Carnegie Mellon **DSP-I** Electrical & Computer ENGINEERING

Digital Signal Processing I (18-791)
Fall Semester, 2005

PROBLEM SET 9

Issued: 11/4/05
Due: 11/10/05

Reminder: Quiz 2 is scheduled for November 17. It will cover material through this problem set (specifically digital filter implementation (following the material we discussed in OSB Chapter 6) but not digital filter design (which corresponds to the material in OSB Chapter 7).

Reading: During the past week we reviewed implementation procedures for IIR and FIR discrete-time systems and we touched on some of the effects of coefficient quantization, following material in OSB 6.0-6.7. Next week we will then begin our next big discussion topic, discrete-time design. We will focus first on design procedures for IIR filters based on continuous-time prototypes, following Secs. 7.0 through 7.2 in OSB. You should also review the material on continuous-time filters in Appendix B of OSB. We will continue with FIR filter design procedures in the following weeks.

Problem 9.1:

An IIR discrete-time digital filter has poles at 1/2, 1/4, $\frac{e^{j\frac{\pi}{4}}}{\sqrt{2}}$ and $\frac{e^{-j\frac{\pi}{4}}}{\sqrt{2}}$ and zeros at -1, -1/2, and -1/4. The magnitude of the transfer function of the filter at DC ($\omega = 0$ or $z = 1$) equals 1.

Sketch the signal flowgraph diagram for the following implementations of this filter:

(a) direct form II

(b) the transposed version of direct form II

(c) the cascade form

(d) the parallel form

Note: Feel free to use MATLAB for some of the calculations in this and the following problem. The commands `roots` and `poly` may be especially useful.

Problem 9.2:

A linear-phase FIR filter has the following unit sample response:

- $h[0] = 1$
- $h[1] = 2$
- $h[2] = 2$
- $h[3] = 1$

Sketch the signal flowgraph diagram for the following implementations of this filter:

(a) the direct form

(b) the cascade form

(c) the linear-phase form

(d) the frequency-sampled form

Problem 9.3: Problem 6.25 in OSB

Problem 9.4: Problem 6.38 in OSB

Problem 9.5: (an old quiz problem)

We discussed in class the *frequency-sampling implementation* of FIR discrete-time filters. As you may recall, this form was obtained by expressing $H[k]$ as

$$H(z) = (1 - z^{-N})\frac{1}{N}\sum_{k=0}^{N-1}\frac{H[k]}{1 - W_N^{-k}z^{-1}} \qquad (9.5\text{-}1)$$

which implies the filter structure shown in Fig. 9.5a.

If N is odd, Eq. (9.5-1) can be written as

$$H(z) = (1 - z^{-N})\frac{1}{N}\frac{H[0]}{1 - z^{-1}} + (1 - z^{-N})\frac{1}{N}\sum_{k=1}^{(N-1)/2}\frac{H[k]}{1 - W_N^{-k}z^{-1}} + \frac{H[N-k]}{1 - W_N^{-(N-k)}z^{-1}} \qquad (9.5\text{-}2)$$

(a) Obtain expressions for $H[N-k]$ in terms of $H[k]$, and $W_N^{-(N-k)}$ in terms of W_N^{-k}, that are valid for $h[n]$ real.

(b) Let us now assume that $h[n]$ actually is real, as it inevitably is in this course. By applying the symmetry properties you developed in part (a) to Eq. (9.5-2), it can be shown that it is possible to realize Eq. (9.5-2) as a discrete-time filter structure of the form shown in Fig. 9.5b on the next page, where all of the coefficients a_{1k}, a_{2k}, b_{0k}, and b_{1k} are real. Obtain expressions for the coefficients a_{1k}, a_{2k}, b_{0k}, and b_{1k} in terms of $H[k]$, assuming that $h[n]$ is real.

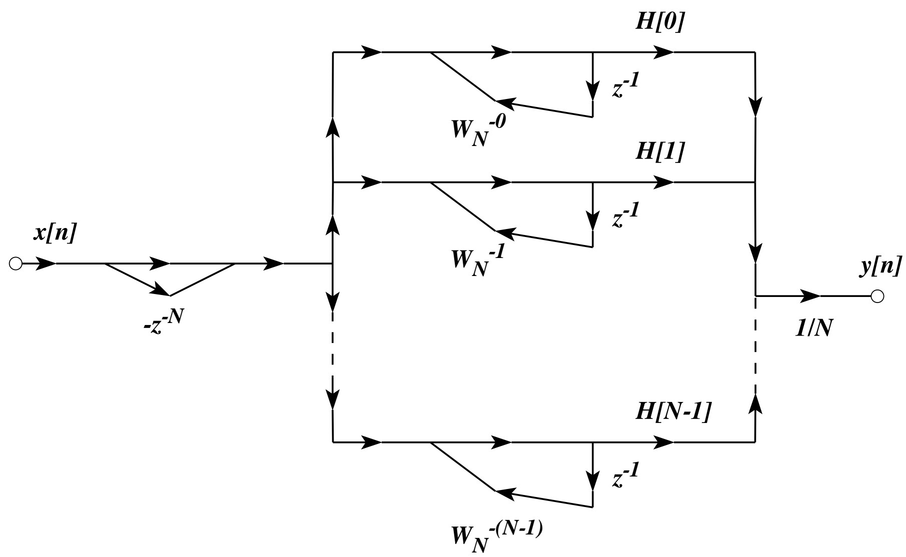

Figure 9.5a. Frequency-sampling FIR filter implementation.

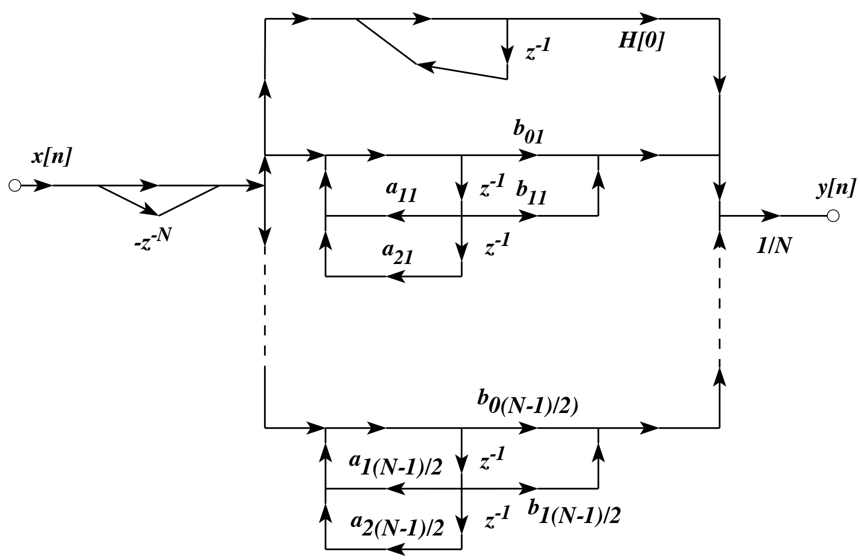

Figure 9.6b. Frequency sampling FIR implementation with real coefficients.

Problem 9.6: Problem 7.4 in OSB. You may wish to wait until the Tuesday lecture before working this and the following problem.

Problem 9.7: Problem 7.10 in OSB