Optimizing Joint Erasure- and Error-Correction Coding for Wireless Packet Transmissions

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Motivation

- Often data has to be transmitted through a series of wireless and wired links
- Performance bottleneck is then the wireless link
  - Unreliable due to fading
  - Less bandwidth
  - Subject to interference
- Powerful and efficient coding possible across the whole data block
- Individual packets are subject to fading when traveling across the wireless link
  - Outage corrupts complete packet, otherwise negligible error rate
  - Not efficient to forward corrupted packets
- Use error-correction coding per packet -> *view as an erasure channel*

Digital Fountain Codes

- Efficient erasure-correction codes now available
  - Digital Fountain principle - generate practically endless streams of encoded packets
  - Reception of a sufficient number of correct packets leads to high decoding probability
  - Small overhead (about 5% for reasonable size)

M. Luby, "LT Codes," in *Proc. 43rd Annual IEEE Symposium on Foundations of Computer Science*, Nov. 2002

M. Shokrollahi, "Raptor Codes," *IEEE Trans. Inform. Theory*

System Model – Assumptions

- End-to-end transport of a finite-size data block
  - Performance is dominated by a wireless link
  - Wireless link is well characterized by block fading model

$$y = hs + w$$

- Average signal-to-noise ratio on wireless link

$$\gamma = E\left[|s|^2\right] / \left[|w|^2\right]$$

- Large feed-back delay
  - Usage of automatic repeat request (ARQ) not possible

System Model – Layered Coding

- Layered coding approach
  - Erasure-correction coding across the data packets
  - Error-correction coding per packet on the physical layer

- Erasure-correction coding
  - Block of N_{data} bits is partitioned into k packets
  - Generate K encoded packets with rate r_n

$$N_{\text{data}} = kN_b$$

$$r_n = \frac{k}{K}$$

- Error-correction coding
  - Each packet of N_s symbols carries N_b bits
  - Define coding rate as non-vanishing fraction of ergodic Capacity C

$$R_{\text{phy}} = \frac{N_b}{N_s}$$

$$r_p = \frac{R_{\text{phy}}}{C}$$

System Model – Nakagami Model

- Capacity on Nakagami-m block fading channel
  - Mutual information assuming Capacity achieving Gaussian codebooks
    $$I = \log_2\left(1+\gamma|h|^2\right)$$
  - Ergodic capacity defined as average mutual information

$$C(\gamma,m) = E\left[\log_2(1+\gamma|h|^2)\right] = \log_2(e)\, e^{m/\gamma} \sum_{k=0}^{m-1} \left(\frac{m}{\gamma}\right)^k \Gamma\left(-k, \frac{m}{\gamma}\right)$$

  - Correct physical layer decoding is achieved, if mutual information is above transmission rate

$$\begin{aligned} p &= \Pr\left(I < R_{\text{phy}}\right) \\ &= \Pr\left(|h|^2 < \alpha = \left(2^{R_{\text{phy}}}-1\right)/\gamma\right) \\ &= 1 - \sum_{k=0}^{m-1} \frac{1}{k!} (m\alpha)^k\, e^{-m\alpha} \end{aligned}$$

System Model – Performance

- Total outage probability of transmission
  - Depends on number of correctly received packets $k' > k$
  - Packet error detection based on CRC is perfect

$$P_{\text{outage}} = \sum_{i=0}^{k'-1} \binom{K}{i} (1-p)^i \cdot p^{K-i}$$

- Define efficiency of data transfer

$$\eta = \frac{N_{\text{data}}}{K N_s C} = \frac{k}{K} \cdot \frac{N_b}{N_s C} = r_n \cdot r_p$$

Problem Statement

- Obvious trade-off necessary between r_p and r_n
  - Smaller physical rate leads to less corrupted packets
  - Low network rate reduces vulnerability to packet loss

- Investigate two dual problems:
  1. Optimizing Performance under Resource Const.
     - Fix overall efficiency
     - Split resources between coding layers
  2. Minimizing Resource under Performance Const.
     - Adhere to prescribed outage probability
     - Combine strengths of coding layers

Optimal Combining of Inter- and Intra-Packet Coding

- Preliminaries
  - For large k' and K approximate P_{outage} as Gaussian

    $$P_{\text{outage}} \approx Q\left(\frac{Kq - k'}{\sqrt{Kpq}}\right)$$

  - Probability of correct transmission q

    $$q = 1 - p$$

  - Define a constant ρ

    $$\rho = k'/k$$

  - Portion of variable redundancy in r_n

    $$\tilde{r}_n = \frac{k'}{K} = \rho r_n$$

  - Simplify P_{outage} using the definitions

    $$P_{\text{outage}} = Q\left(\sqrt{\frac{\rho N}{N_s C}}\frac{q - \tilde{r}_n}{\sqrt{\tilde{r}_n r_p p q}}\right)$$

Optimal Combining of Inter- and Intra-Packet Coding – Solution 1.

- Optimizing performance under resource constraint
  - Use equivalent obj. function

$$\max J(r_p, \tilde{r}_n) := \frac{q - \tilde{r}_n}{\sqrt{r_p \tilde{r}_n (1-q) q}}$$

$$\text{subject to } r_p \tilde{r}_n \geq \tilde{\eta}_0$$

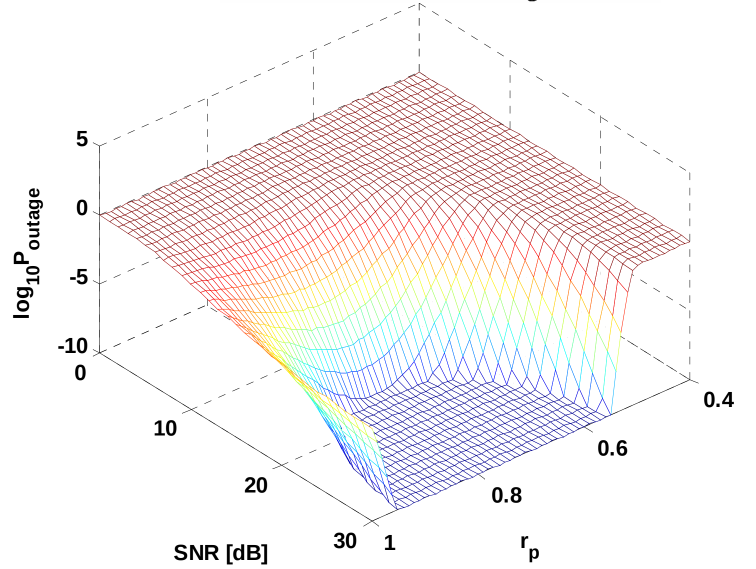
  - The Lagrange approach leads to:

$$\tilde{r}_n = \frac{-r_p q \dot{q}}{2q(1-q) - r_p(2q-1)\dot{q}} \qquad \dot{q} = \frac{\partial q}{\partial r_p}$$

  - Solution is intersection with constraint

- Numerical Example
  - $P_{outage} = 1$ for $r_p < 0.5$
  - Clear minimum for average SNR around $r_p = 0.8$

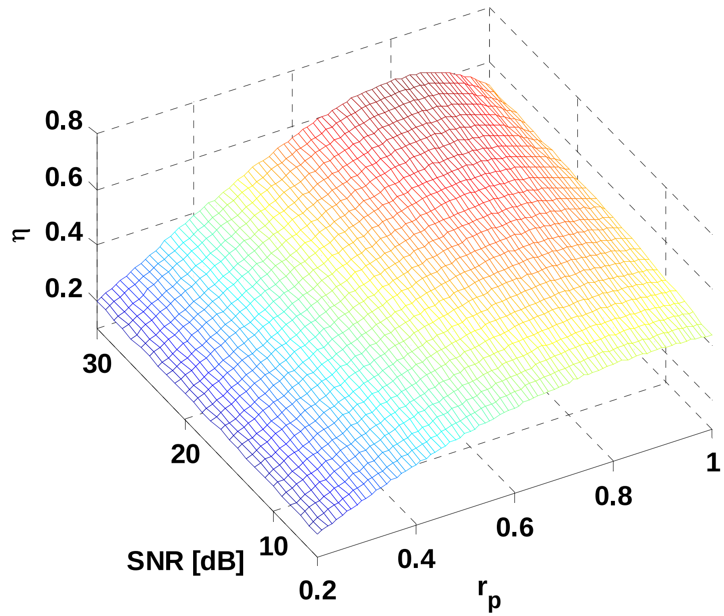
$$\tilde{\eta}_0 = 0.5 \qquad \frac{\rho N}{C N_s} = 2^8$$



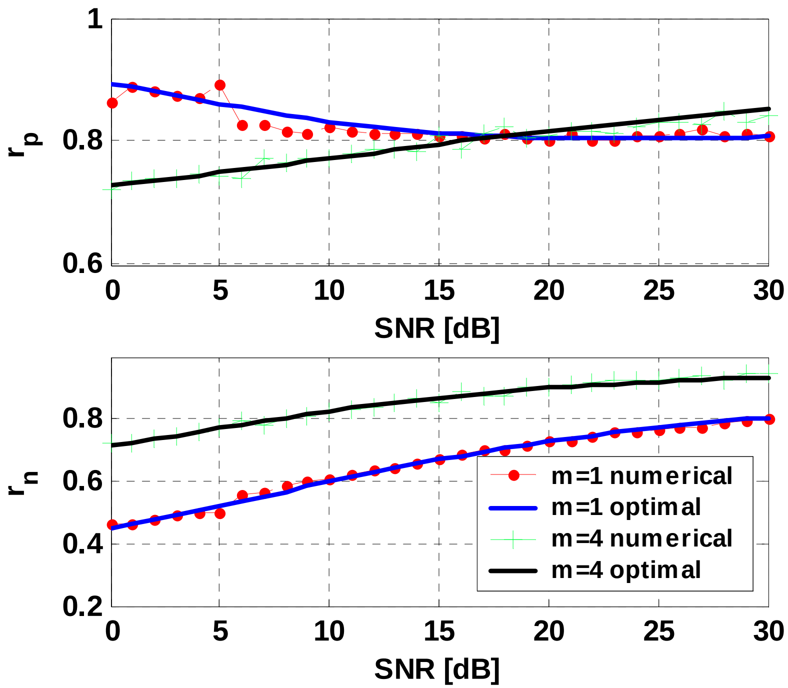
Optimal Combining of Inter- and Intra-Packet Coding – Solution 2.

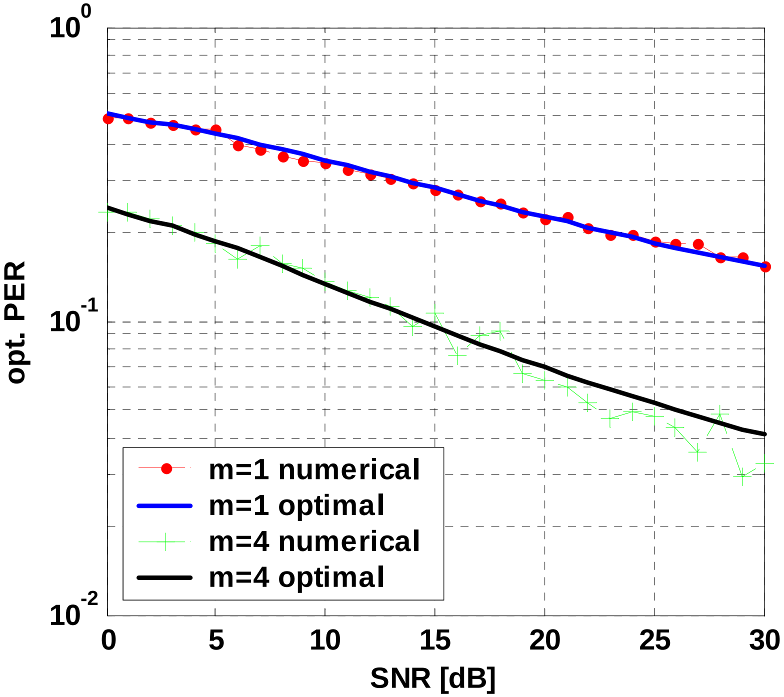
- Minimizing efficiency under performance constraint
  - Constraint and objective exchanged: dual problem
  - Intersect with performance constraint instead
- Numerical Example
  - Plot looks concave with global maximum for all SNR

$$P_{\text{outage}} = 10^{-2} \qquad \frac{\rho N}{C N_s} = 2^8$$



Optimal Combining of Inter- and Intra-Packet Coding – Numerical Example (cont.)





- Optimal rates for Rayleigh and Nakagami-4 fading channel
- Rayleigh has distinctly different behavior

- Comparison of packet error rates
- Rayleigh PER is above 10^{-1} while Nakagami-4 is much lower

Rate Optimization in a Special Case

- Consider an infinite data stream
  - Outage probability goes to zero

$$\lim_{N_{\text{data}}\to\infty} Q\left(\sqrt{\frac{\rho N_{\text{data}}}{N_s C}}\frac{q-\tilde{r}_n}{\sqrt{\tilde{r}_n r_p q(1-q)}}\right) = 0$$

  - If erasure coding rate is below success $r_n < q$

- This leads to a simpler optimization problem
  - Outage problem is zero
  - Erasure coding replaces lost packages

$$\max \eta = \max_{r_p} \; r_p q$$

Rate Optimization in a Special Case Result I

- On a Rayleigh fading channel, the physical rate maximizing $\eta = r_p q$ is given by:

$$r_p = \frac{W(\gamma)}{\ln(2)C}$$

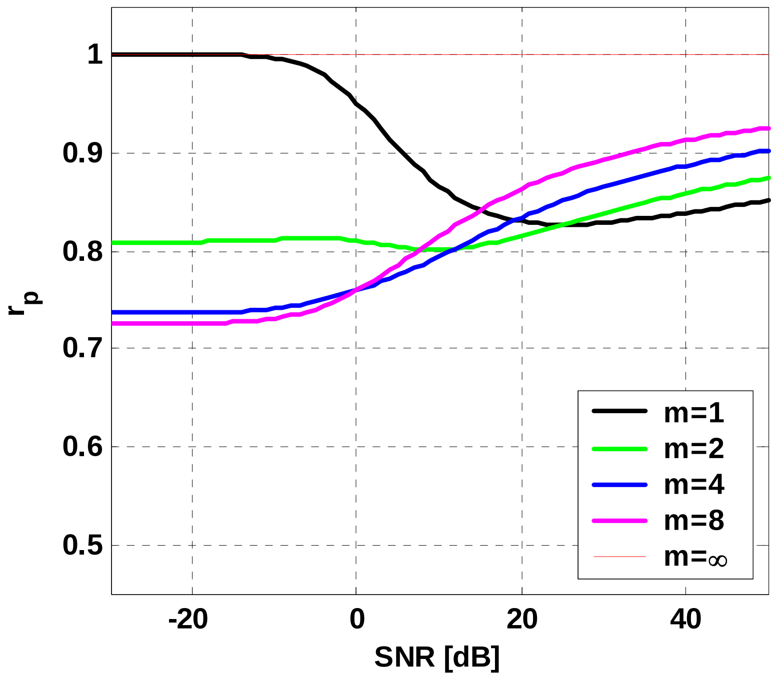
With the Lambert-w function $W(\gamma)$

- This leads to a network rate as:

$$\tilde{r}_n = \exp\left[-\frac{1}{W(\gamma)} + \frac{1}{\gamma}\right]$$

- At high SNR, we have:

$$\lim_{\gamma\to\infty} r_p = 1, \qquad \lim_{\gamma\to\infty} \tilde{r}_n = 1$$

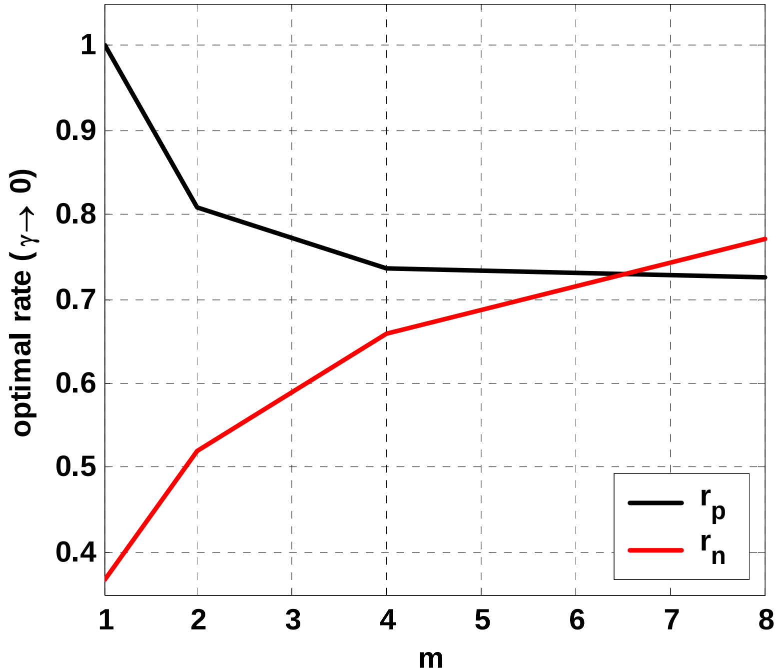


- Using Results I and numerical optimization we plot r_p
- Rayleigh shows distinctly different behavior for $m > 2$

Rate Optimization in a Special Case
Result II

- For general Nakagami-m fading channels, the optimal rates maximizing $\eta = r_p q$ at vanishing SNR are constant

	r_p	r_n
$m = 1$	1	$e^{-1} \approx 0.368$
$m = 2$	0.809	0.519
$m = 3$	0.757	0.660
$m = 4$	0.736	0.729



Conclusion

- Layered coding approach leads to practical and efficient transmission scheme
- A well-defined tradeoff exists, optimally allocating resources to both coding levels
  - On severe fading channels, tendency is to use more erasure coding
  - Investing in physical layer coding has worse payoff
- For infinite data streams, closed form solutions show specific behavior for severe fading channels