Connection between radiation resistances of antenna in rectangular waveguide and in free-space

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Abstract

In this paper, we discuss the connection between radiation resistance formulas for antenna in free-space and in rectangular waveguide. We demonstrate with the example of monopole antenna that a transformation between the angular direction in free-space and mode indices in waveguide makes the formulas for antenna radiation resistance in free-space and in rectangular waveguide equivalent to each other. To the best of our knowledge, this equivalence is shown for the first time.

1 Introduction

Radiation resistance of antennas radiating into free-space has been studied by many researchers [1]. In free-space, expressions can be derived for the contribution to the radiation resistance from the radiation into certain solid angle.

Antennas coupled into a microwave guide (usually operated in one- or two-mode regime) have also been extensively studied [2]. In waveguides, the only direction of field propagation is along the waveguide. An antenna can be characterized by the waveguide mode distribution that it excites and radiation resistance of such antennas can be derived as a function of mode indices.

In free-space any radiation pattern can be approximated with an infinite sum of waves radiated in different directions just as in waveguides the propagating field can always be decomposed into a sum of normal modes. As the waveguide cross-section becomes large, the number of modes increases, approaching a continuous spectrum of free-space. Waveguide propagation in waveguides can be treated using both ray and mode theories [3] and equivalence of the fields derived from both approaches has been previously shown for the parallel plate waveguide [4].

In this paper, we demonstrate that a transformation between the angle and the mode index converts the formula for radiation resistance of antenna in free-space into the formula for radiation resistance in a multimode rectangular waveguide. These two formulas are derived independently from first principles, and, to the best of our knowledge, this connection between them is shown for the first time.

As a demonstration example, we use a monopole antenna over the ground plane in free-space and in multimode rectangular waveguide. The application that stimulated our interest in this topic is study of antenna behavior in HVAC ducts [5].

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2 Monopole antenna

One of the simplest antennas is a monopole probe. Figure 1 shows a monopole antenna positioned above the ground plane in free-space and coupled into a rectangular waveguide.

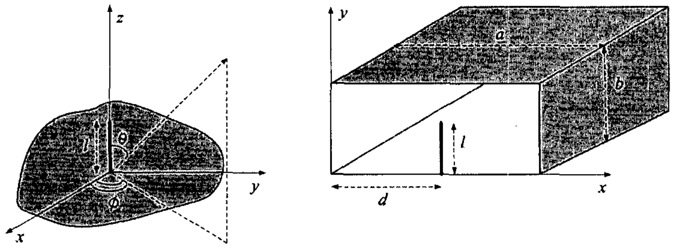

Figure 1: Monopole antenna in free-space and in rectangular waveguide.

2.1 Radiation resistance in free-space

The characteristics of monopole antenna above ground plane in free-space are well known [1]. The contribution to the radiation resistance resulting from the power radiated into the angular direction (θ, ϕ) is:

$$dR(\theta,\phi) = \eta \frac{1}{4\pi^2 \sin^2 kl} \frac{\left[\cos\left(kl\cos\theta\right) - \cos kl\right]^2}{\sin\theta}\, d\theta\, d\phi\,, \tag{1}$$

where l is antenna length, η is the free-space impedance, and $k = \omega/c$ is the wave number.

2.2 Radiation resistance in rectangular waveguide

Coupling of a monopole antenna into a rectangular waveguide has also been well studied [6]. Radiation resistance due to the power radiated into the mode (n, m) can be found to be:

$$R_{nm} = 4\,\frac{p_{nm}\left|C_{nm}\right|^2}{I_o^2}\,, \tag{2}$$

where p_{nm} is the normalized power carried by mode (n, m) in one direction (see [7]) and C_{nm} are mode amplitudes excited by the transmitting antenna that can be expressed as [8]:

$$C_{nm} = -\frac{I_o}{4p_{nm}} \int_l \vec{\varepsilon}_{nm} \cdot \vec{j}\, dl\,, \tag{3}$$

where $\vec{\varepsilon}_{nm}$ is the normalized electric field of mode (n, m) along the antenna, $\vec{j}$ is the normalized surface current density distribution, and the integral is taken along the antenna length. Typically, a sinusoidal current distribution can be assumed on such an antenna [9]. This allows one to calculate excited mode amplitudes and use them in Equation 2.

We derived that the mode radiation resistance in this case is

$$R_{nm} = \eta \frac{\pi^2 \sin^2 \frac{n\pi d}{a}}{2\,\beta\, a\, b\, g^2 \sin^2 kl} \frac{\left[\cos \frac{m\pi l}{b} - \cos kl\right]^2}{1 - \left(\frac{m\pi}{kb}\right)^2} \times \begin{cases} \frac{kn^2}{a^2} & \text{for } TE_{nm} \text{ mode} \\ \frac{\beta m^2}{b^2} & \text{for } TM_{nm} \text{ mode} \end{cases}, \quad (4)$$

where $\beta = \sqrt{k^2 - g^2}$ is the waveguide wave number of the mode and $g = \sqrt{(\frac{n\pi}{a})^2 + (\frac{m\pi}{b})^2}$ is the cutoff wave number. In the special case of a probe completely spanning a rectangular waveguide, our Equation 4 can be reduced to the one published in [2].

3 The transformation and the connection

Consider the following transformation, independent of antenna type:

$$\cos\theta = \frac{m\pi}{kb}. \quad (5)$$

It follows from the known transformation [10] for the angle of incidence ψ of the mode wavefront normal in a rectangular waveguide ($\sin\psi = 1 - (\lambda/\lambda_g)^2$, where $\lambda_g = 2\pi/g$) if $\phi = 0$. Angles θ and ψ are shown in Figure 2. The differential $d\theta$ can be expressed as

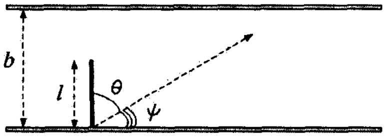

Figure 2: Angles θ and ψ for the mode wavefront normal in a rectangular waveguide.

$$d\theta = \frac{\pi}{kb} \frac{dm}{\sqrt{1 - \left(\frac{m\pi}{kb}\right)^2}}. \quad (6)$$

Let us integrate Equation 1 over ϕ (to retain only dependence on θ) and substitute expressions from Equations 5 and 6 into Equation 1 for free-space. We obtain:

$$dR = \frac{\eta}{2\,k\,b\,\sin^2 kl} \frac{\left[\cos \frac{m\pi l}{b} - \cos kl\right]^2}{1 - \left(\frac{m\pi}{kb}\right)^2} \frac{dm}{\sqrt{1 - \left(\frac{m\pi}{kb}\right)^2}}. \quad (7)$$

On another hand, summing contributions to TE and TM modes in waveguide Equation 4, taking into account the discreteness of mode indices ($dn = dm = 1$), setting $d = a/2$ (only modes with odd n will survive) and making a=b (waveguide is square), we can obtain:

$$dR = \frac{\eta}{2\,k\,b\,\sin^2 kl} \frac{\left[\cos \frac{m\pi l}{b} - \cos kl\right]^2}{1 - \left(\frac{m\pi}{kb}\right)^2} \frac{m^2 + n^2 \frac{k}{\beta}}{m^2 + n^2} \frac{dm}{\sqrt{1 - \left(\frac{m\pi}{kb}\right)^2 - \left(\frac{n\pi}{kb}\right)^2}}. \quad (8)$$

If we now take the limit $kb \rightarrow \infty$ (waveguide dimensions become very large compared to wavelength) then $\beta \rightarrow k$ and formulas given by Equations 7 and 8 become equivalent.

4 Discussion

For illustrative purpose, we considered only a monopole antenna. The transformation we used holds for other types of antennas as well but is limited to rectangular waveguides. One can expect that a similar transformation may exist for cylindrical waveguides, although that case is more complicated. A possible application of the demonstrated connection is the automatic derivation of the radiation resistance for any antenna in a rectangular waveguide if an analytical formula for the radiation resistance of this antenna in free-space is known.

5 Conclusion

In this paper, we for the first time discussed the theoretical equivalence between the radiation resistance of antenna in free-space and in rectangular waveguide. We demonstrated with the example of monopole antenna that a transformation between the angular direction and mode indices serves as a bridge between the formulas for antenna radiation resistance in free-space and in rectangular waveguide.

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