

QUIZ 2

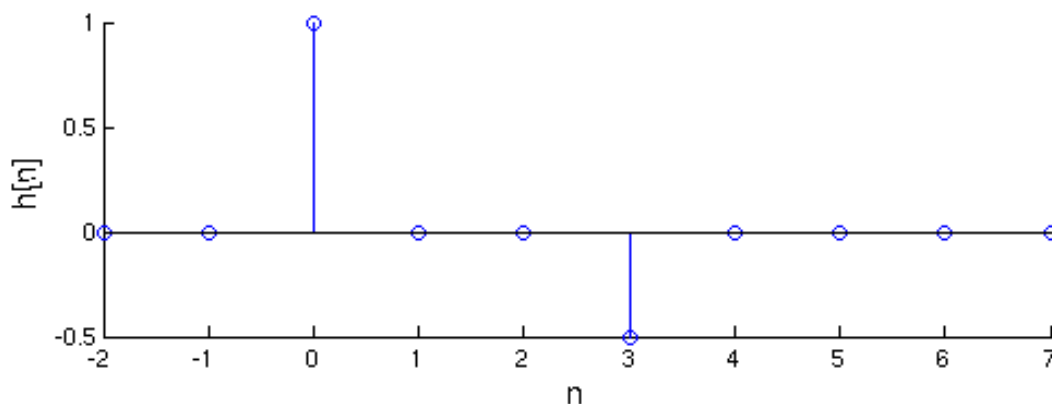
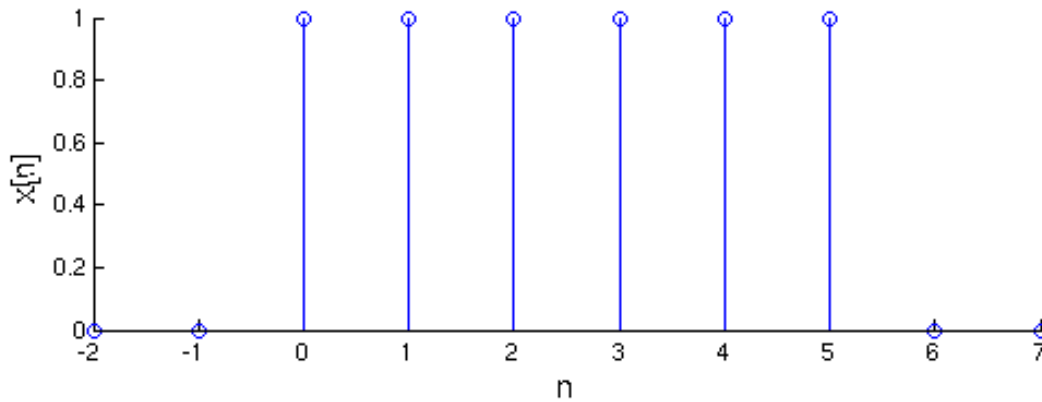
Two hours
November 18, 2004

Closed book
Two sheets of notes

Note: The questions on this exam are not very difficult. Be sure to think about ways of answering the questions using reasoning, transform properties, etc. before resorting to “brute-force” solutions. Although it may seem that there are many questions, most of them are very brief and easy.

The multiple parts of the longer of the questions can be solved separately and will be graded independently.

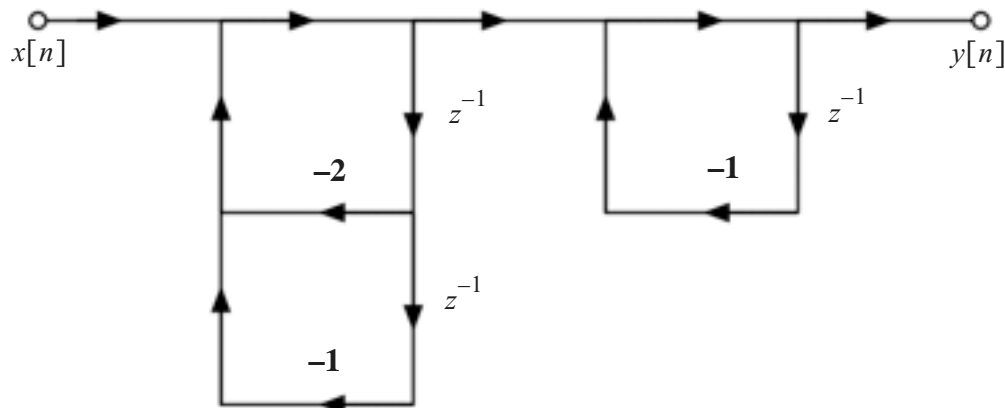
Question 1 (30%):



The discrete-time sequence $x[n] = u[n] - u[n - 6]$ is input to a linear shift-invariant system for which the unit sample response is $h[n] = \delta[n] - 0.5\delta[n - 3]$. (These two functions are depicted above.)

- (a) Carefully sketch and dimension the result of the linear convolution of $x[n]$ with $h[n]$.
- (b) $X[k]$ is the 16-point DFT of $x[n]$ and $H[k]$ is the 16-point DFT of $h[n]$. Sketch and dimension the inverse 16-point DFT of the product of the two sets of DFT coefficients, $X[k]H[k]$ for $0 \leq n \leq 19$.
- (c) Repeat part (b), but using 8-point DFTs and inverse DFTs.
- (d) We would now like to obtain the result of the convolution of the sample response $h[n]$ with a new input $x_2[n]$, which is nonzero only for $0 \leq n \leq 19$, but using only 16-point DFTs and inverse DFTs. Using either the overlap-add or overlap-save techniques discussed in class, specify exactly how the result of the convolution of the 20-point input sequence $x_2[n]$ with the 4-point unit sample response $h[n]$ can be obtained using only 16-point DFTs and inverse DFTs. In order to obtain full credit for this part of this problem you must do the following:
1. Specify exactly how the input sequence $x_2[n]$ is to be broken up into subsequences of length 16 points or less.
 2. Specify how each of these subsequences are combined with $h[n]$.
 3. Specify how the final result (which should be equal to the linear convolution of $x_2[n]$ with $h[n]$) can be obtained from the results of the combination of the subsequences of $x_2[n]$ with the sequence $h[n]$.

Question 2 (20%):



The signal-flow diagram depicts a very simple LSI system in cascade form.

- (a) Obtain the transfer function of the LSI system, $H(z) = Y(z)/X(z)$.
- (b) As you may recall, an **inverse LSI system** is the system that when cascaded with an original LSI sys-

tem produces a transfer function of 1, the transfer function of the identity system. For example, if $H^{-1}(z)$ is the inverse system of $H(z)$, $H(z)H^{-1}(z) = 1$.

1. Obtain the transfer function $H^{-1}(z)$ of the inverse system to $H(z)$.
2. Is the inverse system $H^{-1}(z)$ FIR or IIR? You must explain your reasoning to obtain full credit.
3. Is the inverse system $H^{-1}(z)$ linear phase? Again, please explain your reasoning.

(c) Sketch the signal flow diagram of the inverse system $H^{-1}(z)$. Use an implementation that reduces the number of multiplication operations to the greatest extent possible.

Question 3 (20%):

A single-pole Butterworth filter is a fairly mediocre filter in continuous time. For example, the transfer function of the (continuous-time) lowpass filter with cutoff frequency $(2\pi)(1000)$ radians per second (or 1000 Hz) is

$$H_c(s) = \frac{\Omega_c}{s + \Omega_c}$$

where $\Omega_c = 2\pi 1000$ as before. The corresponding continuous-time Fourier transform is

$$H_c(j\Omega) = \frac{\Omega_c}{j\Omega + \Omega_c} = \frac{1}{1 + j\left(\frac{\Omega}{\Omega_c}\right)}$$

We can convert this filter into a (similarly-mediocre) discrete-time filter using the bilinear transform

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

This is the expression we studied in class, but with the design parameter T_d set equal to 2.

- (a) Write the transfer function $H(z)$ of the resulting discrete-time filter.
- (b) What is the magnitude of the corresponding DTFT $H(e^{j\omega})$ when $\omega = 0$? (You can calculate this, but it is much easier to obtain this result by inspection.)
- (c) What is the frequency Ω for which the response magnitude of $H_c(j\Omega)$ is -3 dB relative to the corresponding response at very low frequencies? In other words, we are looking for the frequency Ω for which

$$\frac{|H_c(j\Omega)|}{|H_c(j0)|} = \frac{1}{\sqrt{2}}$$

(d) Similarly, what is the frequency ω for which the response magnitude of $H(e^{j\omega})$ is -3 dB relative to the corresponding response at very low frequencies? In other words, we are looking for the frequency ω for which

$$\frac{|H(e^{j\omega})|}{|H(e^{j0})|} = \frac{1}{\sqrt{2}}$$

(e) Is the filter $H(z)$ stable? Why or why not?

Question 4 (30%):

We would like to design an FIR bandstop or “notch” filter using the window design technique. The desired frequency response of such a filter is

$$H_d(e^{j\omega}) = \begin{cases} 1, & 0 \leq |\omega| \leq 0.4\pi \\ 0, & 0.4\pi < |\omega| \leq 0.6\pi \\ 1, & 0.6\pi < |\omega| \leq \pi \end{cases}$$

The specifications for a more realistic “practical” filter are as follows:

- Passband ripple¹: $-5 \text{ dB} \leq |H(e^{j\omega})| \leq 5 \text{ dB}$ dB, $0 \leq |\omega| \leq 0.4\pi$ and $0.6\pi < |\omega| \leq \pi$
- Stopband attenuation: $|H(e^{j\omega})| \leq -50 \text{ dB}$ $0.45\pi < |\omega| \leq 0.55\pi$

(a) What is the unit sample response of $h_d[n]$, the inverse DTFT of the ideal filter $H_d(e^{j\omega})$ as specified above? This can be computed analytically, but it can also be obtained by simple linear combinations of the familiar unit sample response for the ideal lowpass filter.

(b) Specify your choice of window shape and size from the five “classic” window types that are likely to satisfy the design specifications. Be sure to explain the reasoning behind your choices. As you know, the classic window shapes are rectangular, Bartlett, Hanning, Hamming, and Blackman. Some characteristics of them are summarized on the last page of this exam.

(c) Write the unit sample response $h[n]$ for the complete FIR notch filter that is believed to satisfy the specifications. Be sure to make any adjustments that are needed for the filter to exhibit true linear phase.

1. Note that the values of passband and stopband attenuation are given in dB. If x has the dimensions of amplitude or magnitude (as opposed to energy or power), the value of x expressed in dB would be $20\log_{10}(x)$

TRANSFORM FORMULAE

Continuous-time Fourier transform (CTFT):

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} d\Omega$$

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$

Continuous-time Fourier series (CTFS):

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\Omega_0 t}; T_0 = 2\pi/\Omega_0$$

$$X[k] = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\Omega_0 t} dt$$

Discrete-time Fourier Transform (DTFT):

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

Z-transform:

$$x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Discrete Fourier Series/Discrete Fourier transform (DFS/DFT):

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}$$

SOME COMMON WINDOW FUNCTIONS

Rectangular window:

$$w[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

Bartlett (triangular) window:

$$w[n] = \begin{cases} 2n/M, & 0 \leq n \leq M/2 \\ 2 - 2n/M, & M/2 < n \leq M \\ 0, & \text{otherwise} \end{cases}$$

Hanning window:

$$w[n] = \begin{cases} 0.5 - 0.5 \cos(2\pi n/M), & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

Hamming window:

$$w[n] = \begin{cases} 0.54 - 0.46 \cos(2\pi n/M), & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

Blackman window:

$$w[n] = \begin{cases} 0.42 - 0.5 \cos(2\pi n/M) + 0.08 \cos(4\pi n/M), & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

Window type	Peak Sidelobe Amplitude (Relative, dB)	Approximate Width of Mainlobe	Peak Approximation Error (dB)
Rectangular	-13	$4\pi/(M+1)$	-21
Bartlett	-25	$8\pi/M$	-25
Hanning	-31	$8\pi/M$	-44
Hamming	-41	$8\pi/M$	-53
Blackman	-57	$12\pi/M$	-74

Comparison of commonly-used windows.