Real-Time Estimation
of the Parameters and Fluxes
of Induction Motors

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OF THE PARAMETERS AND FLUXES
OF INDUCTION MOTORS

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Contents

Acknowledgements 1

1 Introduction 2

2 Induction Motor Model 3

2.1 Standard Model Of the Induction Motor in the Rotor Coordinate Frame 3

2.2 An Alternate Model Useful for Identification 4

2.2.1 Development of the Model 5

2.2.2 Validity of the Model 7

3 Least-Squares Identification 8

3.1 Least-Squares Identification - Basic Algorithm 8

3.2 Error Estimates 10

3.2.1 Residual Error Index 10

3.2.2 Parametric Error Indices 11

4 Identification Scheme for the Induction Motor 14

4.1 Identification of the Electrical Parameters 14

4.2 Flux Reconstruction 14

4.3 Identification of the Mechanical Parameters 15

4.4 Numerical Issues 16

4.4.1 Stator Resistance Measured 17

4.4.2 Iterative Algorithm 17
5 Results

5.1 Implementation Issues ................................................. 19

5.2 Simulation Results .................................................. 20

  5.2.1 Recursive Implementation of the Complete Three Stage Identification Scheme
         with Measured Stator Resistance ........................................ 21

  5.2.2 Batch Implementation of the Identification of the Electrical Parameters with
         Iterative Algorithm .................................................... 24

5.3 Experimental Results .................................................. 24

  5.3.1 Recursive Implementation of the Complete Three Stage Identification Scheme
         with Measured Stator Resistance ........................................ 26

  5.3.2 Batch Implementation of the Identification of the Electrical Parameters with
         Iterative Algorithm .................................................... 28

6 Conclusions ........................................................................ 28

7 References ........................................................................ 31

A List of Symbols .................................................................. 32

B Derivation of Induction Motor Model: Equations (1)-(5) .......... 32

C Proof of Equation (45) ..................................................... 34

List of Tables

  1 Summary of Simulation Results - $R_s$ Measured ......................... 23

  2 Summary of Simulation Results - Iterative Algorithm .................. 25
List of Figures

1 Flux Vector $\psi$ Shown in (a,b) and (x,y) Coordinate Frames .......................... 3
2 Estimate vs. Residual Error Index (y,k scalar) ........................................... 12
3 Set of $\delta K$'s Satisfying Equation (44) .................................................. 13
4 Speed (rad/sec) ......................................................................................... 21
5 Parameter $K_5$ and Estimate with Uncertainty vs. Time (sec) ......................... 22
6 Rotor Flux ($\psi_{\varphi}$) and Estimate vs. Time (sec) ........................................ 22
7 Parameter $K_7$ and Estimate with Uncertainty vs. Time (sec) ......................... 23
8 Speed (rad/sec) ......................................................................................... 24
9 Parameter $K_4$ Estimate with Uncertainty vs. Time (sec) ............................ 27
10 Rotor Flux ($\psi_{\varphi}$) Estimate vs. Time (sec) ................................................ 27
11 Parameter $K_7$ Estimate with Uncertainty vs. Time (sec) ............................ 28
Abstract

This thesis presents a new method for the real-time estimation of the parameters and fluxes of induction motors. Such a procedure is useful for the design of self-commissioning drives, i.e., drives that can adjust controller parameters automatically for a wide range of motors and loads. Another potential application is for the diagnostic of failures. In their recursive form, the algorithms can be used for adaptation to parameters that vary with time, and for estimation of the rotor fluxes in a field-oriented controlled drive.

The estimation method is based on a standard model of the induction motor, expressed in rotor coordinates. It is assumed that current and position (or velocity) measurements are available. The rotor fluxes are not assumed to be measured. The interesting features of the method are that: 1) it does not rely on special tests such as the locked rotor test or the no-load test (instead, a broad range of motor responses can be used), 2) the method provides estimates of the rotor fluxes together with the estimates of the parameters, 3) measures of the uncertainties in the estimated motor parameters are provided (this provides feedback as to the precision of the parameter estimates, as well as some guidance to choose excitation and to optimize the quality of the parameter estimation). Results for both simulated and experimental data are provided.
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1 Introduction

Methods for the real-time estimation of electric machine parameters are vital for the design of intelligent AC drives. Standard methods for the estimation of induction motor parameters include the locked rotor test, the no-load test, and the standstill frequency response test. In [4], an automatic procedure is described in which a sequence of such tests is performed, each designed to isolate and measure a specific parameter. The method is applied for the automatic tuning (self-commissioning) of an induction motor drive. In [3], another procedure is described, based primarily on the identification of the motor transfer function at standstill. The model is then refined to account for magnetic saturation and adaptation is included to compensate for the effects of heating.

In this thesis, a new method for the estimation of induction motor parameters is investigated. The objectives are similar to those of [3] and [4]. However, the emphasis here is on a method that is fast, efficient, and does not require a special configuration of the machine. In addition, the method is suitable for continuous updating of the parameters in the regular operation of the machine, so that tracking of parameter variations is possible. The approach is close in nature to that discussed in [10]. In contrast with [10], we do not consider the velocity estimation problem, but rather simplify the problem by assuming that the velocity is known. On the other hand, the velocity is allowed to vary and the thesis shows how estimates of the rotor fluxes can be simultaneously constructed (for possible use in a field-oriented control drive, see [6] and [7]). Additionally, the method developed in this thesis provides measures of the errors affecting the parameter estimates. Such measures are useful for the monitoring of the estimation procedure itself.

The thesis is organized as follows. Section 2 introduces a standard induction motor model expressed in the rotor coordinates. Then, a simplified model which is linear in the unknown parameters is derived and discussed. Section 3 reviews some basic theory of least-squares identification and details two measures of judging the performance of the algorithm: a residual error index and parametric error indices. Section 4 presents the identification scheme for the induction motor. Specifically, least-squares identification is applied for estimation of the electrical and mechanical motor parameters. The method for reconstruction of the fluxes is also detailed. Additionally, two variations on the scheme are presented, both of which address numerical problems arising because of overparameterization. Section 5 discusses some implementation issues and presents the results of the identification algorithm with simulated data and with experimental data from a small motor.
2 Induction Motor Model

2.1 Standard Model Of the Induction Motor in the Rotor Coordinate Frame

Standard models of induction machines are available in the literature. See [6] for example, where a model suitable for control applications is discussed. Parasitic effects such as hysteresis, eddy currents, magnetic saturation, and others are generally neglected. The algorithm presented here is based on the model in [6], but is expressed in a coordinate frame rotating with the rotor. The current variables are transformed according to:

\[
\begin{pmatrix}
    i_{sx} \\
    i_{sy}
\end{pmatrix} =
\begin{pmatrix}
    
    \cos(n_p\theta) & \sin(n_p\theta) \\
    \cos(n_p\theta) & \cos(n_p\theta)
\end{pmatrix}
\begin{pmatrix}
    i_{sa} \\
    i_{sb}
\end{pmatrix}
\]

where \(\theta\) is the position of the rotor, \(n_p\) is the number of pole pairs, and \(i_{sa}, i_{sb}\) are the stator currents of a 2-phase machine (or equivalent). The transformation simply projects the vectors in the \((a,b)\) frame on the axes of the moving coordinate frame. Figure 1 illustrates this transformation for a one pole pair machine. An advantage of this transformation is that the signals in the moving frame (i.e., the \((x,y)\) frame) typically vary slower than those in the \((a,b)\) frame (at the slip frequency instead of stator frequency). At the same time, the transformation does not depend on any unknown
parameter. The stator voltages and the rotor fluxes are transformed similarly

\[
\begin{pmatrix}
    v_{sx} \\
    v_{sy}
\end{pmatrix}
=
\begin{pmatrix}
    \cos(n_p\theta) & \sin(n_p\theta) \\
    -\sin(n_p\theta) & \cos(n_p\theta)
\end{pmatrix}
\begin{pmatrix}
    v_{sa} \\
    v_{sb}
\end{pmatrix}
\]

\[
\begin{pmatrix}
    \psi_{rx} \\
    \psi_{ry}
\end{pmatrix}
=
\begin{pmatrix}
    \cos(n_p\theta) & \sin(n_p\theta) \\
    -\sin(n_p\theta) & \cos(n_p\theta)
\end{pmatrix}
\begin{pmatrix}
    \psi_{ra} \\
    \psi_{rb}
\end{pmatrix}
\]

and the following model can be obtained from the model in [6] (see appendix B for the derivation):

\[
\frac{di_{sx}}{dt} = \frac{1}{\sigma L_s} v_{sx} - \gamma i_{sx} + \frac{\beta}{T_r} \psi_{rx} + n_p \beta \omega \psi_{ry} + n_p \omega i_{sy}
\]

\[
\frac{di_{sy}}{dt} = \frac{1}{\sigma L_s} v_{sy} - \gamma i_{sy} + \frac{\beta}{T_r} \psi_{ry} - n_p \beta \omega \psi_{rx} - n_p \omega i_{sx}
\]

\[
\frac{d\psi_{rx}}{dt} = \frac{M}{T_r} i_{sx} - \frac{1}{T_r} \psi_{rx}
\]

\[
\frac{d\psi_{ry}}{dt} = \frac{M}{T_r} i_{sy} - \frac{1}{T_r} \psi_{ry}
\]

\[
\frac{d\omega}{dt} = \frac{2M n_{ph}}{J L_r n_{ph}} (i_{sy} \psi_{rx} - i_{sx} \psi_{ry}) - \frac{\tau_L}{J}
\]

In the above model, the angular speed of the rotor is denoted \( \omega \) and \( n_{ph} \) is the number of phases. The (unknown) parameters of the model are the five electrical parameters, \( R_s \) and \( R_r \) (the stator and rotor resistances), \( M \) (the mutual inductance), \( L_s \) and \( L_r \) (the stator and rotor inductances), and the two mechanical parameters, \( J \) (the inertia of the rotor) and \( \tau_L \) (the load torque). The symbols:

\[
T_r = \frac{L_r}{R_r} \quad \sigma = 1 - \frac{M^2}{L_s L_r} \quad \beta = \frac{M}{\sigma L_s L_r} \quad \gamma = \frac{R_s}{\sigma L_s} + \frac{M^2 R_r}{\sigma L_s L_r^2}
\]

have been used to simplify the expressions. \( T_r \) represents the rotor time constant and \( \sigma \) is the total leakage factor. The load torque is assumed constant but the techniques presented in this thesis can be extended to a torque proportional to speed for example.

### 2.2 An Alternate Model Useful for Identification

Measurements of the currents and of the position of the rotor are assumed to be available. Velocity may then be reconstructed from position measurements. The drive is assumed to be controlled through a voltage source inverter, and the commanded voltages are used by the estimation procedure. However, the rotor fluxes are not assumed to be measured.
Standard methods for parameter estimation are based on equalities where known signals depend linearly on unknown parameters. Unfortunately, the induction motor model described above does not fit in this category unless the rotor fluxes are known. To circumvent this problem, an alternate model of the induction machine is introduced. The model is linear in the unknown parameters and involves only measurable or reconstructible signals (i.e., not the rotor fluxes). Therefore, this model allows direct application of the standard methods for parameter estimation.

2.2.1 Development of the Model

To arrive at the simplified model, we begin by rewriting equations (1) through (4) as follows.

\[
\frac{di_{sx}}{dt} + \gamma i_{sx} - \frac{\beta}{T_r} \psi_{rx} - n_p \beta \omega \psi_{ry} - n_p \omega i_{sy} = \frac{1}{\sigma L_s} v_{sx} \tag{6}
\]

\[
\frac{di_{sy}}{dt} + \gamma i_{sy} - \frac{\beta}{T_r} \psi_{ry} + n_p \beta \omega \psi_{rx} + n_p \omega i_{sx} = \frac{1}{\sigma L_s} v_{sy} \tag{7}
\]

\[
\frac{d\psi_{rx}}{dt} = \frac{M}{T_r} i_{sx} - \frac{1}{T_r} \psi_{rx} \tag{8}
\]

\[
\frac{d\psi_{ry}}{dt} = \frac{M}{T_r} i_{sy} - \frac{1}{T_r} \psi_{ry} \tag{9}
\]

Then we take the first derivatives of equations (6) and (7) so that

\[
\frac{1}{\sigma L_s} \frac{dv_{sx}}{dt} = \frac{d^2 i_{sx}}{dt^2} + \gamma \frac{di_{sx}}{dt} - \frac{\beta}{T_r} \frac{d\psi_{rx}}{dt} - n_p \beta \omega \frac{d\psi_{ry}}{dt} - n_p \omega \frac{di_{sy}}{dt} - n_p i_{sy} \frac{dw}{dt} \tag{10}
\]

\[
\frac{1}{\sigma L_s} \frac{dv_{sy}}{dt} = \frac{d^2 i_{sy}}{dt^2} + \gamma \frac{di_{sy}}{dt} - \frac{\beta}{T_r} \frac{d\psi_{ry}}{dt} + n_p \beta \omega \frac{d\psi_{rx}}{dt} + n_p \beta \psi_{rx} \frac{dw}{dt} + n_p \omega \frac{di_{sx}}{dt} + n_p i_{sx} \frac{dw}{dt} \tag{11}
\]

If we sum equation (10) multiplied by \(\frac{1}{T_r}\) and equation (6), and sum equation (11) multiplied by \(\frac{1}{T_r}\) and (7), we can eliminate the flux derivatives using equations (8) and (9). Performing these operations yields:

\[
\frac{1}{\sigma L_s} \frac{dv_{sx}}{dt} + \frac{1}{T_r \sigma L_s} v_{sx} = \frac{d^2 i_{sx}}{dt^2} + \left(\gamma + \frac{1}{T_r}\right) \frac{di_{sx}}{dt} + \left(\frac{\gamma}{T_r} - \frac{\beta M}{T_r^2}\right) i_{sx} - n_p \omega \left(\frac{1}{T_r} + \frac{\beta M}{T_r}\right) i_{sy} - n_p (i_{sy} + \beta \psi_{ry}) \frac{dw}{dt} - n_p \omega \frac{di_{sy}}{dt} \tag{12}
\]
We now have a model in a form where most of the flux variables have been cancelled. However, there still remain two terms which involve $\psi_{rx}$ and $\psi_{ry}$. For both of these terms, the fluxes are multiplied by the acceleration of the motor, $\frac{d\omega}{dt}$. If $\frac{d\omega}{dt} \approx 0$, these terms can be neglected. Then, the model will be:

\[
\begin{align*}
\frac{d^2i_{sx}}{dt^2} &+ K_1 \frac{di_{sx}}{dt} + K_2 i_{sx} - n_p \omega K_3 i_{sy} - n_p \omega \frac{di_{sy}}{dt} = K_4 \frac{dv_{sx}}{dt} + K_5 v_{sx} \\
\frac{d^2i_{sy}}{dt^2} &+ K_1 \frac{di_{sy}}{dt} + K_2 i_{sy} + n_p \omega K_3 i_{sx} + n_p \omega \frac{di_{sx}}{dt} = K_4 \frac{dv_{sy}}{dt} + K_5 v_{sy}
\end{align*}
\]

where a new set of parameters is defined

\[
K_1 = \gamma + \frac{1}{T_r} \quad K_2 = \gamma (\frac{T_r}{T}) - \frac{\beta M}{T_r^2} \quad K_3 = \frac{1}{T_r} + \frac{\beta M}{T_r} \quad K_4 = \frac{1}{\sigma L_s} \quad K_5 = \frac{1}{\sigma L_s T_r}
\]

Note that a nonlinear relationship can be found to exist between these five parameters. Specifically,

\[
K_1 = \frac{K_2 K_4}{K_5} + K_3
\]

In terms of the parameters $R_s$, $L_s$, $\sigma$, and $T_r$,

\[
K_1 = \frac{R_s}{\sigma L_s} + \frac{1}{\sigma T_r} \quad K_2 = \frac{R_s}{\sigma L_s T_r} \quad K_3 = \frac{1}{\sigma T_r} \quad K_4 = \frac{1}{\sigma L_s} \quad K_5 = \frac{1}{\sigma L_s T_r}
\]

Not all five electrical parameters ($R_s$, $L_s$, $R_r$, $L_r$ and $M$) can be retrieved from the K's. The five parameters $K_1$ - $K_5$ determine only the four independent parameters $R_s$, $L_s$, $\sigma$ and $T_r$. Specifically,

\[
R_s = \frac{K_2}{K_5} \quad L_s = \frac{K_3}{K_5} \quad \sigma = \frac{K_5}{K_3 K_4} \quad T_r = \frac{K_4}{K_5}
\]

Since $T_r = \frac{L_r}{R_r}$ and $\sigma = 1 - \frac{M^2}{L_r L_s}$, only $\frac{L_r}{R_r}$ and $\frac{M^2}{L_r}$ can be obtained and not $M$, $L_r$ and $R_r$ independently. This situation is inherent to the identification problem with unknown rotor fluxes and is not a problem with our method. If rotor fluxes are not measured, machines with different $R_r$, $L_r$
and $M$ but identical $\frac{L_x}{R_x}$ and $\frac{M^2}{L_r}$ will have the same input/output (voltage/current) characteristics. However, machines with different $K$ parameters (yet satisfying the nonlinear relationship (15)) will be distinguishable. For further discussion of this question, see [1], where parameter identification is performed using torque-speed and stator current-speed characteristics. The induction motor model is parametrized differently, but the result is the same.

The model may also be expressed in matrix form as is shown below.

\[
\begin{pmatrix}
-\frac{d}{dt}i_{sx} & -i_{sx} & n_p\omega i_{sy} & \frac{dv_{sx}}{dt} & v_{sx} \\
-\frac{di_{sx}}{dt} & -i_{sy} & -n_p\omega i_{sx} & \frac{dv_{sx}}{dt} & v_{sy} \\
\end{pmatrix}
\begin{pmatrix}
K_1 \\
K_2 \\
K_3 \\
K_4 \\
K_5 \\
\end{pmatrix}
= \begin{pmatrix}
\frac{d^2 i_{sx}}{dt^2} - n_p\omega \frac{di_{sx}}{dt} \\
\frac{di_{sx}}{dt} + \frac{d^2 i_{sy}}{dt^2} \\
\end{pmatrix}
\]

Equation (18) is linear in the parameters $K_1$, $K_2$, $K_3$, $K_4$ and $K_5$ and does not involve the unknown rotor flux signals. This linear form of the motor model enables direct application of a least-squares identification algorithm to estimate the electrical parameters. Note that the need to have flux measurements has been avoided by the assumption that the speed of the motor varies slowly. This is an advantage because flux measurements, which require sensors close to the airgap, are impractical to obtain. A drawback is that derivatives of the currents are required. These may be reconstructed by filtered differentiation, see section 5.1. The standard approach of adaptive control using state-variable filters (see Landau, [5]) may also be used.

### 2.2.2 Validity of the Model

One possible use of the model would be to operate the motor at various but constant speed. For any nonzero speed, equation (18) gives two equations, so that with (15), at least two different speed measurements would be required to determine all five parameters. Since it is inconvenient to restrict operation to constant speed for parameter estimation, a natural issue to address is the question of when the motor speed can be considered to be varying slowly. In other words, is there a general guideline to determine when the simplified model adequately represents the motor dynamics? To obtain such a result, we return to equation (10) and rewrite it as follows:

\[
-n_p\beta \frac{d\psi_{ry}}{dt} - n_p\beta \frac{d\omega}{dt} - n_p i_{sy} \frac{d\omega}{dt} = \frac{1}{\sigma L_s} \frac{dv_{sx}}{dt} - \frac{d^2 i_{sx}}{dt^2} - \gamma \frac{di_{sx}}{dt} + \frac{\beta}{T_r} \frac{d\psi_{rx}}{dt} - n_p \frac{di_{sy}}{dt}
\]

(19)
Next, we expand the left-hand side of the equation by using equation (9) to eliminate \( \frac{d\psi_{rx}}{dt} \) as shown below.

\[
-n_p \beta \omega \frac{d\psi_{ry}}{dt} - n_p \beta \psi_{ry} \frac{d\omega}{dt} - n_p i_{sy} \frac{d\omega}{dt} = -n_p \beta \omega \left( \frac{M}{T_r} i_{sy} - \frac{1}{T_r} \psi_{ry} \right) - n_p \beta \psi_{ry} \frac{d\omega}{dt} - n_p i_{sy} \frac{d\omega}{dt}
\]

\[
= -n_p \beta \psi_{ry} \left( \frac{d\omega}{dt} - \frac{\omega}{T_r} \right) - n_p i_{sy} \left( \frac{d\omega}{dt} + \beta M \frac{\omega}{T_r} \right)
\]

We note that if \( \frac{d\omega}{dt} \ll \frac{\omega}{T_r} \) and \( \frac{d\omega}{dt} \ll \frac{M \beta}{T_r} \), then the \( \frac{d\omega}{dt} \) terms will be small with respect to existing terms in the differential equations. This gives a rule of thumb as to when \( \omega \) can be considered to be varying slowly and the model is an adequate representation of the machine's dynamics. The two conditions may be collapsed to only one because \( \beta M = \frac{1}{\omega} - 1 \geq 1 \). Thus, \( \frac{\beta M}{T_r} \omega \geq \frac{1}{T_r} \omega \) and the condition is simply that \( \frac{d\omega}{dt} \ll \frac{1}{T_r} \) \( \omega \).

Note finally that the terms \(-n_p i_{sy} \frac{d\omega}{dt}\) and \(n_p i_{sx} \frac{d\omega}{dt}\) could be included in (18) (as in equation (22)), since \( \omega, i_{sx} \) and \( i_{sy} \) are measured. This would reduce the error between the exact and approximate models. However, since these terms would be negligible under the same conditions as the flux terms, they were similarly dropped.

\[
\begin{pmatrix}
-\frac{di_{sx}}{dt} & -i_{sx} & n_p \omega i_{sy} & \frac{di_{sx}}{dt} & v_{sx} \\
-\frac{di_{sy}}{dt} & -i_{sy} & -n_p \omega i_{sx} & \frac{di_{sy}}{dt} & v_{sy}
\end{pmatrix}
\begin{pmatrix}
K_1 \\
K_2 \\
K_3 \\
K_4 \\
K_5
\end{pmatrix} =
\begin{pmatrix}
\frac{d^2 i_{sx}}{dt^2} - n_p \omega \frac{di_{sx}}{dt} - n_p i_{sy} \frac{d\omega}{dt} \\
\frac{d^2 i_{sy}}{dt^2} + \frac{d^2 i_{sy}}{dt^2} + n_p i_{sx} \frac{d\omega}{dt}
\end{pmatrix}
\]

3 Least-Squares Identification

3.1 Least-Squares Identification - Basic Algorithm

Equation (18) can be rewritten as:

\[
w^T(n)K_N = y(n)
\]

where \( n \) is the time instant at which a measurement is taken and \( K_N \) is the vector of unknown parameters. An exact unique solution for the unknown parameter vector, \( K_N \), may be determined
after several time instants. However, several factors contribute to errors which make equation (23) only approximately satisfied in practice. In modelling the motor, the terms involving $\frac{d\omega}{dt}$ have been neglected. Additionally, both $y(n)$ and $w(n)$ are measured through signals that are noisy (in particular due to quantization and differentiation). These sources of error result in an inconsistent system of equations. To find a solution for such a system, the least-squares algorithm is used. Specifically, given $y(n)$ and $w(n)$ where $y(n) = w^T(n)K$, we define

$$Re(K) = \sum_{n=1}^{N} \left| y(n) - w^T(n)K \right|^2$$

as the residual error associated to a vector $K$. Then, $K^*$ is chosen such that $Re(K)$ is minimized for $K = K^*$. The function $Re(K)$ is quadratic and therefore has a unique minimum at the point where $\frac{\partial Re(K)}{\partial K} = 0$. Solving this expression for $K^*$ yields the least-squares solution to $y(n) = w^T(n)K$.

$$\frac{\partial}{\partial K} \left[ \sum_{n=1}^{N} \left| y(n) - w^T(n)K \right|^2 \right] = 0$$

$$\sum_{n=1}^{N} \frac{\partial}{\partial K} \left[ (y(n) - w^T(n)K)^T(y(n) - w^T(n)K) \right] = 0$$

$$\sum_{n=1}^{N} \left[ 2w(n)(y(n) - w^T(n)K) \right] = 0$$

and therefore

$$K^* = \left[ \sum_{n=1}^{N} w(n)w^T(n) \right]^{-1} \left[ \sum_{n=1}^{N} w(n)y(n) \right]$$

is the least-squares estimate of the vector of parameters.

Additionally, the solution may be easily implemented in a recursive manner to allow real-time calculations of $K^*$. Specifically, we rewrite equation (28) as:

$$K^*(N) = P^{-1}(N)R(N)$$
Then, $P(N)$ and $R(N)$ are determined at each time sample according to:

$$P(N) = P(N-1) + w(N)w^T(N)$$  \hspace{1cm} (30)$$

$$R(N) = R(N-1) + w(N)y(N)$$  \hspace{1cm} (31)$$

Two options are then possible. The standard recursive least-squares algorithm, [2], avoids the inversion of the matrix $P(N)$ by building a recursion for $P^{-1}(N)$ instead of $P(N)$. However, since the equation (18) has two components, the approach still requires the inverse of a (in this case $2 \times 2$) matrix. A pseudo-recursive (or pseudo-batch) approach consists in updating equations (30) and (31) at every time instant and equation (29) at a lower rate. This reduces the computational requirements and allows for continuous monitoring of the estimation procedure, as shown in the next section.

### 3.2 Error Estimates

Naturally, it is desirable to have a meaningful way to evaluate the confidence in the identification scheme. More specifically, one would want to know how well $w^T(n)K^*$ matches the data $y(n)$ and also how sensitive the error is with respect to $K$, the parameters. To treat these issues, a residual error index and parametric error indices are defined. In judging the performance of the algorithm presented in this thesis, the two measures have been used extensively.

Note that the measures of uncertainties developed hereafter do not rely on typical statistical measures used in system identification (of Soderstrom and Stoica). Because of the nonlinearity of the model, the effect of measurement noise cannot be considered to be an additive term and uncorrelated with the signals $w(n)$ and $y(n)$. For that reason, we will rely on different measures on uncertainties derived hereafter.

#### 3.2.1 Residual Error Index

To address the first question raised above, we first use the following abbreviations:

$$R_w = \sum_{n=1}^{N} w(n)w^T(n)$$

$$R_{wy} = \sum_{n=1}^{N} w(n)y(n)$$

10
So that, \( K^* = R_w^{-1} R_{wy} \).

Next, to develop a measure of how well the data \( y(n) \) fits \( w^T(n)K^* \), we define the residual error at \( K^* \) as:

\[
Re^* = Re(K^*) = \sum_{n=1}^{N} | y(n) - w^T(n)K^* |^2
\]

\[
= \sum_{n=1}^{N} \left( y^T(n)y(n) - 2y^T(n)w^T(n)K^* + K^{*T}w(n)w^T(n)K^* \right)
\]

\[
= R_y - 2R_{wy}^T K^* + K^{*T} R_w K^*
\]  

where

\[
R_y = \sum_{n=1}^{N} y^T(n)y(n)
\]

and \( R_{wy} \) and \( R_w \) are as defined previously. Then, using the relationship \( K^* = R_w^{-1} R_{wy} \) results in:

\[
Re^* = R_y - 2R_{wy}^T R_w^{-1} R_{wy} + R_{wy}^T R_w^{-1} R_{wy} = R_y - R_{wy}^T R_w^{-1} R_{wy}
\]

Note that \( 0 \leq Re^* \leq R_y \). Therefore,

\[
E_I = \sqrt{\frac{Re^*}{R_y}}
\]

is zero when \( Re^* = 0 \) (or \( y(n) = w^T(n)K^* \)), and 1 when \( K^* = 0 \) (or \( Re^* = R_y \)). Therefore, the residual error index, \( E_I \), ranges from 0 to 1, where \( E_I = 0 \) indicates that \( y(n) \) fits the relationship \( w^T(n)K^* \) perfectly. The residual error index \( E_I \) is usually nonzero due to noise, unmodeled dynamics and nonlinearities. In the worse case, \( E_I = 1 \), which would mean that the residual error has a magnitude comparable to that of the measurement of \( y(n) \).

### 3.2.2 Parametric Error Indices

In addressing the issue of sensitivity of \( K^* \) to errors, we recall that
Therefore, it is not possible to use the derivative of the residual error as a measure of how sensitive the error is with respect to $K$. An alternative is to define $\delta K$ as the variation in $K$ such that the increase of error is equal to the residual error, $Re^*$ itself, see figure 2. That is, $\delta K$ tells us how much $K$ must vary to lead to an increase of error equal to the residual error. The result is detailed below.

Recall that:

\[ Re(K^*) = Re^* = R_y - 2R_{wy}^T K^* + K^* R_w K^* \]  \hspace{1cm} (39)

By letting $K^*$ vary by $\delta K$ we arrive at:

\[ Re(K^* + \delta K) = R_y - 2R_{wy}^T (K^* + \delta K) + (K^* + \delta K)^T R_w (K^* + \delta K) \]  \hspace{1cm} (40)

\[ = R_y - 2R_{wy}^T K^* + K^* R_w K^* - 2R_{wy}^T \delta K + 2K^* R_w \delta K + \delta K^T R_w \delta K \]  \hspace{1cm} (41)

\[ = Re^* + \delta K^T R_w \delta K \]  \hspace{1cm} (42)

The $\delta K$'s which satisfy the relationship
Figure 3: Set of $\delta K$'s Satisfying Equation (44)

\[ Re(K^* + \delta K) = 2Re^* \]  

(43)

satisfy

\[ \delta K^T R_w \delta K = Re^* \]  

(44)

When $\delta K \in \mathbb{R}^2$, this equation defines an ellipse, as illustrated in figure 3 (in general, an ellipsoid).

We will define the parametric error associated to the parameter $K_1$, $\delta K_1$, as the worst case error. It can then be shown that in general:

\[ \delta K_i = \sqrt{Re^*(R_w^{-1})_{ii}} \]  

(45)

The proof of equation (45) is detailed in appendix C.

Several comments may be made. $\delta K_i$ indicates the amount by which $K_i$, the $i$th component of $K$, would need to vary to double the residual error. In essence, the parametric error provides a measure of the uncertainty in the estimated motor parameter. A large parametric error indicates that the parameter estimate could vary greatly without a large change in the residual error. Thus, the accuracy of those parameter estimates with high parametric errors would be in doubt. Likewise,
small parametric errors indicate that the residual error index is very sensitive the changes in the parameter estimates. In these cases, the parameter estimates may be considered more accurate. In any case, the error indices should not be considered as actual errors, but rather as orders of magnitude of the errors to be expected, to guide the identification process and to warn about unreliable results.

4 Identification Scheme for the Induction Motor

4.1 Identification of the Electrical Parameters

As detailed in section 2.2.1, the induction motor dynamics may be described by the following model:

\[
\begin{pmatrix}
  -\frac{di_x}{dt} - i_x & n_p \omega i_y & \frac{dv_x}{dt} & v_x \\
  -\frac{di_y}{dt} - i_y & -n_p \omega i_x & \frac{dv_y}{dt} & v_y
\end{pmatrix}
\begin{pmatrix}
  K_1 \\
  K_2 \\
  K_3 \\
  K_4 \\
  K_5
\end{pmatrix}
= \begin{pmatrix}
  \frac{d^2i_x}{dt^2} - n_p \omega \frac{di_y}{dt} \\
  n_p \omega \frac{di_x}{dt} + \frac{d^2i_y}{dt^2}
\end{pmatrix}
\]

The equation is linear in the parameters \( K_1, K_2, K_3, K_4 \) and \( K_5 \) so that the application of the least-squares algorithm is straightforward. From the obtained estimates, the system parameters may be calculated easily using the following relationships.

\[
R_s = \frac{K_2}{K_5}, \quad L_s = \frac{K_3}{K_5}, \quad \sigma = \frac{K_5}{K_3 K_4}, \quad T_r = \frac{K_4}{K_5}
\] (46)

In theory, the estimated parameters \( K_1 - K_5 \) should satisfy the nonlinear relationship (15). In practice, it may be necessary to enforce this relationship. We will defer discussion of this question to section 4.4.1.

4.2 Flux Reconstruction

The second phase of the identification scheme is the reconstruction of the rotor fluxes \( \psi_{rx} \) and \( \psi_{ry} \). This procedure is based on the estimates of the electrical parameters \( R_s, \sigma, L_s \) and \( T_r \) and on the derivatives calculated in the identification of these parameters. However, no further integration needs to be performed (i.e., no state observer is needed). To achieve this, the original model in the
(x,y) frame is returned to. In particular, the two equations (1) and (2) are rewritten such that the fluxes are expressed as sums of the voltage and current vectors and of the current derivatives. It is then possible to solve for the two flux vectors. This is detailed below.

\[
\begin{align*}
\frac{\beta}{T_r} \psi_{rx} + \beta n_p \omega \psi_{ry} &= \frac{d}{dt} i_{sx} - \frac{1}{\sigma L_s} v_{sx} + \gamma i_{sx} - n_p \omega i_{sy} \\
- n_p \beta \omega \psi_{rx} + \frac{\beta}{T_r} \psi_{ry} &= \frac{d}{dt} i_{sy} - \frac{1}{\sigma L_s} v_{sy} + \gamma i_{sy} + n_p \omega i_{sx} 
\end{align*}
\]  

so that

\[
\begin{pmatrix}
\psi_{rx} \\
\psi_{ry}
\end{pmatrix} = \frac{1}{\beta \left( \frac{1}{T_r} \right)^2 + n_p^2 \omega^2} \begin{pmatrix}
\frac{1}{T_r} & -\omega \\
\omega & \frac{1}{T_r}
\end{pmatrix} \begin{pmatrix}
\frac{d}{dt} i_{sx} - \frac{1}{\sigma L_s} v_{sx} + \gamma i_{sx} - n_p \omega i_{sy} \\
\frac{d}{dt} i_{sy} - \frac{1}{\sigma L_s} v_{sy} + \gamma i_{sy} + n_p \omega i_{sx}
\end{pmatrix}
\]

or, equivalently,

\[
\begin{align*}
\frac{M}{L_r} \begin{pmatrix}
\psi_{rx} \\
\psi_{ry}
\end{pmatrix} &= \sigma L_s \frac{1}{\left( \frac{1}{T_r} \right)^2 + n_p^2 \omega^2} \begin{pmatrix}
\frac{1}{T_r} & -\omega \\
\omega & \frac{1}{T_r}
\end{pmatrix} \begin{pmatrix}
\frac{d}{dt} i_{sx} - \frac{1}{\sigma L_s} v_{sx} + \gamma i_{sx} - n_p \omega i_{sy} \\
\frac{d}{dt} i_{sy} - \frac{1}{\sigma L_s} v_{sy} + \gamma i_{sy} + n_p \omega i_{sx}
\end{pmatrix}
\end{align*}
\]

Since the parameters \( M \) and \( L_r \) cannot be estimated separately, it is impossible to solve for the rotor fluxes themselves. However, it is possible to calculate scaled versions of the fluxes, \( \frac{M}{L_r} \psi_{rx} \) and \( \frac{M}{L_r} \psi_{ry} \), using the estimated parameters \( R_s, L_s, \sigma \) and \( T_r \). Since the fluxes appear with this scaling factor in the torque equation, this is a natural choice. The scaled versions of the fluxes may be used directly for control, or for estimating the two mechanical parameters.

### 4.3 Identification of the Mechanical Parameters

The third and final stage of the estimation scheme is the identification of the two mechanical parameters, \( J \) and \( \tau_L \). Once the fluxes have been reconstructed, a linear form of the mechanical equation can be obtained. As previously, this enables a direct application of a least-squares algorithm to estimate the two last parameters, \( K_6 \) and \( K_7 \). From \( K_6 \) and \( K_7 \), the inertia and load torque are calculated. This is straightforward and the details follow.

The torque equation is:

\[
\frac{d\omega}{dt} = \frac{2M}{JL_r n_{ph}} (i_{sy} \psi_{rx} - i_{sx} \psi_{ry}) - \frac{\tau_L}{J} \]

or

\[
\frac{2n_p}{J n_{ph}} \left( \frac{M}{L_r} i_{sy} \psi_{rx} - \frac{M}{L_r} i_{sx} \psi_{ry} \right) - \frac{\tau_L}{J}
\]
or, in matrix form:

\[
\frac{d\omega}{dt} = \begin{pmatrix} \frac{M}{L_x} \psi_{rx} i_{sy} - \frac{M}{L_x} \psi_{ry} i_{sx} & -1 \end{pmatrix} \begin{pmatrix} K_6 \\ K_7 \end{pmatrix}
\]

where

\[
K_6 = \frac{2n_p}{Jn_{ph}} \quad K_7 = \frac{\tau_L}{J}
\]

Once the parameters \(K_6\) and \(K_7\) are estimated, \(\tau_L\) and \(J\) may be calculated according to:

\[
J = \frac{2n_p}{K_6n_{ph}} \quad \tau_L = \frac{2K_7n_p}{K_6n_{ph}}
\]

Note that a load torque, \(\tau_L\), proportional to speed, (i.e. \(\tau_L = B\omega\)) can also be estimated by replacing \(-1\) by \(-\omega\) in equation (53), \(\frac{\psi}{J}\) by \(\frac{\psi}{J}\) in (54) and \(B = \frac{2K_7n_p}{K_6n_{ph}}\) in (55).

### 4.4 Numerical Issues

As mentioned earlier, the parameters \(K_1 - K_5\) are supposed to satisfy the nonlinear relationship (15). In other words, the system is overparameterized. As is usually the case in such instances, simulations (see section 5.1) showed that the solution of equation (28) is poorly conditioned. Incorporation of the nonlinear relationship (15) is therefore necessary. To avoid this problem, two approaches were taken. The first solution is to assume that \(R_s\) is measured independently and to replace \(v_s\) by \(v_s - R_s i_s\). The resulting response is then the same as that of a machine with the same parameters, but \(R_s = 0\). As shown below, the nonlinear relationship becomes a trivial linear relationship and is much easier to enforce.

An alternative solution consists in fixing one of the parameters and solving the least-squares problem for the remaining four parameters. The fixed parameter is then replaced by the value given by equation (15) and the procedure is repeated until the iterations converge. Since it was found in the simulations that the contributions of the terms \(-K_2 i_{sx}\), and \(-K_2 i_{sy}\) in equation (18) were small, the iteration was performed with the parameter \(K_2\) initially set to zero and updated using (15). These two approaches are discussed in detail in the following subsections.
4.4.1 Stator Resistance Measured

One way to deal with the difficulty of estimating $K_2$ is to assume that $R_s$ is known. This assumption is often acceptable because it is possible to measure the resistance of the stator independently by applying a DC voltage to the motor and deriving the voltage to current ratio (applying $R = \frac{V}{I}$). The assumption that $R_s$ is known can be incorporated in the algorithm by letting $R_s = 0$ in the original five parameter $K$ matrix and by replacing $v_s$ by $v_s - R_s i_s$. Then, the new parameters become:

$$K'_1 = \frac{1}{T_r \sigma}, \quad K'_2 = 0, \quad K'_3 = \frac{1}{T_r \sigma}, \quad K'_4 = \frac{1}{\sigma L_s}, \quad K'_5 = \frac{1}{\sigma L_s T_r}$$ (56)

An important observation to make is that $K'_2$ is now zero, and thus need not be identified. Additionally, the constraint between the parameters is now simply that $K'_1$ equals $K'_3$. Therefore, there are only three parameters to be identified and the overall identification problem is:

$$
\begin{pmatrix}
-\frac{di_{sx}}{dt} + n_p \omega i_{sy} & \frac{dv_{sx}}{dt} - R_s \frac{di_{sx}}{dt} & v_{sx} - R_s i_{sx} \\
-\frac{di_{sy}}{dt} - n_p \omega i_{sx} & \frac{dv_{sy}}{dt} - R_s \frac{di_{sy}}{dt} & v_{sy} - R_s i_{sy}
\end{pmatrix}
\begin{pmatrix}
K'_3 \\
K'_4 \\
K'_5
\end{pmatrix}
= \begin{pmatrix}
\frac{d^2 i_{sx}}{dt^2} - n_p \omega \frac{di_{sx}}{dt} \\
n_p \omega \frac{di_{sx}}{dt} + \frac{d^2 i_{sx}}{dt^2}
\end{pmatrix}
$$ (57)

Once again, the problem is linear in the parameters $K'_3$, $K'_4$ and $K'_5$ and of the form $y = w^T K$. Thus, the application of the least-squares algorithm is straightforward. Additionally, the nonlinear relationship is automatically enforced by the fact that $K'_1 = K'_3$ can be directly accounted for. The electrical parameters can be derived from the $K'_i$s according to:

$$L_s = \frac{K'_4}{K'_5}, \quad \sigma = \frac{K'_5}{K'_3 K'_4}, \quad T_r = \frac{K'_4}{K'_5}$$ (58)

4.4.2 Iterative Algorithm

As discussed earlier, a second solution to the difficulties in estimating $K_2$ is to set $K_2$ equal to zero. We then have a new identification problem with a different $K$ vector and $w(n)$ matrix than previously. The system becomes:

$$
\begin{pmatrix}
-\frac{di_{sx}}{dt} & n_p \omega i_{sy} & \frac{dv_{sx}}{dt} & v_{sx} \\
-\frac{di_{sy}}{dt} & -n_p \omega i_{sx} & \frac{dv_{sy}}{dt} & v_{sy}
\end{pmatrix}
\begin{pmatrix}
K_1 \\
K_3 \\
K_4 \\
K_5
\end{pmatrix}
= \begin{pmatrix}
\frac{d^2 i_{sx}}{dt^2} - n_p \omega \frac{di_{sx}}{dt} \\
n_p \omega \frac{di_{sx}}{dt} + \frac{d^2 i_{sx}}{dt^2}
\end{pmatrix}
$$ (59)
where the $K$ parameters are the same as defined earlier:

$$
K_1 = \gamma + \frac{1}{T_r} \quad K_3 = \frac{1}{T_r} + \frac{\beta M}{T_r} \quad K_4 = \frac{1}{\sigma L_s} \quad K_5 = \frac{1}{\sigma L_s T_r} \quad (60)
$$

Just as before, it is clear that the problem is linear in the parameters $K_1$, $K_3$, $K_4$ and $K_5$ and of the form $y = w^T K$. Thus, the application of the least-squares algorithm is again straightforward. Since $K_2$ does not equal zero in actuality, it does contribute to the overall system of equations, (18). Thus, it is desirable to attempt to incorporate this information in our solution to the overall problem. Once finishing the identification using (59), equation (15) is used to solve for $K_2$ in terms of the other four parameters. Then, the entire identification algorithm is repeated, again only for estimating $K_1$, $K_3$, $K_4$ and $K_5$ but incorporating the terms depending on $K_2$. The system equations for this scheme and the main steps are given below:

$$
\begin{pmatrix}
-\frac{d^2i_{sx}}{dt^2} & n_p \omega i_{sy} & \frac{dv_{sx}}{dt} & v_{sx} \\
-\frac{d^2i_{sy}}{dt^2} & -n_p \omega i_{sx} & \frac{dv_{sy}}{dt} & v_{sy}
\end{pmatrix}
\begin{pmatrix}
K_1 \\
K_3 \\
K_4 \\
K_5
\end{pmatrix} + 
\begin{pmatrix}
-i_{sx} \\
-i_{sy}
\end{pmatrix}
K_2 = 
\begin{pmatrix}
\frac{d^2i_{sx}}{dt^2} - n_p \omega \frac{di_{sy}}{dt} \\
\omega \frac{d^2i_{sx}}{dt^2} + n_p \frac{d^2i_{sy}}{dt^2}
\end{pmatrix}
(61)
$$

rewriting these equations yields:

$$
\begin{pmatrix}
-\frac{d^2i_{sx}}{dt^2} & n_p \omega i_{sy} & \frac{dv_{sx}}{dt} & v_{sx} \\
-\frac{d^2i_{sy}}{dt^2} & -n_p \omega i_{sx} & \frac{dv_{sy}}{dt} & v_{sy}
\end{pmatrix}
\begin{pmatrix}
K_1 \\
K_3 \\
K_4 \\
K_5
\end{pmatrix} = 
\begin{pmatrix}
\frac{d^2i_{sx}}{dt^2} - \omega \frac{di_{sy}}{dt} + i_{sx} K_2 \\
\omega \frac{d^2i_{sx}}{dt^2} + \frac{d^2i_{sy}}{dt^2} + i_{sy} K_2
\end{pmatrix}
(62)
$$

To solve for $K_2$, equation (15) is rewritten as follows:

$$
K_2 = \frac{K_5}{K_4} (K_1 - K_3)
(63)
$$

Although there is no guarantee that this procedure will converge, it was found to do so, and quickly, in experiments.
5 Results

5.1 Implementation Issues

The three-stage identification scheme outlined in the previous sections was implemented in both a batch and recursive form. To execute the scheme recursively, all filtering and reconstruction of the derivatives of the signals were performed using difference equations (as opposed to batch filtering). The parameters were updated at specified time intervals using a pseudo-recursive algorithm (see section (3.1)) while the fluxes were reconstructed continuously once the initial parameter estimates were available.

To perform the filtering, lowpass digital Butterworth filters were designed. For the simulated data, one filter, a third order filter with a cut-off frequency of 100 Hz, was sufficient for all the signals. With the experimental data, however, a more complex filter proved to be necessary. The frequency of the voltage and current signals varied over time, beginning at 400 Hz and decreasing to 50 Hz. A single lowpass filter would not sufficiently reduce the noise over the entire signal. To address this problem, a variable cut-off frequency lowpass digital Butterworth filter was designed. The cut-off frequency of the filter was changed for each 10th of the data, with an initial value of 515 Hz and a final value of 62.5 Hz. This filter worked well, preserving the necessary information of the signals while reducing the noise.

Filtered differentiation (using digital filters) was used for calculating the acceleration and the derivatives of the voltages and currents. More specifically, the signals were filtered with a lowpass digital Butterworth filter followed by reconstruction of the derivatives using

\[
\frac{dx(t-1)}{dt} = \frac{x(t) - x(t-2)}{2\Delta t}
\]

where \(\Delta t\) is the sampling interval. This reconstruction may be implemented through a filter or iteratively using the difference equation. Since the signals were only filtered once, before any differentiation, these derivatives (especially the second derivatives), for both simulated and experimental data, were noisy. Another concern was that both filtering and differentiation introduced delays in the signals, and the signals were adjusted to account for the delays. This was particularly crucial for the experimental data, which had higher frequency signals than the simulations and was sampled at a lower rate (3125 Hz). Without adjustment for the delays, the voltages and currents were out of phase.
The original algorithm (i.e.- with \((K_1, K_2, K_3, K_4, K_5)\)) was developed and tested on data from simulations of a 2-pole, 3-phase induction machine model. In practice, \(K_1, K_3, K_4\) and \(K_5\) were identified to within a reasonable range of error. However, the parameter \(K_2\) proved to be difficult to estimate. Parametric errors for \(K_2\) were large, and the condition numbers of the matrix \(R_w\) were regularly in the range of \(10^9\). Additionally, the second row and column of the matrix were near zero, suggesting that estimates of \(K_2\) would be poor (as was seen) due to high sensitivity to the noisy vector \(y\). The behavior of the algorithm with the experimental data was consistent with these observations. These numerical problems led to the development of variations on the identification scheme, see section 4.4. It is these variations on the scheme which were actually implemented for parameter estimations. In implementing these variations, condition numbers were reduced to between \(10^2\) and \(10^4\) and the numerical problems described above were avoided.

5.2 Simulation Results

Simulations were performed using the simulation language Simnon. The simulations were useful to determine the usefulness of the parametric indices. A 2-pole, 3-phase induction motor model was simulated in the \((a,b)\) coordinate frame with parameter values chosen to be:

\[
R_s = 9.7\Omega \quad R_r = 8.6\Omega \quad L_s = L_r = .67H \quad M = .64H
\]

\[
\sigma = .0875 \quad J = .011 \frac{kg}{m^2} \quad \tau_L = 3.7\frac{N}{m}
\]

These physical parameter values correspond to the following \(K\) parameter values:

\[
K_1 = 312.0 \quad K_2 = 2122.6 \quad K_3 = 146.6 \quad K_4 = 17.05 \quad K_5 = 218.8 \quad K_6 = 121.2 \quad K_7 = 336.4
\]

The input voltages were 50 Hz sinusoids of peak amplitude 466.7V and the current signals reached a steady-state peak amplitude of 3A (the same frequency). Figure 4 shows an experiment consisting of an acceleration from zero speed to steady-state speed, \(150\text{rad/s}\), in less than a second. As discussed in section 2.2.2, the model is an adequate representation of the machine’s dynamics if \(|\frac{d\omega}{dt}| \ll \frac{1}{\tau_r} |\omega|\). For this experiment, the condition requires that \(|\frac{d\omega}{dt}| \ll \frac{800\text{rad}}{s^2}\) and \(|\omega| \gg \frac{80\text{rad}}{s}\). Though these conditions are not satisfied initially (during the first \(.05s\) of operation), the results are satisfactory. All the results provided in this section are for this experiment. Two sets of results are provided. First, the results for the complete three-stage identification scheme when \(R_s\) is measured
are given. The second set of results shows the performance of the first stage of the identification scheme in the case when $R_s$ is unknown.

![Figure 4: Speed (rad/sec)](image)

5.2.1 Recursive Implementation of the Complete Three Stage Identification Scheme with Measured Stator Resistance

The form of the identification scheme when $R_s$ is measured is considered first. 4000 sample points were taken, and a mixed batch/recursive approach was used where the estimates were updated every 100 points (i.e., every 0.025s). Figure 5 shows the response of the estimator for the first stage of the identification scheme (identification of the electrical parameters) as a function of time. The true parameter (i.e., the one used in the simulation) is shown, together with its estimate and the envelope given by the uncertainty estimates. The figure indicates the effectiveness of the parameter estimator as well as of the measure of uncertainty. Figure 6 shows the result of the flux estimation. As discussed previously, the flux estimation is begun once the initial parameter estimates are made. The results of the third stage of the identification scheme (identification of the mechanical parameters) are provided in figure 7. In this case, the identification of the mechanical parameters was performed using estimates of the fluxes based on final electrical parameter estimates. To perform this stage of estimation truly recursively, a further acceleration or deceleration of the motor is necessary. A summary of the results of the algorithm for $R_s$ measured is provided in table 1.
Figure 5: Parameter $K_5$ and Estimate with Uncertainty vs. Time (sec)

Figure 6: Rotor Flux ($\psi_{r_y}$) and Estimate vs. Time (sec)
Figure 7: Parameter $K_7$ and Estimate with Uncertainty vs. Time (sec)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Value</th>
<th>Estimated Value</th>
<th>Estimated Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_3$</td>
<td>146.62</td>
<td>150.12</td>
<td>7.11</td>
</tr>
<tr>
<td>$K_4$</td>
<td>17.05</td>
<td>17.46</td>
<td>.48</td>
</tr>
<tr>
<td>$K_5$</td>
<td>218.83</td>
<td>220.90</td>
<td>24.96</td>
</tr>
<tr>
<td>$K_6$</td>
<td>121.21</td>
<td>134.03</td>
<td>6.64</td>
</tr>
<tr>
<td>$K_7$</td>
<td>336.36</td>
<td>357.23</td>
<td>23.19</td>
</tr>
</tbody>
</table>

Error Index = 2.7%  $(K_3-K_5)$
Error Index = 4.7%  $(K_6-K_7)$

Table 1: Summary of Simulation Results - $R_s$ Measured
5.2.2 Batch Implementation of the Identification of the Electrical Parameters with Iterative Algorithm

Also provided are results for a batch run of the first stage of the identification scheme when \( R_s \) is not assumed to be measured. As detailed in section 4.4.2, this algorithm is unique in that it estimates the four parameters which are identifiable \((K'_1, K'_3, K'_4 \text{ and } K'_2)\) and then uses the nonlinear relationship between the parameters to calculate the remaining "troublesome" parameter, \( K_2 \). The algorithm is iterated until the parameters converge. For this experiment, the parameters converged in three iterations. A summary of the algorithm's results is found in table 2.

As would be expected, the scheme for which \( R_s \) is measured behaved slightly better than the scheme for which knowledge of \( R_s \) is not required. This makes sense since more information about the system is known in the former case.

5.3 Experimental Results

The experiments used a small, 60W, 3-pole, 2-phase induction motor. The input voltages were 400 Hz sinusoids of amplitude 74.2V. The currents reached a steady-state amplitude of 2A. Figure 8 shows the speed, accelerating from zero to steady-state speed in less than a second. 1.28 seconds of data, sampled at 3125 Hz (4000 sample points), was used. All the results provided in this section...
are for this experiment. As with the simulations, two sets of results are provided, the results for the complete three stage identification scheme when \( R_s \) is measured and the results of the first stage of the identification scheme in the case when \( R_s \) is unknown.

**1st Iteration:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Value</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1 )</td>
<td>311.99</td>
<td>337.26</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>146.62</td>
<td>151.59</td>
</tr>
<tr>
<td>( K_4 )</td>
<td>17.05</td>
<td>17.97</td>
</tr>
<tr>
<td>( K_5 )</td>
<td>218.83</td>
<td>211.95</td>
</tr>
</tbody>
</table>

Calculated \( K_2 = 2189.6 \), True value = 2122.6

**2nd Iteration:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Value</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1 )</td>
<td>311.99</td>
<td>327.40</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>146.62</td>
<td>152.95</td>
</tr>
<tr>
<td>( K_4 )</td>
<td>17.05</td>
<td>17.48</td>
</tr>
<tr>
<td>( K_5 )</td>
<td>218.83</td>
<td>224.51</td>
</tr>
</tbody>
</table>

Calculated \( K_2 = 2240.9 \), True value = 2122.6

**3rd Iteration:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Value</th>
<th>Estimated Value</th>
<th>Estimated Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1 )</td>
<td>311.99</td>
<td>327.16</td>
<td>15.36</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>146.62</td>
<td>152.99</td>
<td>14.50</td>
</tr>
<tr>
<td>( K_4 )</td>
<td>17.05</td>
<td>17.47</td>
<td>.59</td>
</tr>
<tr>
<td>( K_5 )</td>
<td>218.83</td>
<td>224.81</td>
<td>31.75</td>
</tr>
</tbody>
</table>

Calculated \( K_2 = 2241.9 \), True value = 2122.6

Error Index = 3.21%

**Table 2: Summary of Simulation Results - Iterative Algorithm**
5.3.1 Recursive Implementation of the Complete Three Stage Identification Scheme with Measured Stator Resistance

The form of the identification scheme when $R_s$ is known is considered first. By applying a DC voltage to the motor and deriving the voltage to current ratio, $R_s$ was calculated to be $1.70\Omega$. As with the simulations, a mixed batch/recursive approach was used. The estimates were updated every 200 points (i.e. - every $0.0640s$). Figure 9 shows the response of the estimator for the first stage of the identification scheme (identification of the electrical parameters) as a function of time. As previously, the estimate and the envelope given by the uncertainty estimates are shown. Figure 10 shows the result of the flux estimation. The results of the third stage of the identification scheme (identification of the mechanical parameters) are provided in figure 11.

A summary of the results of the algorithm for $R_s$ measured is provided in table 3. Based on these results, the actual motor parameters were calculated to be:

$$R_s = 1.7\Omega \quad L_s = 0.0137H \quad \sigma = 0.2981 \quad T_r = 0.0036s$$

$$\gamma = 1.0770 \quad J = 0.000110\frac{kg}{m^2} \quad \tau_L = 0.0190\frac{N}{m}$$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Estimated Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>K3</td>
<td>939.89</td>
<td>565.13</td>
</tr>
<tr>
<td>K4</td>
<td>245.44</td>
<td>48.82</td>
</tr>
<tr>
<td>K5</td>
<td>68763.30</td>
<td>72961.00</td>
</tr>
<tr>
<td>K6</td>
<td>26595.86</td>
<td>6218.41</td>
</tr>
<tr>
<td>K7</td>
<td>168.05</td>
<td>235.68</td>
</tr>
</tbody>
</table>

Error Index = 17.9% (K3-K5)
Error Index = 16.1% (K6-K7)

Table 3: Summary of Experimental Results - $R_s$ Measured
Figure 9: Parameter $K_4$ Estimate with Uncertainty vs. Time (sec)

Figure 10: Rotor Flux ($\psi_{rx}$) Estimate vs. Time (sec)
5.3.2 Batch Implementation of the Identification of the Electrical Parameters with Iterative Algorithm

The second set of results are those for a batch run of the first stage of the identification scheme when $R_s$ is not assumed to be known. A summary of the algorithm's results is found in table 4. The algorithm proved to converge quickly, in five iterations, though slightly slower than for the simulated data.

6 Conclusions

In this thesis, a new method for the automatic identification of induction motor parameters is presented. The method is unique in that it depends only on a short acceleration experiment to perform the identification. The trajectory was chosen for convenience. The algorithm does not depend on a specific acceleration profile (as long as it carries enough information to determine the parameters). The algorithm is fast and simple and may easily be implemented in real-time with existing hardware. In addition to providing parameter estimates, the algorithm calculates measures of the uncertainty of each estimate. An additional unique feature of the method is that it simultaneously provides estimates of the rotor fluxes.

The method is applicable for the design of self-tuning (self-commissioning) AC drives (i.e. - drives that can adjust controller parameters in response to a range of motors and loads) to optimize
their performance. Additionally, in its recursive form, the scheme provides real-time tracking of variations in motor parameters and may be used directly in adaptive controllers of induction motors. The rotor fluxes are also estimated in real-time and thus, the scheme is applicable for use in a field-oriented controlled drive.

One of the challenges of the method involves the reconstruction of the fluxes from the estimated parameters. The parameter estimates depend on second derivatives of the current signals, which may be very noisy. Therefore, it is necessary to use high-order filters and careful filter design to eliminate the noise.

Finally, the error estimates defined in the thesis are useful for judging the performance of the identification scheme. Results indicate that error indices less than 20% are acceptable and that the procedure in these cases is accurate. Likewise, error indices above 20% suggest low confidence in the accuracy of the parameter estimation. The parametric indices are also very useful. They provide order-of-magnitude figures of parameter confidence and may be used guide the identification process and warn about unreliable results.
### Table 4: Summary of Experimental Results - Iterative Algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Estimated Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>1357.50</td>
<td>599.95</td>
</tr>
<tr>
<td>K3</td>
<td>648.22</td>
<td>973.37</td>
</tr>
<tr>
<td>K4</td>
<td>236.56</td>
<td>52.87</td>
</tr>
<tr>
<td>K5</td>
<td>48141.73</td>
<td>86342.46</td>
</tr>
</tbody>
</table>

Calculated K2 = 144341.26

Error Index = 17.47%
7 References


A List of Symbols

\[
\begin{align*}
    s &= \text{stator} \\
    r &= \text{rotor} \\
    (a,b) &= \text{components of a vector in the fixed stator coordinate frame} \\
    (x,y) &= \text{components of a vector in the rotating coordinate frame} \\
    i_{sa}, i_{sb} &= \text{components of the stator current vector in the (a,b) frame} \\
    \psi_{\tau a}, \psi_{\tau b} &= \text{components of the rotor flux vector in the (a,b) frame} \\
    v_{sa}, v_{sb} &= \text{components of the stator input voltage vector in the (a,b) frame} \\
    i_{sx}, i_{sy} &= \text{components of the stator current vector in the (x,y) frame} \\
    \psi_{\tau x}, \psi_{\tau y} &= \text{components of the rotor flux vector in the (x,y) frame} \\
    v_{sx}, v_{sy} &= \text{components of the stator input voltage vector in the (x,y) frame} \\
    \omega &= \text{angular speed of the rotor} \\
    J &= \text{inertia} \\
    \tau_L &= \text{load torque} \\
    n_{ph} &= \text{number of phases} \\
    n_p &= \text{number of pole pairs} \\
    R_s &= \text{stator resistance} \\
    R_r &= \text{rotor resistance} \\
    M &= \text{mutual inductance} \\
    L_s &= \text{stator inductance} \\
    L_r &= \text{rotor inductance}
\end{align*}
\]

B Derivation of Induction Motor Model: Equations (1)-(5)

The induction motor model presented in section 2.1, (equations (1) - (5)), is based on a standard model in [6]. With a slight change of notation, to be consistent, the model in [6] is, in complex variable notation,

\[
\begin{align*}
    R_s i_s + L_s \frac{d}{dt} i_s + M \frac{d}{dt} (i_r \cos(n_p \theta) + j i_r \sin(n_p \theta)) &= v_s \\
    R_r i_r + L_r \frac{d}{dt} i_r + M \frac{d}{dt} (i_s \cos(n_p \theta) - j i_r \cos(n_p \theta)) &= 0 \\
    \frac{j}{n_{ph}} \frac{d\omega}{dt} &= \frac{M n_p}{n_{ph}} \text{Im} (i_s i_r \cos(n_p \theta) - j i_r i_r \cos(n_p \theta)) - \tau_L
\end{align*}
\]
Where \( \text{Im} \) refers to the imaginary part of the complex number. Then, rewriting these equations using the vector representations,

\[
\begin{align*}
i_s &= i_{sa} + ji_{sb} & i_r &= i_{ra} + ji_{rb} & \nu_s &= \nu_{sa} + j\nu_{sb} \quad (68)
\end{align*}
\]

and separating the real and imaginary parts results in a model expressed in the stator, (a,b), frame. This model is shown below.

\[
L_s \frac{di_{sa}}{dt} = \nu_{sa} - R_s i_{sa} - M \frac{d}{dt} (i_{ra} \cos(n_p \theta) - i_{rb} \sin(n_p \theta)) \quad (69)
\]

\[
L_s \frac{di_{sb}}{dt} = \nu_{sb} - R_s i_{sb} - M \frac{d}{dt} (i_{ra} \sin(n_p \theta) + i_{rb} \cos(n_p \theta)) \quad (70)
\]

\[
L_r \frac{di_{ra}}{dt} = -R_r i_{ra} - M \frac{d}{dt} (i_{sa} \cos(n_p \theta) + i_{sb} \sin(n_p \theta)) \quad (71)
\]

\[
L_r \frac{di_{rb}}{dt} = -R_r i_{rb} - M \frac{d}{dt} (-i_{sa} \sin(n_p \theta) + i_{sb} \cos(n_p \theta)) \quad (72)
\]

\[
J \frac{dw}{dt} = \frac{M n_p}{n_{ph}} (i_{sb} (i_{ra} \cos(n_p \theta) - i_{rb} \sin(n_p \theta)) - i_{sa} (i_{ra} \sin(n_p \theta) + i_{rb} \cos(n_p \theta))) - \tau_L \quad (73)
\]

The model may be simplified by expressing the rotor currents in the stator frame according to:

\[
i_{ra} = i_{ra} \cos(n_p \theta) - i_{rb} \sin(n_p \theta) \quad i_{rb} = i_{ra} \sin(n_p \theta) + i_{rb} \cos(n_p \theta) \quad (74)
\]

The model now becomes:

\[
L_s \frac{di_{sa}}{dt} = \nu_{sa} - R_s i_{sa} - M \frac{d}{dt} (\frac{d i_{ra}}{dt}) \quad (75)
\]

\[
L_s \frac{di_{sb}}{dt} = \nu_{sb} - R_s i_{sb} - M \frac{d}{dt} (\frac{d i_{rb}}{dt}) \quad (76)
\]

\[
L_r \frac{di_{ra}}{dt} = -R_r i_{ra} - M \frac{d}{dt} (\frac{d i_{sa}}{dt}) - M n_p \omega i_{sb} - L_r n_p \omega i_{rb} \quad (77)
\]

\[
L_r \frac{di_{rb}}{dt} = -R_r i_{rb} - M \frac{d}{dt} (\frac{d i_{sb}}{dt}) + M n_p \omega i_{sa} + L_r n_p \omega i_{ra} \quad (78)
\]

\[
J \frac{dw}{dt} = \frac{M n_p}{n_{ph}} (i_{sb} i_{ra} - i_{sa} i_{rb}) - \tau_L \quad (79)
\]

Rearranging,

\[
\frac{di_{sa}}{dt} = \frac{1}{\sigma L_s} (\nu_{sa} - R_s i_{sa}) - \frac{M}{\sigma L_s L_r} (-R_r i_{ra} - n_p \omega L_r i_{rb} - n_p \omega M i_{sb}) \quad (80)
\]

\[
\frac{di_{sb}}{dt} = \frac{1}{\sigma L_s} (\nu_{sb} - R_s i_{sb}) - \frac{M}{\sigma L_s L_r} (-R_r i_{rb} + n_p \omega L_r i_{ra} + n_p \omega M i_{sa}) \quad (81)
\]

\[
\frac{di_{ra}}{dt} = \frac{M}{\sigma L_s L_r} (\nu_{sa} - R_s i_{sa}) + \frac{1}{\sigma L_r} (-R_r i_{ra} - n_p \omega L_r i_{rb} - n_p \omega M i_{sb}) \quad (82)
\]

\[
\frac{di_{rb}}{dt} = \frac{M}{\sigma L_s L_r} (\nu_{sb} - R_s i_{sb}) + \frac{1}{\sigma L_r} (-R_r i_{rb} + n_p \omega L_r i_{ra} + n_p \omega M i_{sa}) \quad (83)
\]

\[
\frac{dw}{dt} = \frac{M n_p}{J n_{ph}} (i_{sb} i_{ra} - i_{sa} i_{rb}) - \frac{\tau_L}{J} \quad (84)
\]
A further model transformation is now performed, with the rotor flux variables introduced. The total rotor flux is expressed in the stator coordinates using:

$$\psi_{ra} = L_r i_{ra} + M i_{sa}$$  \hspace{1cm} $$\psi_{rb} = L_r i_{rb} + M i_{sb}$$  \hspace{1cm} (85)$$

With this change of variables, the resulting model is:

$$\frac{d i_{sa}}{dt} = \frac{1}{\sigma L_s} v_{sa} - \left( \frac{R_s}{\sigma L_s} + \frac{M^2 R_r}{\sigma L_s L_r} \right) i_{sa} + \frac{M R_r}{\sigma L_s L_r} \psi_{ra} + \frac{M n_p}{\sigma L_s L_r} \omega \psi_{rb}$$  \hspace{1cm} (86)$$

$$\frac{d i_{sb}}{dt} = \frac{1}{\sigma L_s} v_{sb} - \left( \frac{R_s}{\sigma L_s} + \frac{M^2 R_r}{\sigma L_s L_r} \right) i_{sb} + \frac{M R_r}{\sigma L_s L_r} \psi_{rb} - \frac{M n_p}{\sigma L_s L_r} \omega \psi_{ra}$$  \hspace{1cm} (87)$$

$$\frac{d \psi_{ra}}{dt} = M \frac{R_r}{L_r} i_{sa} - \frac{R_r}{L_r} \psi_{ra} - n_p \omega \psi_{rb}$$  \hspace{1cm} (88)$$

$$\frac{d \psi_{rb}}{dt} = M \frac{R_r}{L_r} i_{sb} - \frac{R_r}{L_r} \psi_{rb} + n_p \omega \psi_{ra}$$  \hspace{1cm} (89)$$

$$\frac{d \omega}{dt} = \frac{M n_p}{J L_r n_{ph}} (i_{sb} \psi_{ra} - i_{sa} \psi_{rb}) - \frac{\tau_L}{J}$$  \hspace{1cm} (90)$$

A final transformation is necessary. As detailed in section 2.1, the stator voltages and currents and rotor fluxes are transformed into the moving coordinate, \((x,y)\), frame. Additionally, the symbols \(T_r, \beta\) and \(\gamma\) are used to simplify the model. With these last substitutions, the model is derived.

$$\frac{d i_{sx}}{dt} = \frac{1}{\sigma L_s} v_{sx} - \gamma i_{sx} + \frac{\beta}{T_r} \psi_{rx} + n_p \beta \omega \psi_{ry} + n_p \omega i_{sy}$$  \hspace{1cm} (91)$$

$$\frac{d i_{sy}}{dt} = \frac{1}{\sigma L_s} v_{sy} - \gamma i_{sy} + \frac{\beta}{T_r} \psi_{ry} - n_p \beta \omega \psi_{rx} - n_p \omega i_{sx}$$  \hspace{1cm} (92)$$

$$\frac{d \psi_{rx}}{dt} = \frac{M}{T_r} i_{sx} - \frac{1}{T_r} \psi_{rx}$$  \hspace{1cm} (93)$$

$$\frac{d \psi_{ry}}{dt} = \frac{M}{T_r} i_{sy} - \frac{1}{T_r} \psi_{ry}$$  \hspace{1cm} (94)$$

$$\frac{d \omega}{dt} = \frac{2 M n_p}{J L_r n_{ph}} (i_{sy} \psi_{rx} - i_{sx} \psi_{ry}) - \frac{\tau_L}{J}$$  \hspace{1cm} (95)$$

### C  Proof of Equation (45)

As detailed in section 3.2.2, the parametric error associated to the parameter \(K_i\) is defined as the worst case error vector, \(\delta K\). \(\delta K_i\) is the \(i\)th component of \(\delta K\) such that \(|\delta K_i|\) is maximized given the constraint \(\delta K^T R_w \delta K = R e^*\). To find the maximum \(\delta K_i\), we look for the \(\delta K\) maximizing \((\delta K_i)^T \delta K_i\) subject the constraint.

Letting
\[
e_i = \begin{pmatrix}
0 \\
\vdots \\
0 \\
1 \\
0 \\
\vdots \\
0
\end{pmatrix} \quad \leftarrow i
\]

Then \( \delta K_i = e_i^T \delta K \). We now define a Lagrangian augmented function

\[
f(\lambda) = (\delta K_i)^T \delta K_i + \lambda (Re^* - \delta K^T R_w \delta K)
\]

and set its derivatives equal to zero to solve for \( \delta K_i \).

\[
\frac{\partial}{\partial K} \left( \delta K^T e_i e_i^T \delta K + \lambda (Re^* - \delta K^T R_w \delta K) \right) = 0
\]

so that

\[
2e_i e_i^T \delta K - 2\lambda R_w \delta K = 0
\]

with the constraint

\[
Re^* - \delta K^T R_w \delta K = 0
\]

Solving the system of equations (equations (98) and (99)) to find \( \delta K_i \) we proceed as follows. Multiplying equation (98) on the left by \( \delta K^T \) and using equation (99) we arrive at:

\[
\delta K^T e_i e_i^T \delta K = \lambda \delta K R_w \delta K = \lambda Re^*
\]

Then, multiplying equation (98) on the left, first by \( R_w^{-1} \), and then by \( e_i^T \), yields:

\[
e_i^T R_w^{-1} e_i e_i^T \delta K = \lambda e_i^T \delta K
\]
Since \( e_i^T \delta K = \delta K_i \) is scalar,

\[
\left( R_w^{-1} \right)_{ii} = \lambda \tag{102}
\]

and,

\[
(\delta K_i)^2 = \lambda R e^* = (R_w^{-1})_{ii} R e^* \tag{103}
\]

Since the Lagrange multiplier, \( \lambda \), is positive \( (R_w \geq 0) \), this solution is the maximum.