
HW SET #8 (DUE BEFORE CLASS ON APR 14, WED)

Problem 1 (30 points) Construct a filter bank with the following specifications (Due to symmetry, only positive frequency is shown.):

Subband #0: $[0, p/2)$
Subband #1: $[p/2, 5p/8)$
Subband #2: $[5p/8, 11p/16)$
Subband #3: $[11p/16, 3p/4)$
Subband #4: $[3p/4, p)$

First draw the tree-structure implementation. Show all the analysis filters, downsamplers, upsamplers, and the synthesis filters. Then, use the Nobel Identities to transform the tree-structure implementation into a parallel implementation. Clearly express the equivalent analysis filter and the equivalent synthesis filter in each subband, in terms of filters in the tree-structure implementation.

Problem 2 (30 points) Repeat Problem 1 with the following specifications:

Subband #0: $[0, p/3)$
Subband #1: $[p/3, p/2)$
Subband #2: $[p/2, 5p/9)$
Subband #3: $[5p/9, 11p/18)$
Subband #4: $[11p/18, 2p/3)$
Subband #5: $[2p/3, p)$

Note: You may have to use a filter bank that has more than two bands.

Problem 3 (40 points) Write a simple program to simulate a two-channel filter bank. Let the analysis filters be $H_0(z) = 1 + z^{-1}$ and $H_1(z) = 1 - z^{-1}$. It can be shown that $H_0(z)$ is a half-band lowpass filter and $H_1(z)$ is a highpass filter. (They don't have very sharp transition bands, though.) Let the synthesis filters be $F_0(z) = 1 + z^{-1}$ and $F_1(z) = 1 - z^{-1}$. Let $x(n)$, $x_0(n)$, $x_1(n)$, and $\hat{x}(n)$ represent the input signal, the lowpass subband signal, the highpass subband signal, and the output signal, respectively. You don't have to hand in your code. Instead, do the following:

1. Let $x(n) = 20\cos(w_0 n + q_0)$ where $w_0 = p/4$ and $q_0 = p/10$. Plot $x(n)$, $x_0(n)$, $x_1(n)$, and $\hat{x}(n)$ for $n = 0 \ldots 100$.

2. Let $x(n) = 10 + 40\cos(w_0 n + q_0) + 20\cos(w_1 n + q_1)$ where $w_0 = p/15$, $q_0 = 0$, $w_1 = 11p/12$ and $q_1 = p/3$. Plot $x(n)$, $x_0(n)$, $x_1(n)$, and $\hat{x}(n)$ for $n = 0 \ldots 100$.

3. Do you think the filter bank have the "perfect reconstruction property," i.e., $x(n) = \hat{x}(n)$ up to a scale factor and a delay? If yes, what is the scale factor and what is the delay? (You don't have to prove this.)