Homework Assignment #6

Due: May 3, 4:00pm
(in HH A305)

1. (25%) Following data represents a collection of vectors each containing three features. Determine the best linear combination of the three features using the K-L transform. What is the resulting mean squared error?

$$\begin{bmatrix}0\\0\\0\end{bmatrix} \begin{bmatrix}1\\1\\1\end{bmatrix} \begin{bmatrix}-1\\-1\\-1\end{bmatrix} \begin{bmatrix}1\\-1\\1\end{bmatrix} \begin{bmatrix}-1\\1\\-1\end{bmatrix}$$

2. (25%) The convex hull of a set of vectors $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$ is the set C_X where

$$C_X = \{\mathbf{x} = a_1\mathbf{x}_1 + a_2\mathbf{x}_2 + \ldots + a_N\mathbf{x}_N, \quad a_i \geq 0, \quad a_1 + a_2 + \ldots + a_N = 1\}$$

Given two training vector sets $\mathbf{Y} = \{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_N\}$ and $\mathbf{Z} = \{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_N\}$ representing two classes, prove that either **y** and **z** are linearly separable or that their convex hulls intersect.

3. (25%) Following 4 training vectors represent two classes.

$$\text{class } \omega_1 = \left\{\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix}\right\}$$

$$\text{class } \omega_2 = \left\{\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix}\right\}$$

Use single-sample perceptron method with $\rho_n = 1$ to determine a solution vector. Use the all-zero vector as the initial weight vector.

4. (25%) Test the linear separability for the following 2-class training data using Ho-Kashyap method. Assume that all initial safety margins are set to 1.

$$\text{class } \omega_1 = \left\{\begin{bmatrix}-1\\-1\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}\right\}$$

$$\text{class } \omega_2 = \left\{\begin{bmatrix}1\\-1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\right\}$$