On Manifold Graph Embedding

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Introduction

Graph embedding is an insightful method of data analytics [1]. It is typically performed in flat Euclidean space. This work expands graph embedding to **curved space.**

Current approaches to finding geodesic paths on curved surfaces are limited to deformations of a sphere [2]. Use of piecewise surfaces can be added to generalize to **arbitrary spaces** with boundaries and holes. This method also needs to generalize to higher dimensions.

Triangulation on the Earth

Figure 1.

Method

1. **Fit spheres** to clusters of points Fig. 2.
2. Construct a **graph** from local points on spheres.
3. Find the **shortest path** on the graph and arcs on each sphere.
4. **Straighten** geodesics based on arcs Eq.1, Fig. 3.
5. Repeat steps 3-4 to perform a **graph embedding** using Eq.2

$$c_{t+1} = c_t + (\nabla P_i - \nabla P_{i+1})$$

Equation 1.

$$p_{t+1} = p_t + \sum_{i=1}^{n} (d_a - d)\nabla P$$

Equation 2.

Surface of Piecewise Spheres

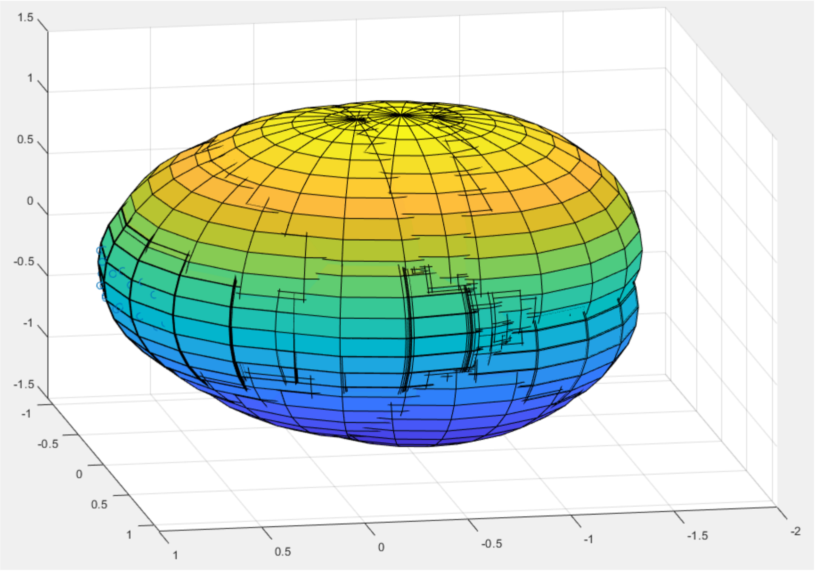

Figure 2.

Initial Geodesic Path | **Smoothed Geodesic Path**

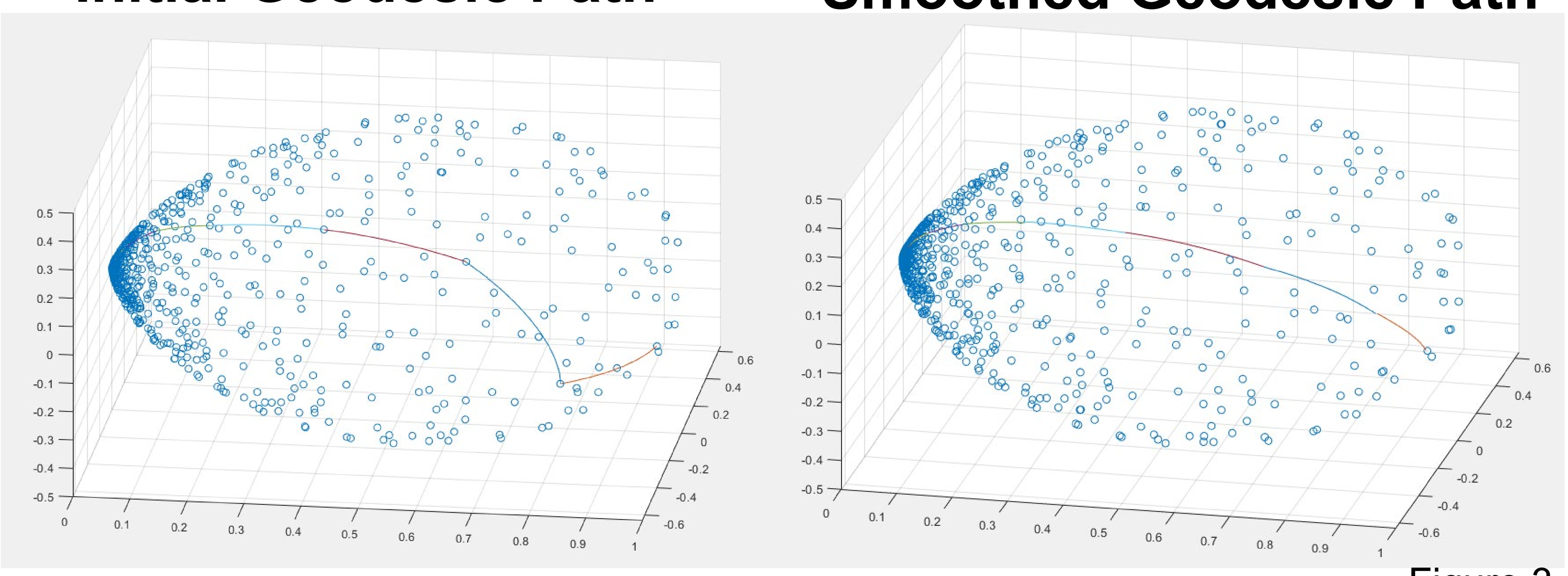

Figure 3.

Geodesic Distances from Origin

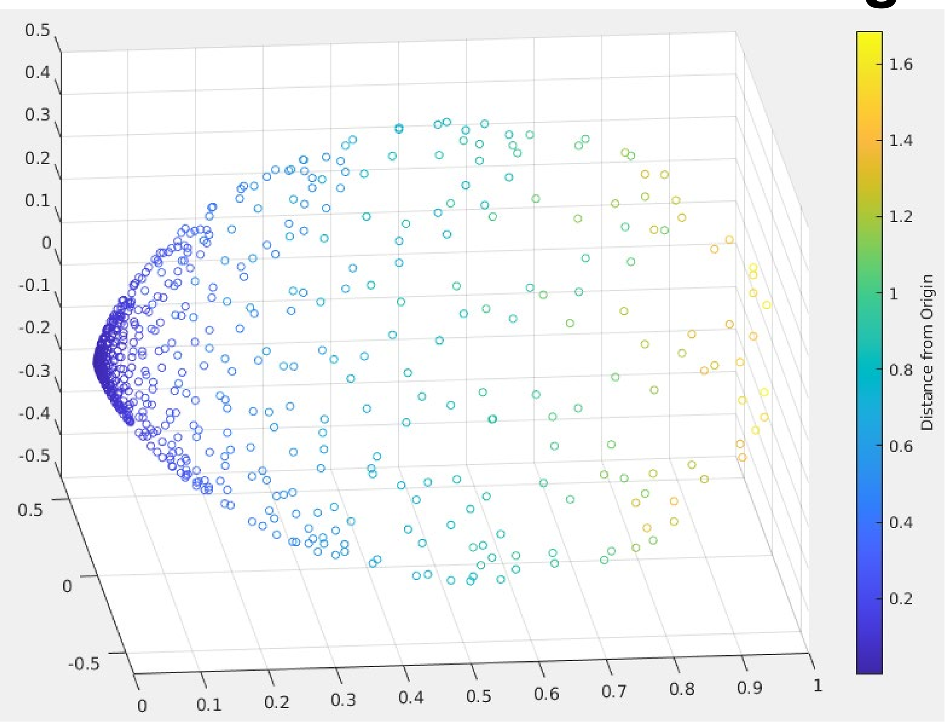

Figure 4.

Euclidean Space Embedding

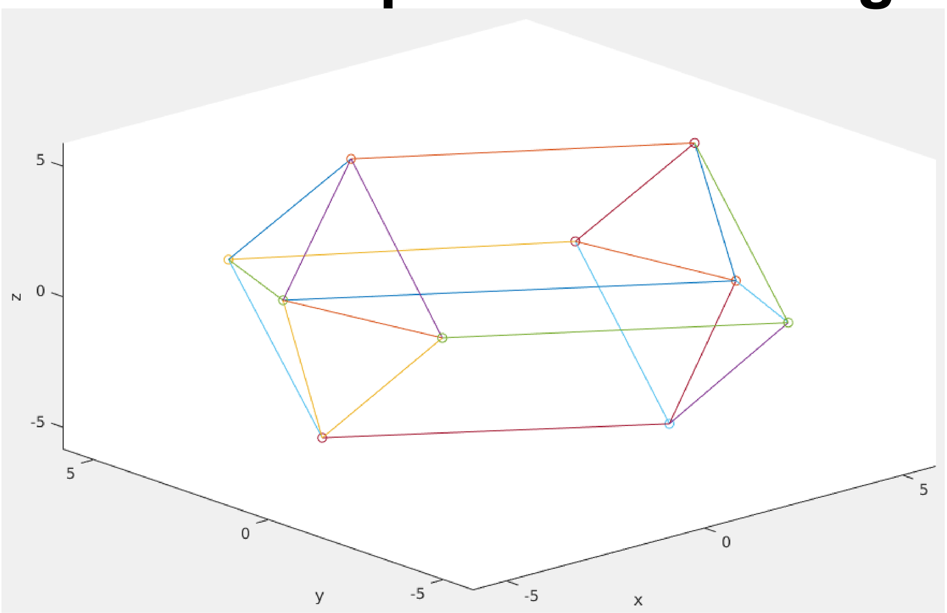

Figure 5.

Experimental Setup

Given **3D data** points and a graph of **geodesic distances** find graph node locations on the manifold.

Example case: Ellipsoid distorted nonlinearly and a graph of ten nodes spread across the manifold with geodesic distances.

Initial locations of nodes were found by embedding in Euclidean space and projecting onto the manifold Fig. 5.

Results

The **graph embedding** on manifold successfully found consistent positions for the nodes given the data cloud and graph of distances. The points arrive at the expected locations except rotated about the X-axis.

Each point in the embedding remains in the same color section as the ground truth Fig 6. Given this example graph coloring problem with manifold data this method **correctly classifies** them.

Future work will look towards fitting more complex quadrics and finding geodesics on them. Applying piecewise surfaces in higher dimensions will also be explored

Ground Truth | **Graph Embedding**

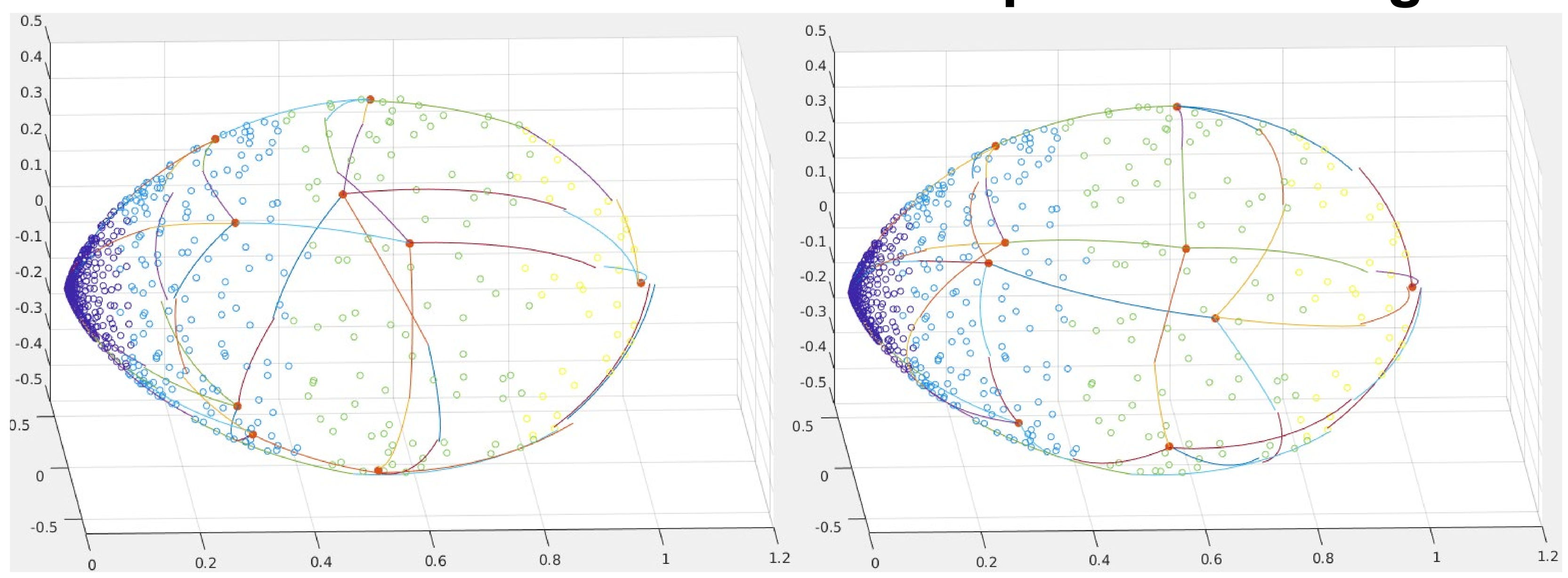

Figure 6.

References

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