Simplified Models for Use in the Analysis of Future Power Systems

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Development of a Simplified Motor-Drive System Model

- Model is based on [1]:
  - Induction motor equivalent circuits
  - Input-output relationships of the voltage source inverter
  - Power balance at the input and output terminals of the voltage source inverter
The Simplified Model Can be Used in other Topologies

Balanced Three-Phase AC Source (Utility) → Three-Phase Diode Rectifier → Voltage Source Inverter → Three-Phase Induction Motor → Load
- Develop an analytical method to analyze this system.
- Avoid using simulation packages such as PSPICE and Simulink.

- Develop a simplified model of the system.
- Extend the simplified model to a DC power system containing multiple motor-drive loads.
Three-Phase Voltage Source Inverter

(S1, S4), (S3, S6), (S5, S2) are switched as pairs.
Inverter Output Voltage Waveform

Six-Step with 180° Conduction

Space Vector PWM

Two-Level Sinusoidal PWM

Six-Step with 120° Conduction
Three-Phase Inverter Block Model

- Develop a Fourier series for:
  - Line-to-neutral voltage
The Fourier series can be expressed in the following form for two-level sine PWM and space vector PWM [2]:

\[ v_{as}(t) = \sum_{n=1}^{\infty} C_n \cos\left(\frac{2\pi n}{T} t + \delta_n\right) \]

\[ C_n = \sqrt{a_n^2 + b_n^2} \quad \delta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right) \]
The Fourier series of the six-step inverter phase $a$ line-to-neutral voltage waveform with $120^\circ$ conduction can be expressed as [3, 4]:

$$v_\phi(t) = \frac{\sqrt{3}}{\pi} V_i \left[ \cos(\omega t + 30^\circ) - \frac{1}{5} \cos(5\omega t + 30^\circ) - \frac{1}{7} \cos(7\omega t + 30^\circ) + \frac{1}{11} \cos(11\omega t + 30^\circ) + \ldots \right]$$

The Fourier series of the six-step inverter phase $a$ line-to-neutral voltage waveform with $180^\circ$ conduction can be expressed as [3, 4]:

$$v_\phi(t) = \frac{2}{\pi} V_i \left[ \cos \omega t + \frac{1}{5} \cos 5\omega t - \frac{1}{7} \cos 7\omega t - \frac{1}{11} \cos 11\omega t + \ldots \right]$$
Induction Motor Circuit Models
Inverter Power Balance

- If a value of $V_i$ is assumed at the input terminals of the inverter, a corresponding $I_i$ can be found using a power balance.

- Inverter power balance:

$$V_i I_i = \sum_{k=1}^{\infty} \frac{3}{2} V_k I_k \cos \theta_k$$
V-I Characteristic Curves

- Varying $V_i$ over a range of values produces a V-I load characteristic curve having the following form:

$$V(I_i) = aI_i^2 + bI_i + c$$

- The *polyfit* command in MATLAB can be used to curve fit the generated V-I data.
V-I Characteristic Curve Developed for a 50 Hp Motor and Inverter

- 50 Hp Motor-Drive System was analyzed using MATLAB
- The source voltage was varied over a range of 401V-500V with all other parameters remaining unchanged.
- Induction motor parameters: $f = 60$ Hz, $P = 4$, $R_1 = 0.087 \Omega$, $R_2 = 0.228 \Omega$, $X_1 = 0.302 \Omega$, $X_2 = 0.302 \Omega$, and $X_m = 13.08 \Omega$
V-I Characteristic Curves Developed for a 50 Hp Motor and Inverter

Two-Level Sinusoidal PWM Inverter

\[ m_a = 1.4 \quad m_f = 15 \quad T_L = 100 \text{ N-m} \]

Space Vector PWM Inverter

\[ M = 0.7 \quad m_f = 15 \quad T_L = 80 \text{ N-m} \]
Simplified Motor-Drive System Model

The inverter drive system can now be replaced by a current-controlled voltage source [1].
Multiple Motor-Drive Systems

- The simplified model can be extended to a system containing more than one motor-drive.
- Use the V-I characteristic curve of each motor-drive load.
- Incorporate the simplified model into an iterative procedure based on the Newton-Raphson method.

The network can be represented as [5]:

$$\vec{I} = G\vec{\tilde{V}}$$
The Bus Voltages

Since the system studied contains motor-drive loads, each bus element of \( \tilde{V} \) will have the following form:

\[
\tilde{V} = \begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
\vdots \\
V_n
\end{bmatrix} = \begin{bmatrix}
V_1 \\
a_2 I_2^2 + b_2 I_2 + c_2 \\
a_3 I_3^2 + b_3 I_3 + c_3 \\
\vdots \\
a_n I_n^2 + b_n I_n + c_n
\end{bmatrix}
\]

(where, \( V_1 \) is the swing bus, and \( n \) is the number of buses)
A Newton-Raphson Based Method

- The network is a system of simultaneous nonlinear algebraic equations.
- An iterative procedure based on the Newton-Raphson method can be used to solve for the load currents [1].
- After the currents have converged, the bus voltages can be found by substitution into the voltage vector.
10-Bus System with Two-Level Sinusoidal PWM Inverter Drives

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Load Torque (N-m)</th>
<th>Line Number</th>
<th>Section</th>
<th>Line Resistance (mΩ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1 - 2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>2 - 3</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>65</td>
<td>2 - 4</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>2 - 5</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>2 - 6</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>2 - 7</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>2 - 8</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>2 - 9</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>80</td>
<td>2 - 10</td>
<td>90</td>
<td></td>
</tr>
</tbody>
</table>

Diagram: 10-Bus System with Two-Level Sinusoidal PWM Inverter Drives
The swing bus voltage was chosen to be 550V.
The inverter parameters used for all of the inverters in the system were: \( f_i = 60 \text{ Hz}, \ m_a = 1.4, \) and \( m_f = 15. \)
The induction motor equations, the two-level PWM inverter relationships, and the power flow equations were all coded in MATLAB.
The voltage at each load bus of the system was varied over the range of 496V-595V, with all other parameters in the system remaining unchanged.
Induction motor parameters: 50Hp, \( f = 60 \text{ Hz}, \ P = 4, \ R_1 = 0.087\Omega, \ R_2 = 0.228\Omega, \ X_1 = 0.302 \Omega, \ X_2 = 0.302 \Omega, \) and \( X_m = 13.08 \Omega \)
## Power Flow Results for the Two-Level Sine PWM Inverter

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Current from PSPICE (A)</th>
<th>Converged Current from PSPICE (MATLAB) (A)</th>
<th>ΔI Current Error (% of PSPICE)</th>
<th>Voltage from PSPICE (V)</th>
<th>Converged Voltage from PSPICE (MATLAB) (V)</th>
<th>ΔV Voltage Error (% of PSPICE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>24.6824</td>
<td>24.6512</td>
<td>0.0312</td>
<td>547.7546</td>
<td>547.7583</td>
<td>0.00370</td>
</tr>
<tr>
<td>4</td>
<td>22.9430</td>
<td>22.9104</td>
<td>0.0326</td>
<td>547.5599</td>
<td>547.564</td>
<td>0.00410</td>
</tr>
<tr>
<td>5</td>
<td>35.2968</td>
<td>35.253</td>
<td>0.0438</td>
<td>546.8364</td>
<td>546.8412</td>
<td>0.00480</td>
</tr>
<tr>
<td>6</td>
<td>21.2136</td>
<td>21.1773</td>
<td>0.0363</td>
<td>547.1876</td>
<td>547.1924</td>
<td>0.00480</td>
</tr>
<tr>
<td>7</td>
<td>17.7230</td>
<td>17.6864</td>
<td>0.0366</td>
<td>547.1849</td>
<td>547.1901</td>
<td>0.00520</td>
</tr>
<tr>
<td>8</td>
<td>14.2392</td>
<td>14.2022</td>
<td>0.0370</td>
<td>547.2515</td>
<td>547.2571</td>
<td>0.00560</td>
</tr>
<tr>
<td>9</td>
<td>10.7636</td>
<td>10.726</td>
<td>0.0376</td>
<td>547.3871</td>
<td>547.3932</td>
<td>0.00610</td>
</tr>
<tr>
<td>10</td>
<td>28.3157</td>
<td>28.2638</td>
<td>0.0519</td>
<td>545.6998</td>
<td>545.7076</td>
<td>0.00780</td>
</tr>
</tbody>
</table>
Advantages of the Analytical Method Presented

- Faster than simulation packages such as PSPICE
- Produces results that are comparable to other simulation packages
- The analytical method presented can be used regardless of the switching scheme employed in the inverter (two-level sinusoidal PWM, space vector PWM, etc.)
Advantages of the Analytical Method Presented

- The simplified model developed can be utilized in the analysis of a multiple-bus power system containing multiple motor-drive loads.

- The simplified model can be used in DC or AC system analysis.
References


Questions???