Demystifying Electric Energy Systems Dynamics, Modeling and Control

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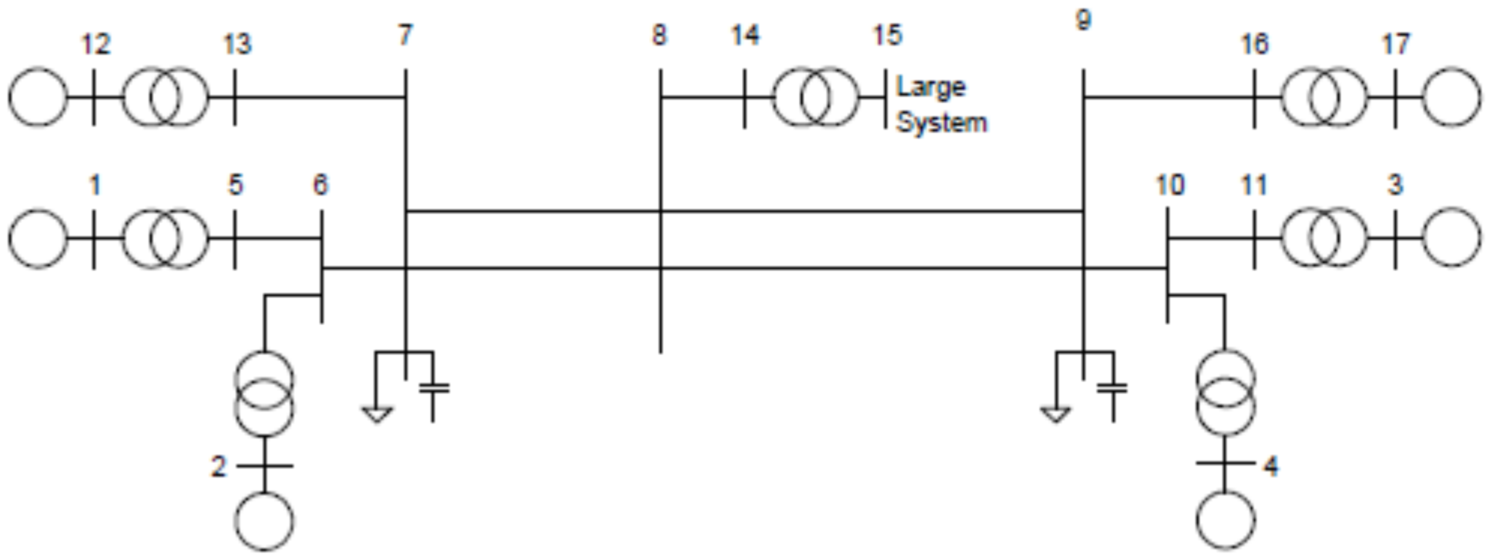
Nature of long-term Power System Oscillations

- During system-wide electro-mechanical oscillations, the swing energy flows through the power lines back and forth between the rotating masses of the different generators with a frequency of typically 0.1-1 Hz.
- In the case of insufficient damping, small disturbances may trigger growing oscillations, that lead to loss of synchronization between groups of generators and possibly blackouts.
- Two types of slow oscillations:
  - Local
  - **Interarea**

Real world example of inter-area oscillation form Portugal distribution network



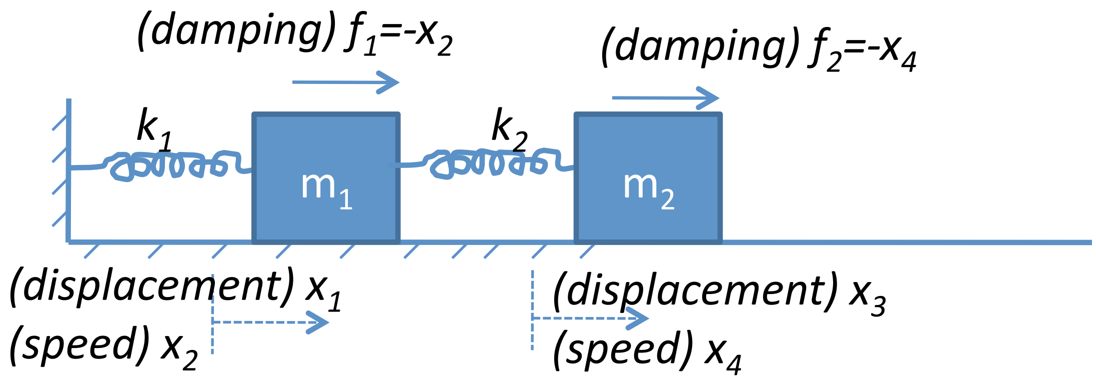
- Each area has synchronous machines and wind farms
- There is an electromechanical mode of oscillatory between two areas
- To resolve the problem Power System stabilizer is implemented
- Understanding the nature of inter-area oscillation in power systems is not an easy task

Outline

- Understanding interactions (inter-area dynamics) in power systems by drawing analogies with mechanical systems and electrical circuits
  - Mechanical systems
    - Two mass spring system (Interconnected system)
  - Electrical circuits
    - Two RLC circuits
  - Governor control of synchronous generators for frequency control
  - Excitation control of electromagnetic dynamics for voltage/reactive power support [Working Paper]

Mechanical Systems
Two-Mass-Spring System

Double Mass Spring System



$m_1 = m_2 = m$
$k_1 = k_2 = k$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{m_1}(f_1 - k_1 x_1 + k_2(x_3 - x_1))$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{1}{m_2}(f_2 - k_2(x_3 - x_1))$$

Possible System Decomposition

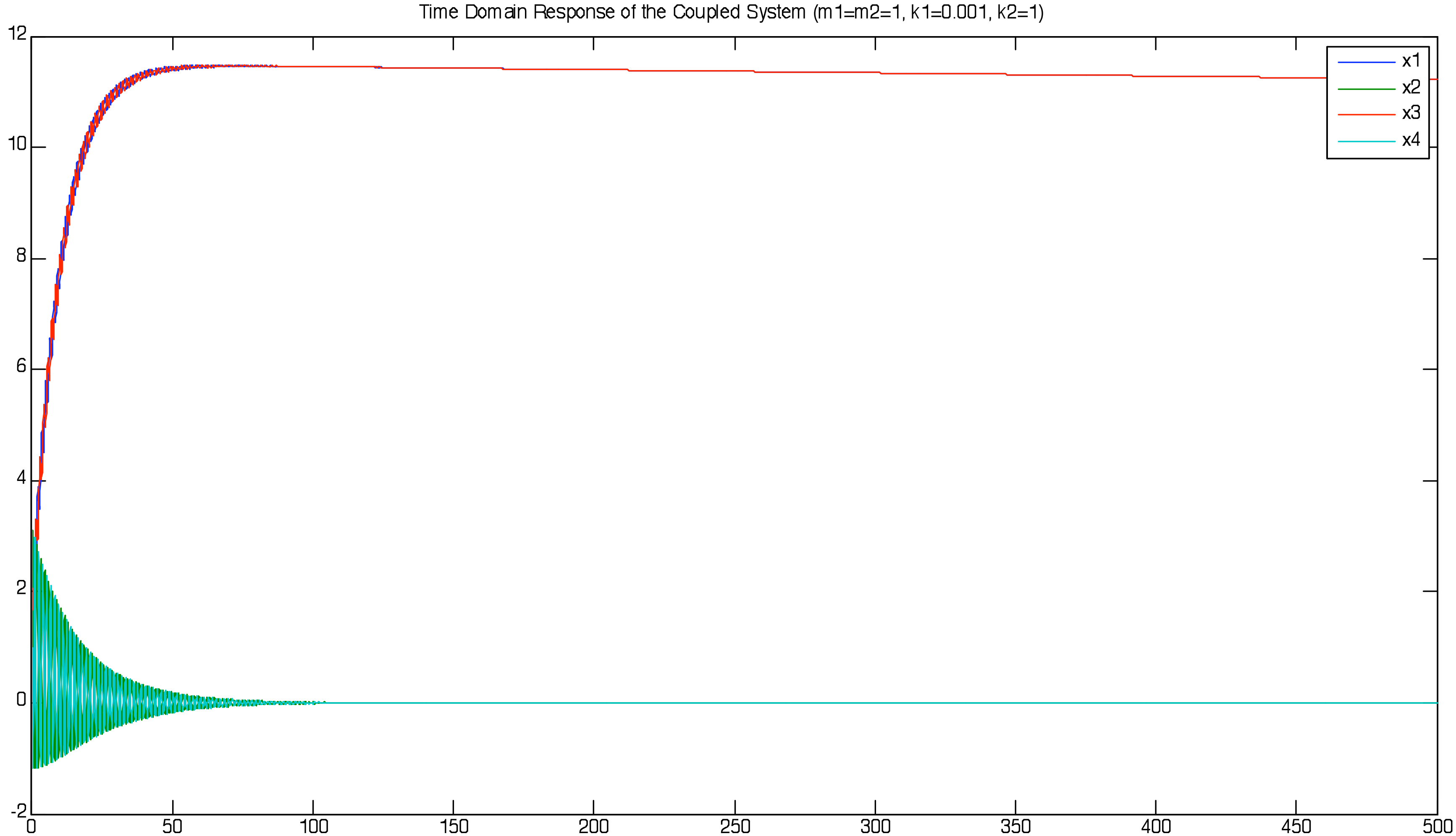
The whole system can be represented as:

$$S_1 : \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_1 + k_2}{m_1} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{k_2}{m_1} & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + f_1$$

$$S_2 : \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_2}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{k_2}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + f_2$$

$$f_1 = -\frac{D_1}{m_1} x_2 \qquad f_2 = -\frac{D_2}{m_2} x_4$$

Time-domain Response of the Coupled System (Smaller friction $f = -0.1x_2$ $m_1 = m_2 = 1$, $k_1 = 0.001$, $k_2 = 1$)



Non-standard Singularly Perturbed Form Intepretation

$k_1 = \varepsilon = 0.001$

Couples system S takes on the form

$$\varepsilon \dot{X} = A(\varepsilon) X$$

Rank(A(0)) = 3

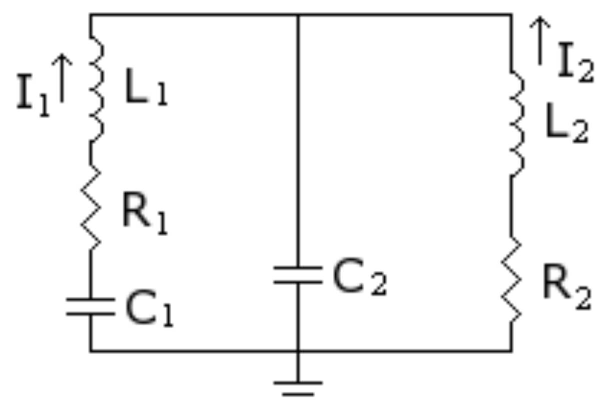
That is, in this case $\lambda\left(A(0)\right) = \{-0.05 + 4.44i,\ -0.05 - 4.44i,\ 0,\ -0.1\}$

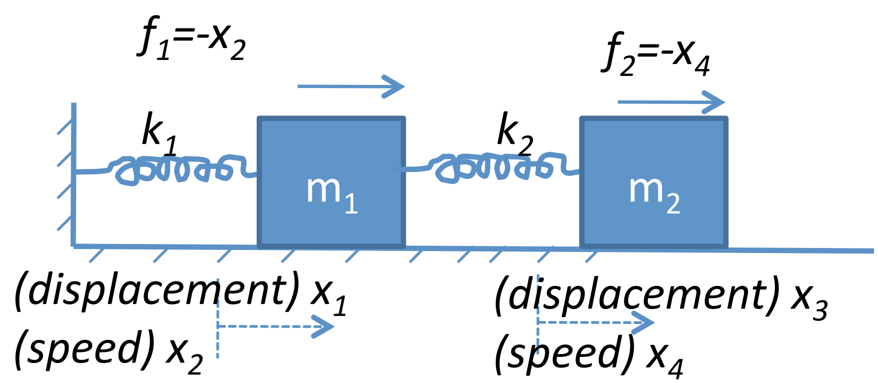
Electrical Circuit Analogy to Mass-Spring System

Analogous Systems

Mechanical Quantity	**Electrical System Analogue**
Displacement, x	Charge, Q
Velocity, v	Current, I
Force, F	Voltage, e
Friction, D	Resistance, R
Spring Constant, K	Inverse of Capacitance, 1/C
Mass, M	Inductance, L
Potential Energy in Spring, $(Kx^2)/2$	Energy stored by Capacitor, $Q^2/(2C)$
Kinetic Energy in Mass, $(Mv^2)/2$	Energy in Inductor, $(Li^2)/2$

Double LC, with resistance





Take

$$x_1 = Q_1$$
$$x_2 = I_1$$
$$x_3 = Q_2$$

Thus

$$x_4 = I_2 + I_1$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{m_1}(f_1 - k_1 x_1 + k_2(x_3 - x_1))$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{1}{m_2}(f_2 - k_2(x_3 - x_1))$$

Possible System Decomposition

The whole system can be represented as:

$$S_1 : \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{L_1 C_1} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{1}{L_1 C_2} & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + f_1$$

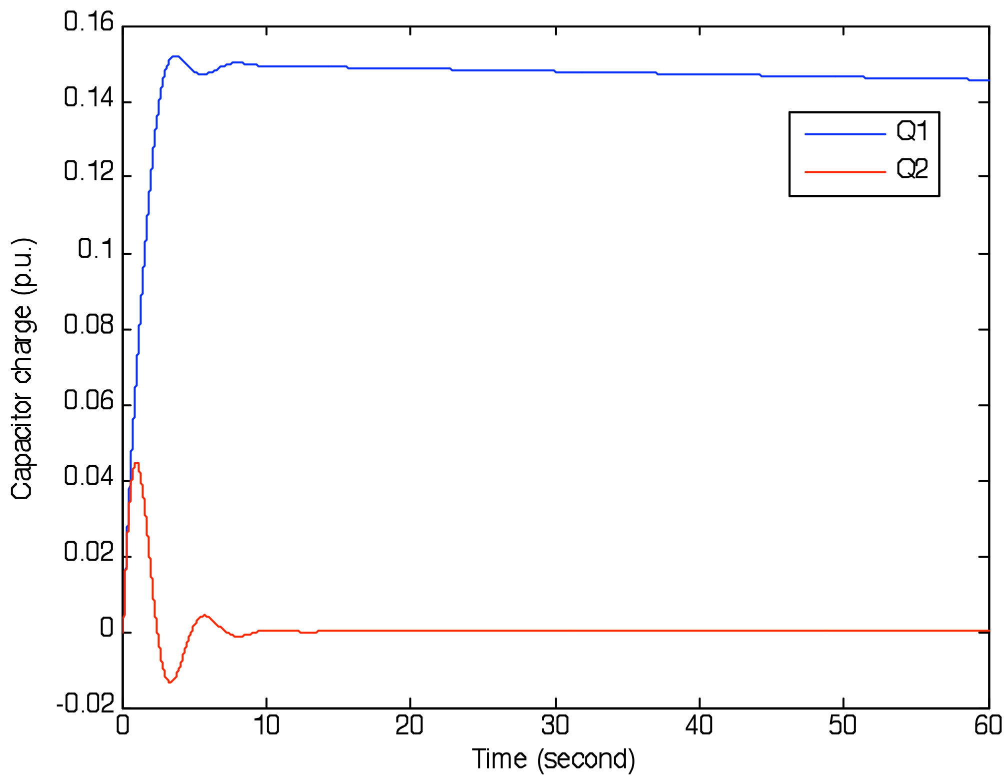
$$S_1 : \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{C_2}\left(\dfrac{1}{L_1} + \dfrac{1}{L_2}\right) & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{1}{L_1 C_1} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + f_2$$

$$f_1 = -\frac{R_1}{L_1} x_2$$

$$f_2 = -\frac{R_2}{L_2} x_4 + \left(\frac{R_1}{L_1} - \frac{R_2}{L_2}\right) x_2$$

Time-domain Responses of Coupled S1 and S2

- L_1=1; C_1=1000; R_1=1; L_2=1; C2=$_1$; R_2=1



Non-standard Singularly Perturbed Form Intepretation

$$\frac{1}{C_1} = \varepsilon = 0.001$$

Couples system takes on the form

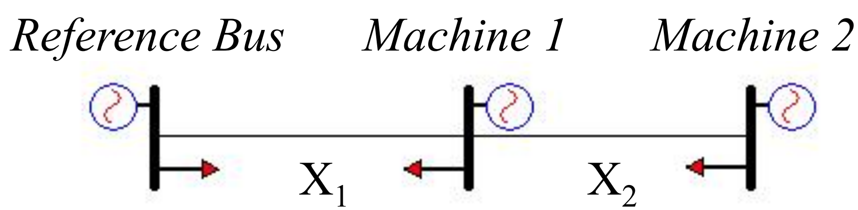
$$\varepsilon \dot{X} = A(\varepsilon) X$$

Rank(A(0)) = 3
That is, in this case

$$\lambda\left(A(0)\right) = \{0,\ -0.5 + 1.32i,\ -0.5 - 1.32i,\ -0.9995\}$$

Governor Control of Synchronous Machines Analogy to Mass-Spring

Dynamic Model of Two Machines Infinite bus



$$\begin{cases} \dot{\delta}_{G1} = \omega_{G1} \\ \dot{\omega}_{G1} = \dfrac{1}{M_1}\left[-K_1\omega_{G1} + \left(\dfrac{\delta_{G2}}{X_2} - \left(\dfrac{X_1 + X_2}{X_1 X_2}\right)\delta_{G1}\right)\right] \\ \dot{\delta}_{G2} = \omega_{G2} \\ \dot{\omega}_{G2} = \dfrac{1}{M_2}\left[-K_2\omega_{G2} + \left(\dfrac{\delta_{G2}}{X_2} - \dfrac{\delta_{G1}}{X_2}\right)\right] \end{cases}$$

$$\begin{cases} x_1 \leftrightarrow \delta_{G1} \\ x_2 \leftrightarrow \omega_{G1} \\ x_3 \leftrightarrow \delta_{G2} \\ x_4 \leftrightarrow \omega_{G2} \end{cases}$$

Decoupled model

Power flow: $$\begin{cases} P_{G1} = \frac{1}{x_1}\sin\delta_{G1} + \frac{1}{x_2}\sin(\delta_{G1}-\delta_{G2}) \\ P_{G2} = \frac{1}{x_2}\sin(\delta_{G2}-\delta_{G1}) \end{cases}$$

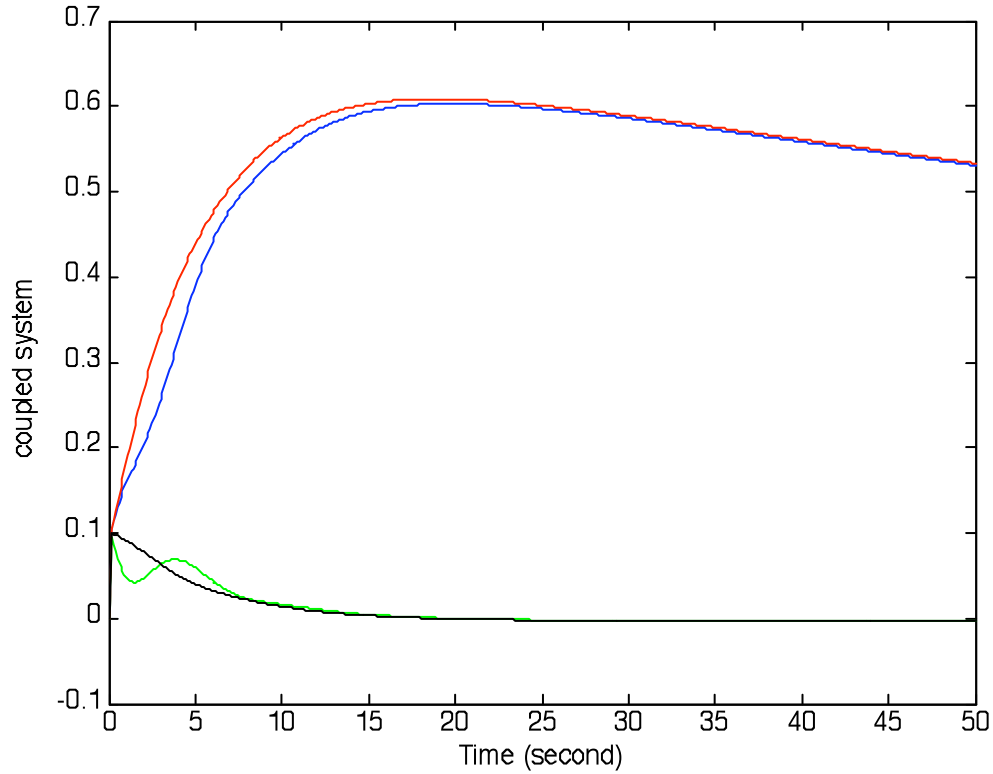
$$S_1 : \begin{bmatrix} \dot{\delta}_{G1} \\ \dot{\omega}_{G1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{x_1 + x_2}{M_1 x_1 x_2} & 0 \end{bmatrix} \begin{bmatrix} \delta_{G1} \\ \omega_{G1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_1 x_2} & 0 \end{bmatrix} \begin{bmatrix} \delta_{G2} \\ \omega_{G2} \end{bmatrix}$$

$$S_2 : \begin{bmatrix} \dot{\delta}_{G2} \\ \dot{\omega}_{G2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{M_2 x_2} & -\frac{K_2}{M_2} \end{bmatrix} \begin{bmatrix} \delta_{G2} \\ \omega_{G2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_2 x_2} & 0 \end{bmatrix} \begin{bmatrix} \delta_{G1} \\ \omega_{G1} \end{bmatrix}$$

$$f_1 = -\frac{K_1}{M_1}\omega_{G1} \qquad f_2 = -\frac{K_2}{M_2}\omega_{G2}$$

Time-domain Response of the Coupled System

- $M_1=1$; $K_1=1$; $X_1=100$; $M_2=10$; $K_2=1$; $X_2=1$
- Slow inter-area oscillation (singular characteristic of system matrix)



Concluding remarks

- Understanding analogies between: (1) mechanical systems and electrical circuits; and, (2) electric power systems, is a good way to introduce power systems problems (2) to those familiar with (1) [WP…];
- In this presentation we illustrated how can one interpret slow inter-area dynamics in power systems by understanding simpler systems (1).
- We are preparing an extensive paper on these analogies;