L1 minimization method for estimating sparsely represented errors in electric energy system:
Simulation based investigation of IEEE 300 bus test case with a linearized model

Euiseok Hwang
Advisors: Rohit Negi. Vijayakumar Bhagavatula

Mar. 10, 2010

*Acknowledgements: Zhijian Liu, Marija Ilić
Outline

- **Motivation**

- *L1 minimization based state estimation for a linear measurement model*

- **Sparsely represented error simulations**
  - Sparse errors in node voltage angle measurements

- **Summary**
Motivation

- State estimation of electric energy systems
  - Estimate electrical states of the network from noisy measurements based on given network topology and parameters

- State estimation for sparsely represented measurement error
  - i.e., arbitrarily large values for a few elements whose sensor may have failed, while zero errors for others

- L0-min. method may be efficient, but L0-min. is an NP-hard problem

- L1-min., polynomial-time, is equivalent to L0-min. when the measurement matrix satisfies a restricted isometric prop. (RIP)

- Investigation of L1-min. methods using numerical simulation
State Estimation based on a Linear Measurement Model

- **General state estimation problem**
  - **Nonlinear measurement model**, \( y_j = h_j(x) + e_j, \ e_j : \text{mean 0, variance } \sigma_j^2 \)

- **Weighted least square (WLS) state estimation**

  \[
  \min_x J(x) = \sum_{j=1}^{m} \left( y_j - h_j(x) \right)^2 / \sigma_j^2
  \]
  \[
  \text{s.t. } g_i(x) = 0; \ i = 1, \ldots, n_g
  \]
  \[
  c_i(x) \leq 0; \ i = 1, \ldots, n_c
  \]

- **State estimation for a linear measurement model**
  - **(Linearized Model between node active powers, P, & voltage angles, } \delta \)**

  \[
  P_i = -\sum_{j=1, j \neq i}^{n} \left[ Y_{yj} \right] (\delta_j - \delta_i) \quad \rightarrow P = W\delta
  \]

  \[
  y = Ax + e, \quad x = \begin{bmatrix} P \\ \delta \end{bmatrix}
  \]

  \[
  \min_x (y - Ax)^T \ C^{-1} (y - Ax), \quad C = \text{diag} \left( \sigma_j^2 \right)
  \]

  \[
  \text{s.t. } \begin{bmatrix} 1 & 0 \\ 0 & -W \end{bmatrix} x = 0
  \]


Estimation of Sparsely Represented Measurement Errors

- **Problem statement**
  - **Linear measurements**
    \[ y = Ax + e, \quad e \text{ is } 2n \times 1 \text{ and sparse, near zero values except } k \text{ elements, } k << 2n \]
    \[ A \text{ is a } m \times 2n \text{ matrix, } m < 2n \]
  - **Estimate** \( e \) from \( y \)

- **L1 minimization method for estimating a sparse error**
  - **Sparse errors in node voltage angle measurements and noiseless node active power measurements**
    \[
    y = \begin{bmatrix} P_s \\ \delta \end{bmatrix} = \begin{bmatrix} I_s & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} P \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta \delta \end{bmatrix}, \quad A = \begin{bmatrix} I_s & 0 \\ 0 & I \end{bmatrix}, e = \begin{bmatrix} 0 \\ \Delta \delta \end{bmatrix}
    \]
    \[ I_s \text{ is a } m \times n \text{ matrix, } m < n, \text{ where } P_s \text{ is a subset of } P, P_s = I_s \cdot P \]
    \[ \Delta \delta \text{ is } n \times 1 \text{ and sparse} \]

*Least square method*:
\[
\min_{\Delta \delta} \left\| \Delta \delta \right\|_2 \quad \text{s.t. } W_s \Delta \delta = b
\]

\[ b = W_s \delta - P_s \]

\[ W_s = I_s \cdot W \]
Estimation of Sparsely Represented Measurement Errors

- **Estimating sparsely represented error, $\Delta \delta$, from**

\[
\min_{\Delta \delta} \| \Delta \delta \|_1 \quad \text{s.t.} \quad W_s \Delta \delta = b
\]

\[\Delta \delta: n \times 1, \quad \| \Delta \delta \|_0 << n\]

\[b: m \times 1, \quad m < n\]

\[W_s: m \times n, \quad m < n\]
Simulation Results

- **Sparse errors in bus voltage angle meas. (IEEE 300 test case)**
  - 290 power measurements (noiseless)
  - 32 voltage angle errors, $\Delta \delta_j \sim N (0, 0.25^2 (= \text{var}(\delta)))$

**Histogram of voltage angle estimation error**

(290 power meas., 32 voltage angle errors, …)

**Injected and estimated voltage angle errors**
Simulation Results

- **Sparse errors in bus voltage angle meas. (IEEE 300 test case)**
  - 4, 8, 16 and 32 errors, degrees of voltage angle error columns of $W \geq 4$

Estimation failure rates as a function of the number of voltage angle errors

- 290 P meas. (out of 299), deg(W(:,Δδ~0)) ≥ 4
- Random phase error, $Δδ \sim N(0,0.25^2)$
- Fail if $\max(Δδ^-Δδ) > 0.05$, $σ_δ = 0.25$

- $\min \| Δδ \|_2$
- $\min \| Δδ \|_1$
Summary

- **Evaluation of IEEE 300 test case with a linearized model**
  - Sparse voltage angle errors can be better estimated by L1 minimization method than LS method (290 noiseless power meas. and 4~32 voltage angle errors)

- **Further investigations**
  - Sparse errors in node voltage angle measurements and Gaussian noise in node active power measurements
  - Subset of node voltage angle measurements with sparsely represented errors