(✓)
Generates counter examples	✓	✓	-	-	(✓)
Type of model	TA	LHA	LDHA	NHA	NHA

IV. THE STATUS OF MODEL CHECKING TOOLS FOR HYBRID SYSTEMS

On the one hand, the example considered in Sec. III as well as experiences gained from other applications show that the problem of analyzing discrete controllers for hybrid systems can be solved by available tools. On the other hand, the example reveals also that the modelling step demands some insight into the specific problem and the chosen modelling paradigm, and that it requires appropriate simplifications of the continuous dynamics for most of the tools.

The first step in verifying a given controlled system is to choose a suitable accuracy of the model. Clearly, the choice of a TA model is sufficient if the behavior of the controlled process can be accurately approximated by timed transitions. But for systems such as the reactor example, the determination of guards and invariants is only possible by simulation of a more complex model or by experiments – certainly both methods can in general not ensure the completeness of the TA model. Hence, the use of verification techniques for hybrid systems with linear/nonlinear continuous dynamics is required. If linearizations of an originally nonlinear model are involved, the completeness issue has to be considered also.

Having determined a suitable type of model, the best tool for analysis is usually that which is designed for that specific class of models. For example, if timed automata are used, there is no need to choose a tool for nonlinear hybrid systems, since UPPAAL and KRONOS provide special data structures that allow an efficient analysis of TA.

Apart from the type of model, criteria such as modularity (and communication between modules), the type of state space partitioning, and the user-friendliness of the modelling environment are important for choosing a tool. With respect to the latter, a GUI that makes it possible to build the model graphically certainly enhances the acceptance by industrial users. Another very useful feature of a verification environment is a simulation facility. The usual course of analysis is to explore the system behavior by computing some single trajectories first. Based on the results, parameters as the partitioning, the time steps, tolerances etc. are chosen such that an efficient verification is possible. Table II summarizes some properties and features of the tools that were described in Sec. III-A to III-E. The type of models are denoted by: TA - timed automata, LHA - linear hybrid automata, LDHA - hybrid automata with linear ODEs, NHA - hybrid automata with nonlinear ODEs. (For VERDICT, brackets are used if the feature depends on the chosen model checker).

With respect to the shortcomings of available tools, the following two points seem to be most important. First,

the complexity of the computation restricts the applicability to fairly small systems. The verification of systems with around five continuous variables with nonlinear dynamics usually require some hours of computation and also the memory consumption is enormous for some approaches. This problem immediately points to a second important shortcoming, namely, the lack of modularity in verification. The analysis of large and distributed systems may become possible if they can be decomposed into small modules for which model checking is possible. Techniques like deduction or theorem proving can then be used to derive global properties from the model checking results. A tool which follows this idea for reactive systems is MOCHA [4] – however, the extension of this approach to hybrid systems is an open problem.

Another shortcoming of current tools is the interpretation of the analysis results. If the verification fails (i. e. the required property is not fulfilled), most of the tools report a trajectory/trace that violates the specification. In most cases, it is not obvious how to redesign the controller such the requirements are met. Even more severe, there is no hint whether the violation results from an over-approximation of the real behaviour or indeed from a wrong controller design. Usually, a refinement of the partitioning or the computation tolerances must reveal the actual cause of the violation.

V. Making Tools Useful for Industry

The assessment in the previous section identifies challenges that are well known to the research community. There are additional issues, however, that need to be addressed if tools for hybrid system verification are going to move out of research laboratories and become widely used in industry. Here we identify three key issues that have arisen often in our discussions with industrial colleagues.

- *Connecting with Existing Models.* Tools for computer-aided control system design are now used extensively. This means that computer models of plants and controllers are readily available. It is unlikely that tools that require the construction of completely new models will gain wide acceptance, since the model building process is time consuming and can introduce errors that could make the verification results irrelevant to the actual system being developed.
- *Tools for Exploring Models and Results.* Experience to date with tools for hybrid system verification shows that useful results can be obtained for nontrivial systems only when the user is involved in directing the verification process. Because of the difficulties that arise in computing and representing reachable sets for the continuous dynamics, tools that help the user gain insight into the system behavior are an absolute necessity.
- *Tools for Building Verification Specifications and Interpreting Results.* One of the biggest barriers to the use of formal verification methods is the difficulty of translating requirement specifications into the formal specifications to be verified. Here industry needs tools that facilitate this translation, and also make it possible to interpret the results of the verification procedure in terms of the original application.

Although these issues are motivated by practical considerations, we believe each of them leads to significant research problems that need to be addressed by the developers of the tools for hybrid system verification. For example, there are interesting and challenging research problems in developing methods to abstract formal models appropriate for verification from existing models of hybrid control systems. Our hope is that the assessment of the current state of the tools in this paper, and the other papers in this invited session, will stimulate discussion of these issues and provide impetus for new research directions that will eventually make hybrid system verification a standard tool for control system design.

Acknowledgements

The authors gratefully acknowledge the financial support by the National Science Foundation (NSF) and the German Academic Exchange Council (DAAD). B. Izaias Silva is supported by a grant from CNPq/Brazil - 20.0079/92. The research of B.H. Krogh was also supported in part by Ford Motor Co., General Electric Company, and DARPA.

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