Combining PMUs with Conventional Measurements for Distributed Phasor Estimation:

New Concept and Illustration on the IEEE 30-Bus System

Usman Khan, Soummya Kar, Nermeen Tallat, Zhijian Liu, Marija Ilic and Jose Moura

Electrical and Computer Engineering Department
Carnegie Mellon University

Phasor Estimation Problem

- Estimate phasors at each node in power grid
  - To ensure system stability
  - For control design
  - Predicting catastrophic behaviors, e.g., blackouts

Conventional State Estimator

- Centralized procedures have their limitations in terms of
  - Single point of failure
  - Requires a lot of measurements
  - Convergence/observability are key issues
  - Collecting/Computation data at a single location
    - Communication/Computation intense

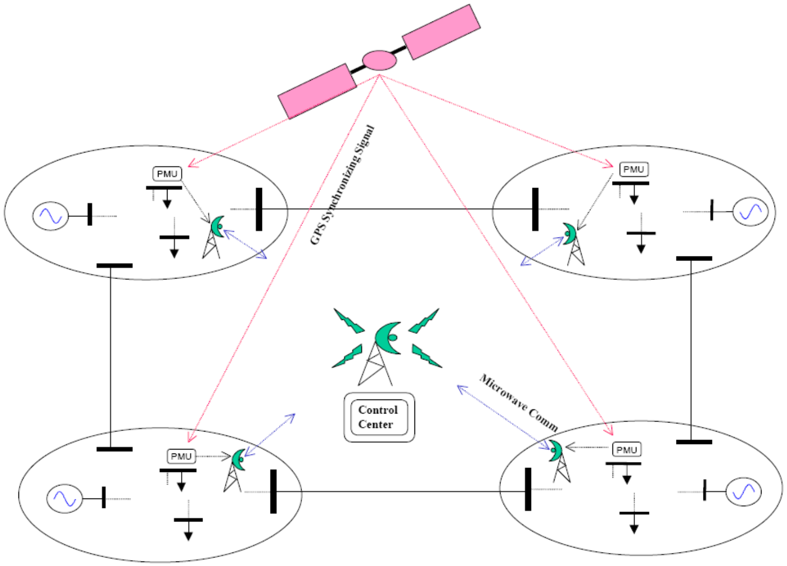
Key Idea

- Distributed Algorithm
  - Requires no global knowledge of any system parameter
  - Only relies on local measurements and local communication
- Minimal number of PMUs placed at optimal locations
- Low accuracy measurements already available from relays
- Our algorithm combines a few highly accurate PMU measurements with low accuracy measurements
- In the context of the given algorithm: given any network and its operating point, a minimal number of PMUs can be obtained that guaranteed convergence of the estimator

Phasor Measurement Unit (PMU)

- Synchronized Phase Measurement Unit (PMU) is a monitoring device, which was first introduced in mid-1980s.

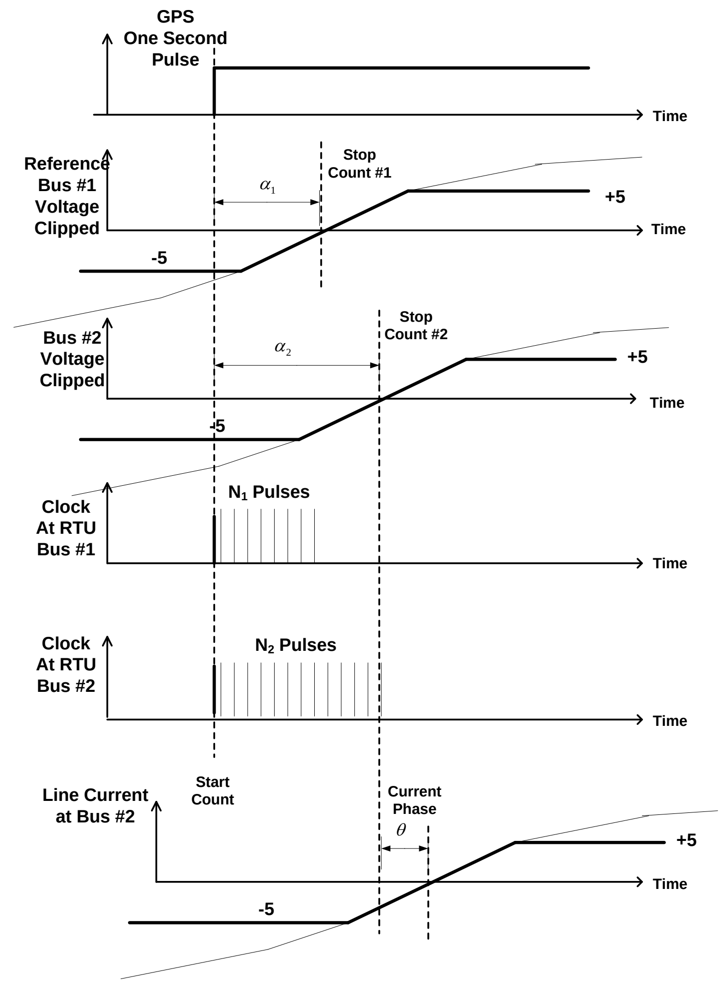
- Phasor measurement units
  - use synchronization signals from the GPS satellites
  - provide the phasors of voltage and currents measured at a given substation, generator or load bus.



THE BASICS Of PMUs

- The absolute time reference from the GPS is simultaneously (within nanoseconds) transmitted to transducers in power system generating stations, substations, and field bus locations.
- Each of these locations communicates with the system Units (RTU's).
- The RTU's are clocked computers that can be synchronized within several milliseconds to prepare for the advent of a specific GPS timing pulse.
- The one second pulse is from the IRIG-B train of one second pulses [12].
- The arrival of the GPS time pulse starts a pulse counter within the RTU to measure the zero crossing of the voltages at remote locations as shown schematically in figure 1 .
- control center by means of Remote Terminal

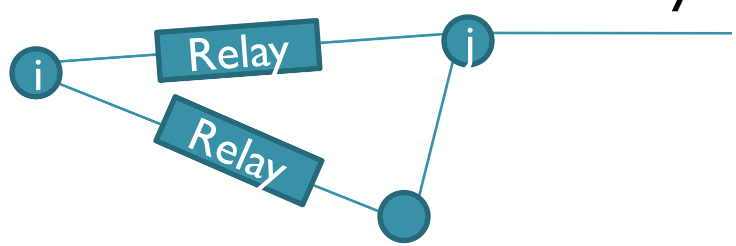
$$\delta = \alpha_1 - \alpha_2 = \left(\frac{N_1 - N_2}{N} \right) * 180^0$$



Low Accuracy Relays

- Actual physical devices (relays) available at the lines to measure line flows

- Line flows are inaccurate and noisy



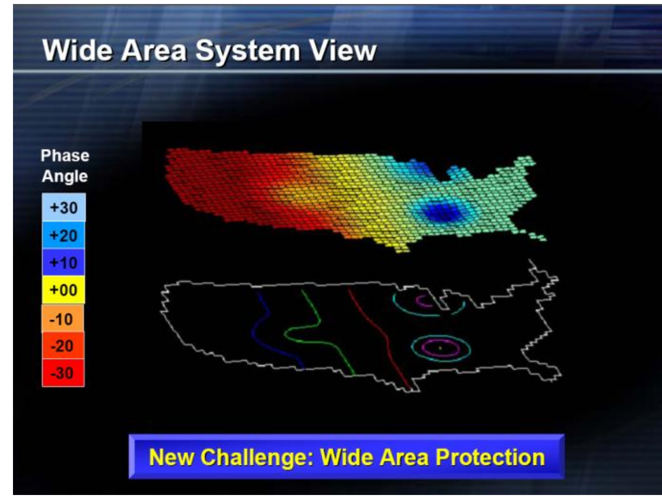
- Relays measure a noisy version of $\underline{y}_{ij}$ (current in the ij line)
  - $\underline{y}_{ij}$ œ $\underline{\theta}_{ij}$

- Simulation purposes (for a specified injection)
  - Compute θ_{ij} from the decoupled power flow
  - Add noise to simulate the inaccuracy
    - $\underline{\theta}ij = \theta_{ij}$ +noise

AC Decoupled Power Flow

$$\left[\frac{\partial P}{\partial \delta}\right]\Delta\delta = \Delta P$$

$$\begin{bmatrix} -B_{22} & -B_{23} & \cdots & -B_{2N} \\ . & -B_{33} & \cdots & -B_{3N} \\ & & & . \\ . & & & . \\ -B_{N1} & \cdots & & -B_{NN} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ . \\ . \\ \Delta\delta_N \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ . \\ . \\ \Delta P_N \end{bmatrix}$$

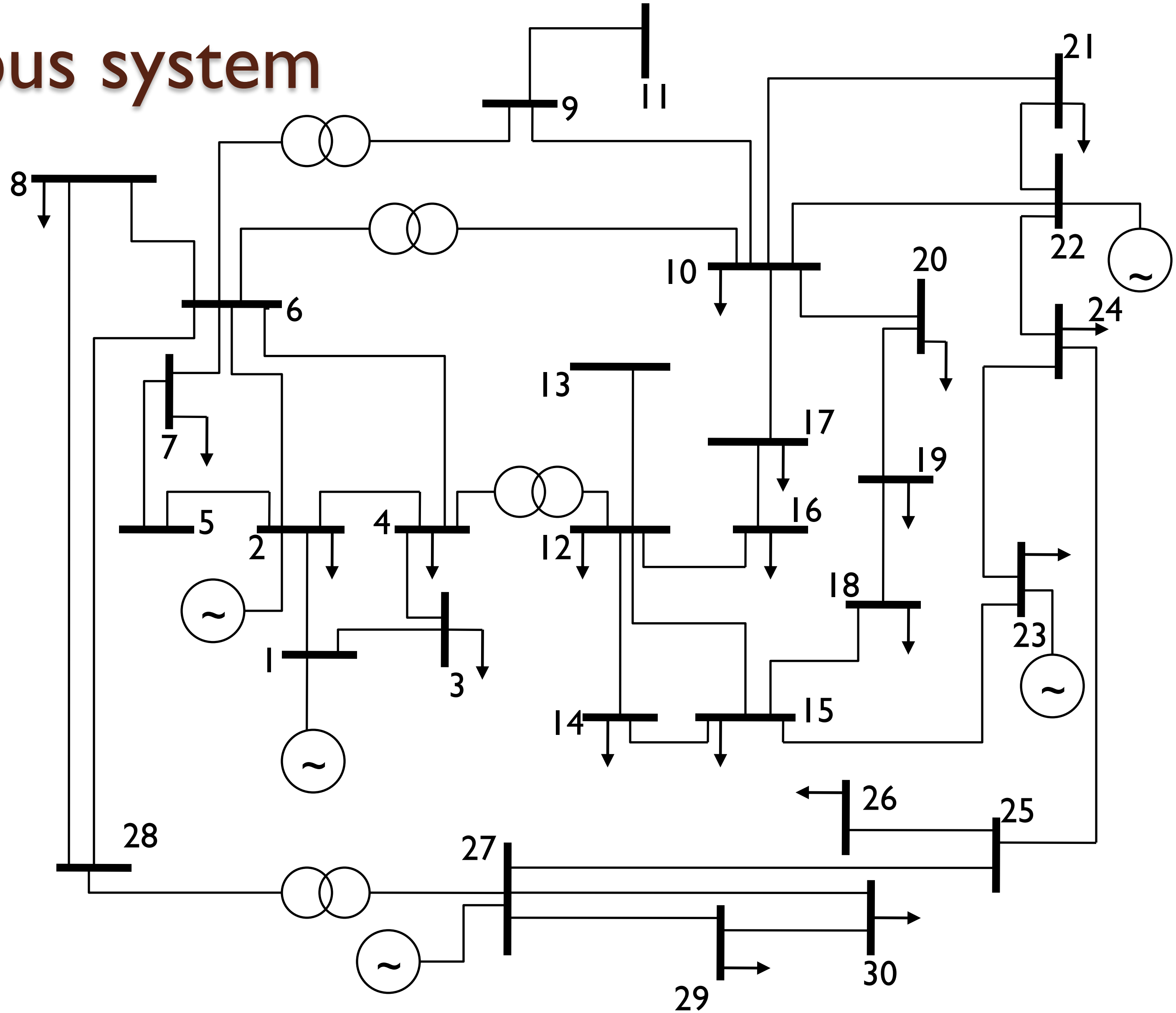


Figure(2): [2]

- Vector ΔP is the change of system real power
- Vector $\Delta\delta$ is the change of voltage phase angle in each bus
- B_{ij} is the susceptance between bus *i* and bus *j*
- $\left[\frac{\partial P}{\partial \delta}\right]$ is part of the system Jacobian Matrix $\begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial P}{\partial V} \end{bmatrix}$

[2] http://phasors.pnl.gov/Meetings/2004%20January/Phasor%20Measurement%20Overview.pdf

IEEE 30-bus system



IEEE 30-bus system
Graphical Representation

- T_{max}

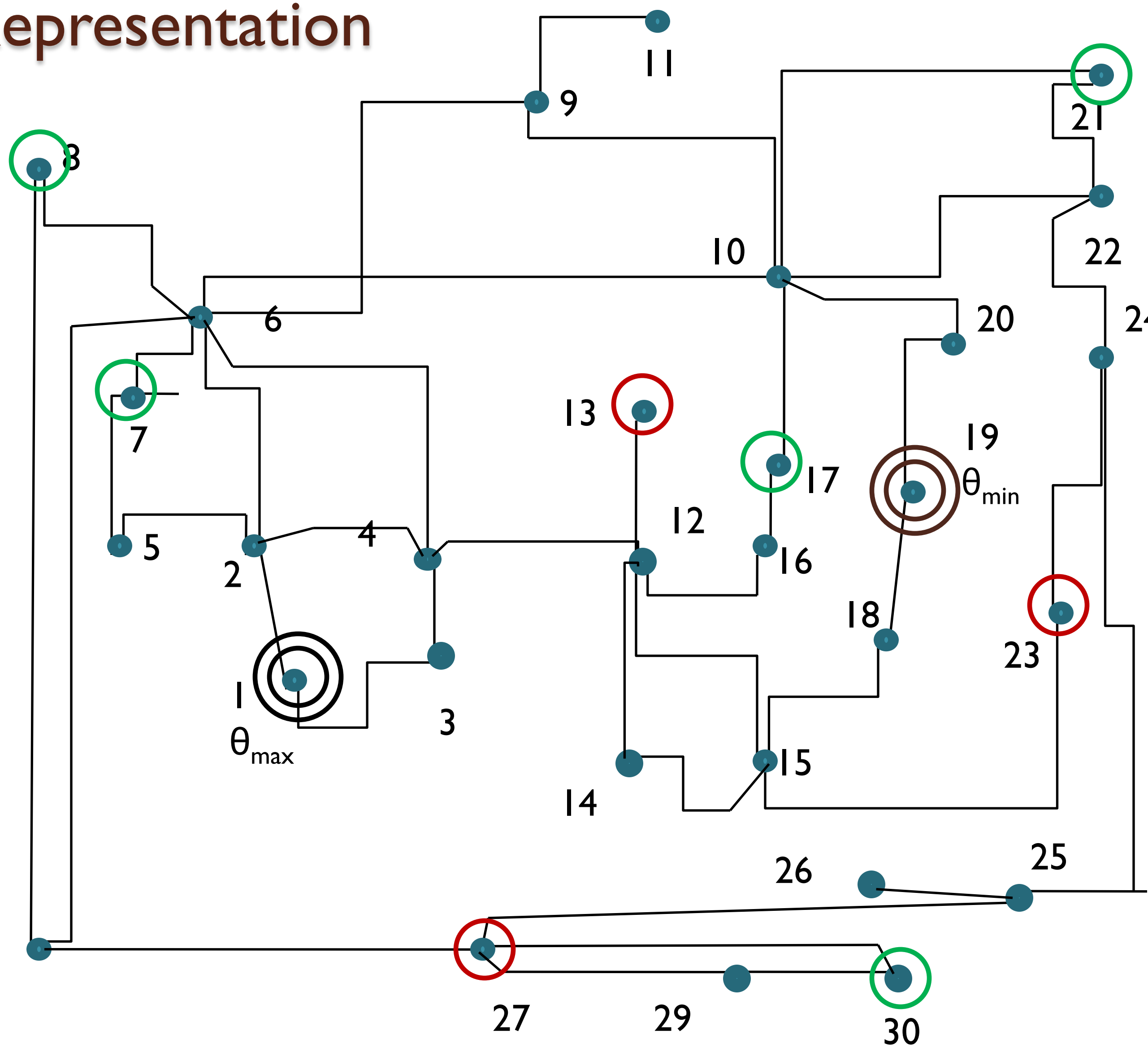
  $i = T_{max}$, if $\underline{\theta}_i = \underline{\theta}_{max}$
- T_{min}

  $i = T_{min}$, if $\underline{\theta}_i = \underline{\theta}_{min}$
- Source, S: Larger phasor than all the neighbors

  $i \in S$, if $\underline{\theta}_{ij} > 0$ for all $j \in N(i)$
- Sink, D: Smaller phasor than all the neighbors

  $i \in D$, if $\underline{\theta}_{ij} < 0$ for all $j \in N(i)$
- Node, N: Neither source or a sink

  $i \in N$, if i not $\in S \cup D$



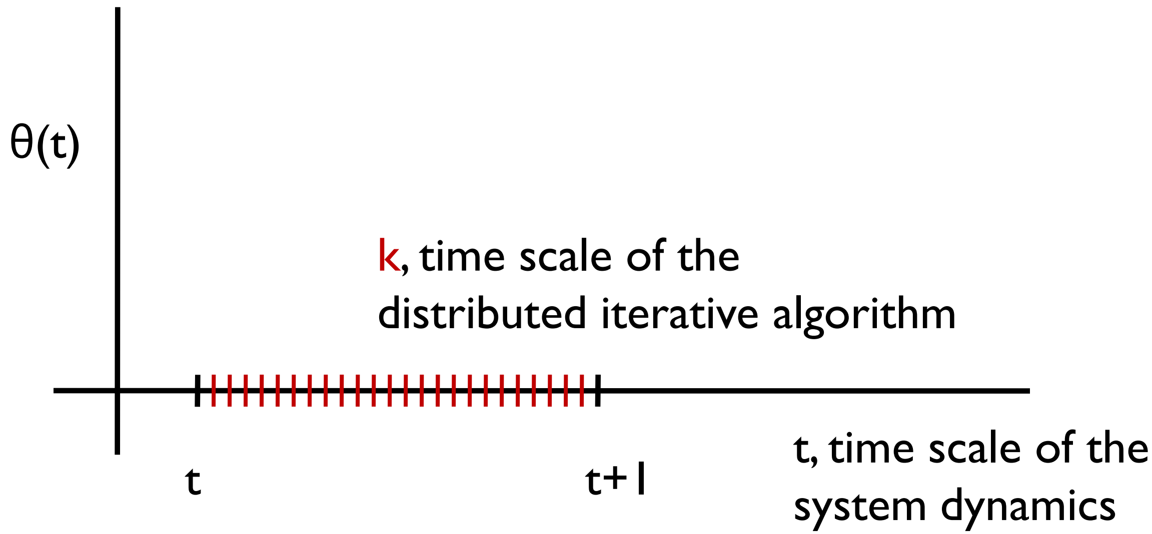
Distributed Phasor Estimation Algorithm

- The algorithm is given by
  - $\theta_i(k+1) = [\ 1-b(k)\ ]\ \theta_i(k) + b(k)\ [\ a_{ij}\theta_l(k) + a_{il}\theta_j(k)\ ]$,
    - $i \in N$, $j,l \in \mathcal{N}(i)$
  - $\theta_i(k+1) = \theta_j(k) + \underline{\theta}_{ij}$,
    - $i \in \underline{S} \cup \underline{D}$, $j \in \mathcal{N}(i)$
    - $\underline{S} \cup \underline{D}$ is the set of sources and sinks whose removal keeps the information network connected
  - $\theta_i(k+1) = \theta_i(k)$, $i \in \{T_{max}, T_{min}\} \cup S \cup D$
    - $S \cup D$ is the set of sources and sinks whose removal make the information network disconnected
  - where

$a_{ij} = |\underline{\theta}_{il}|/(|\underline{\theta}_{il}|+|\underline{\theta}_{ij}|)$

$a_{il} = |\underline{\theta}_{ij}|/(|\underline{\theta}_{il}|+|\underline{\theta}_{ij}|)$

Distributed Phasor Estimation Algorithm: Time-scale



Distributed Phasor Estimation Algorithm: Information network

- The information network is different from the physical network
- The information network consists of
  - nodes (buses)
  - the interconnections among the nodes are the neighbors of each node that sends information to it
  - For the given setup: each node in the information network has at most two neighbors (highly sparse)

Distributed Phasor Estimation Algorithm

- Result 1: Given
  - PMUs at T_{max} and T_{min}, (only 2 PMUs)
  - $\underline{\theta}_{ij}$`s for at least two neighbors at each node, and
  - some conditions on information network connectivity,

  the phasor at each other node can be estimated in a distributed way.

- Result 2: Given
  - PMUs at T_{max} , T_{min}, and
  - on those sources and sinks whose removal make the information network disconnected, and
  - $\underline{\theta}_{ij}$`s for at least two neighbors at each node,

  the phasor at each other node can be estimated in a distributed way.

Distributed Phasor Estimation Algorithm: Remarks

- Result 1 is an ideal case since it requires stringent conditions on network connectivity
- Result 2 is more practical as adding PMUs at some sources and some sinks keeps the network connected

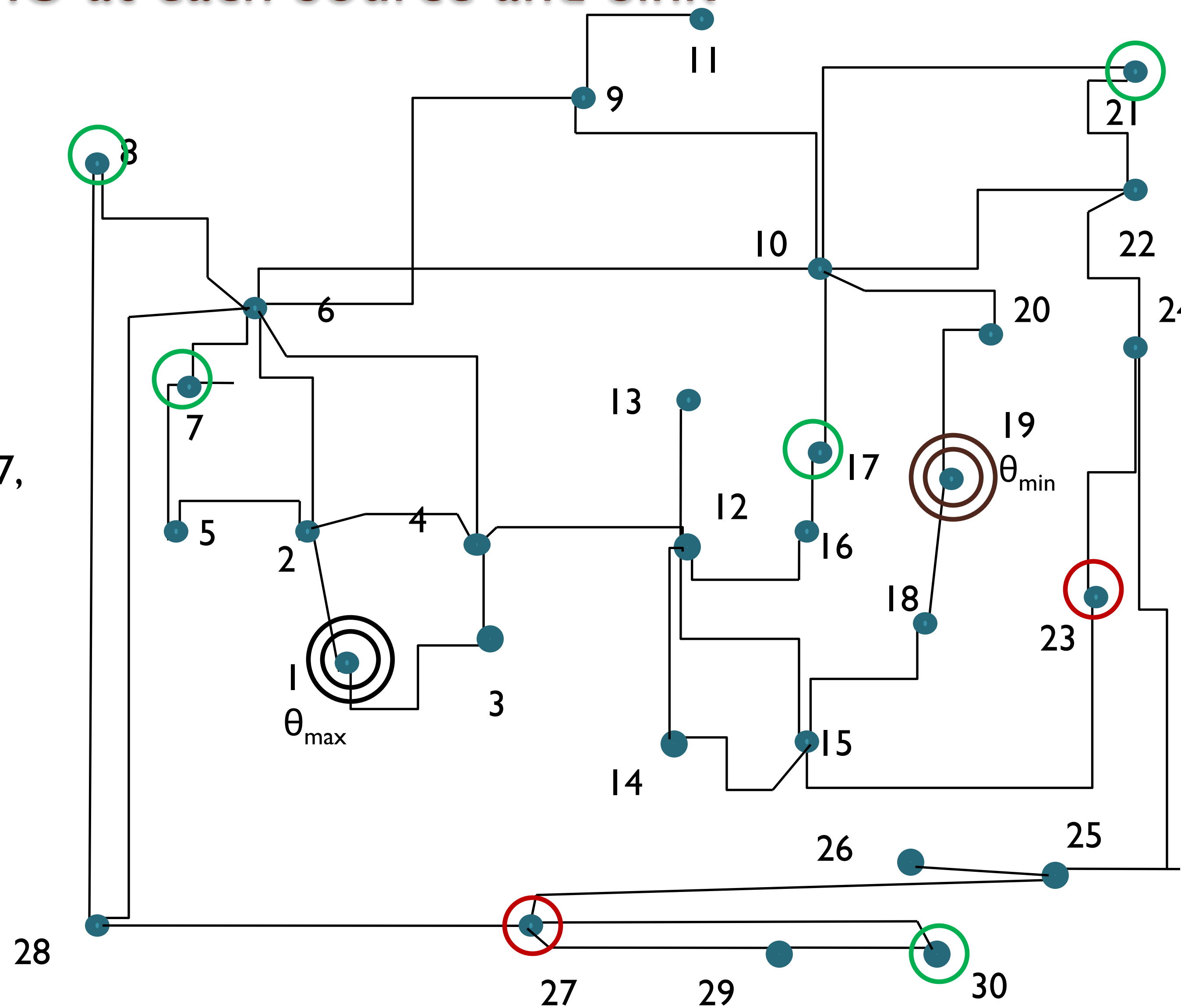
- Worst case: A PMU is required at each source and sink, equivalent to a highly sparse information network

Example 1: PMU at each Source and Sink

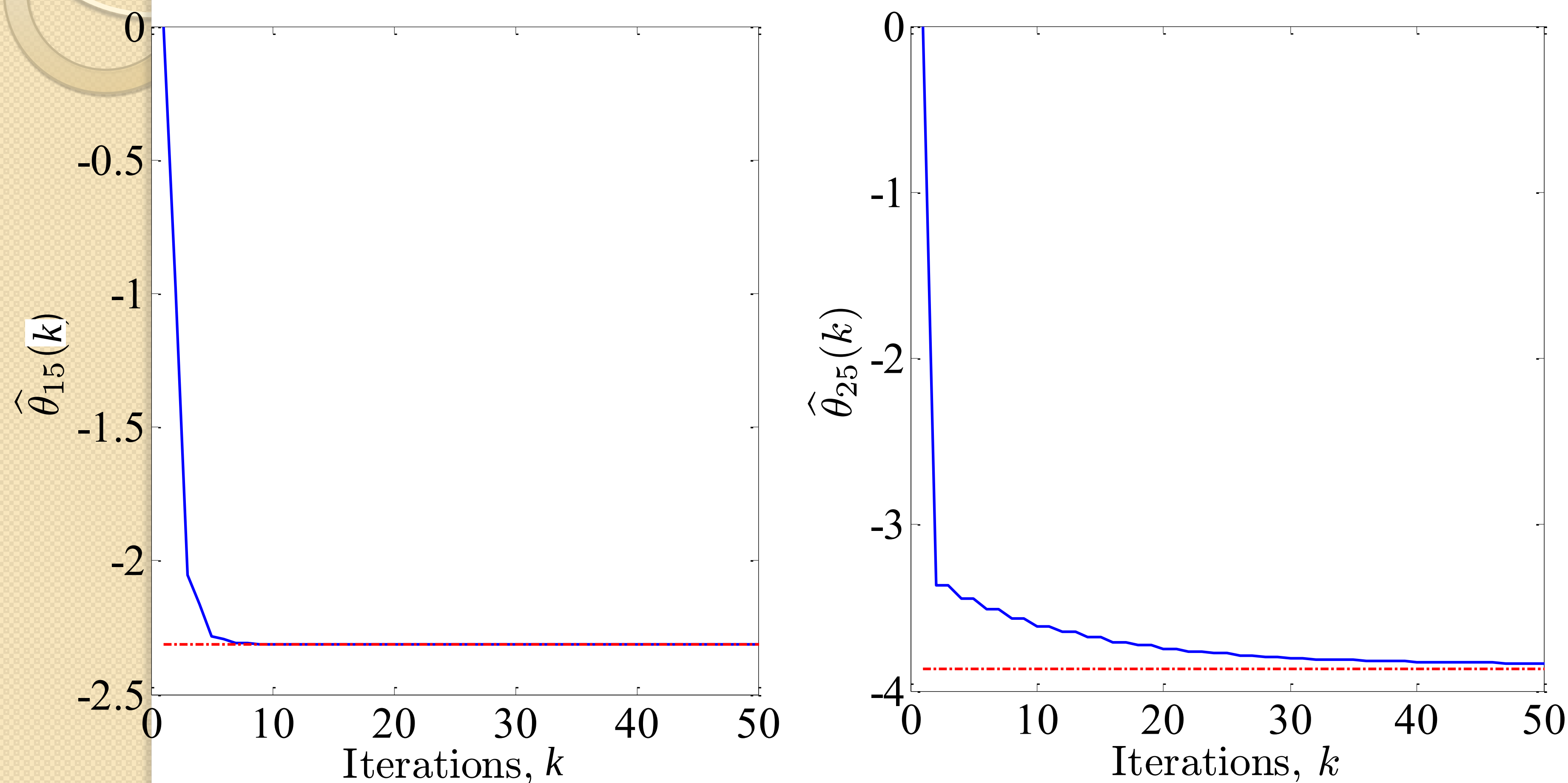
- $T_{max} = 1$
- $T_{min} = 19$
- Source,

  S = {23, 27}
- Sink,

  D = {7, 8, 17, 21, 30}
- 21 nodes
- Requires 9 PMUs



Example 1: PMU at each Source and Sink

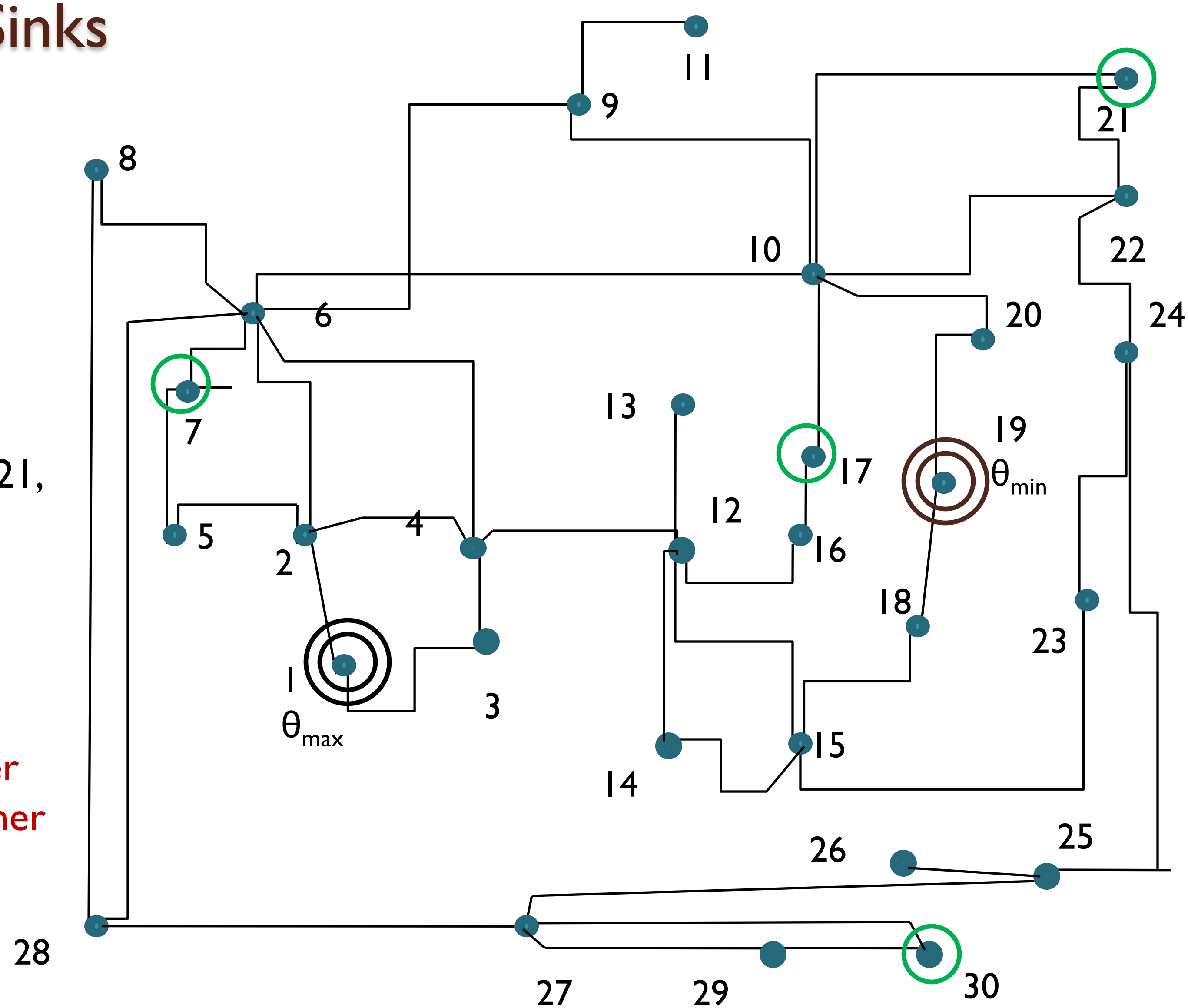


Example 2: PMU at some Sources and Sinks

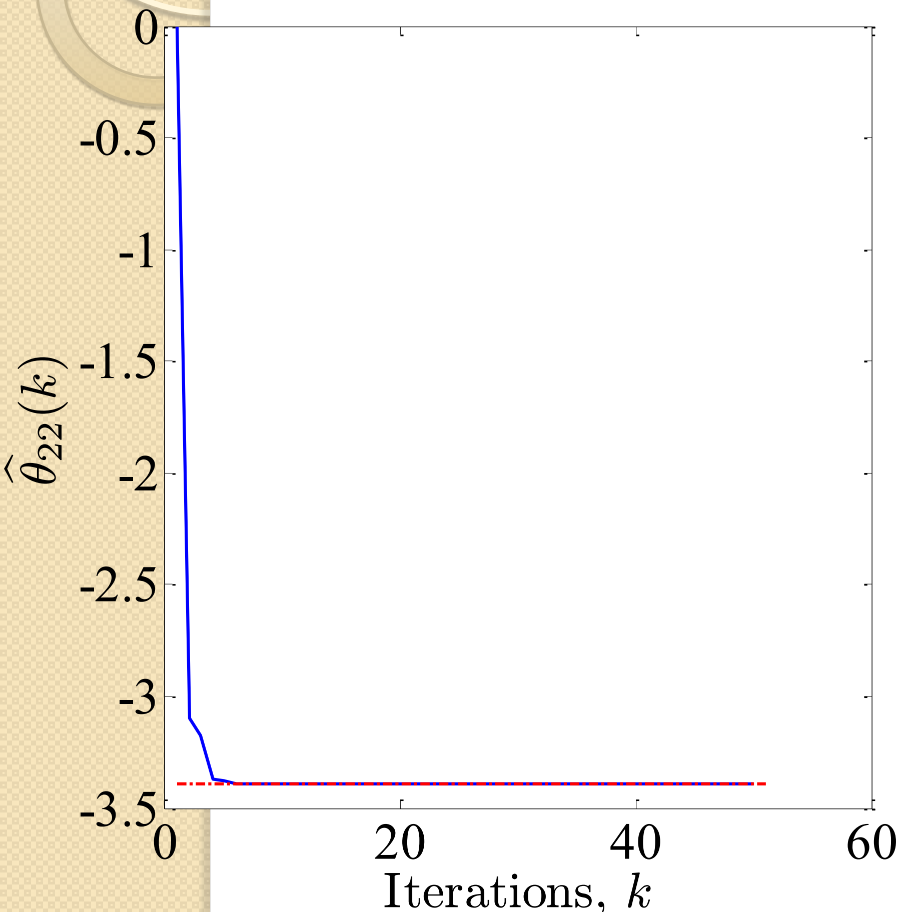
- $T_{max} = 1$
- $T_{min} = 19$
- Source,

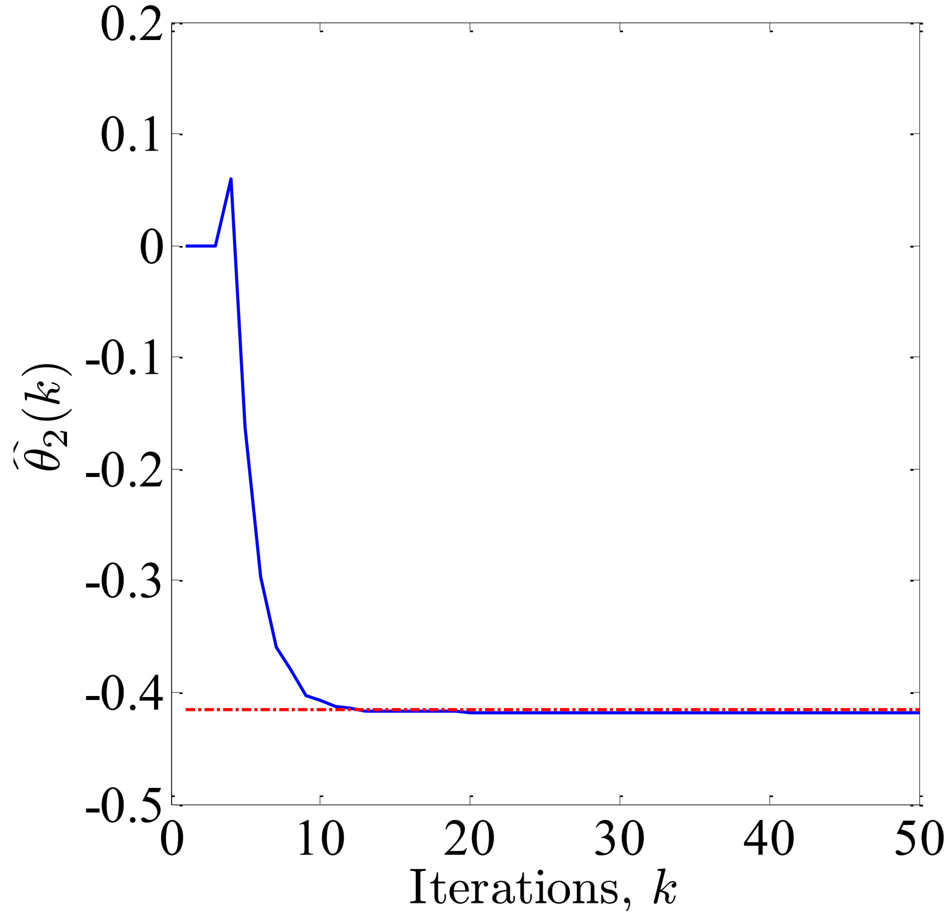
  S = { }
- Sink,

  D = {7, 17, 21, 30}
- 24 nodes
- Requires 6 PMUs
- This number can be further reduced



Example 2: PMU at each Source and Sink





Robustness Issues

- Information network is driven by inter-node communication
- With inaccurate measurements, some conditions on the noise distribution
- noisy communication and
- random link failures
  - the algorithm can be shown to converge a.s. to the exact phasors at each node in the network

Conclusions

- Combine inaccurate already existing measurements with a few accurate PMUs
- The entire phase vector for the grid can be estimated
- Future work
  - Improve the reduction of sources and sinks
  - Information network can have the same edges as in the physical network that can be exploited
    - results into a denser network and number of sources/sinks can be further reduced
  - Extended simulations in the noisy environment
  - Testing on larger grids