Virtual Temperature Measurement for Smart Buildings via Bayesian Model Fusion

(Invited Paper)

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***Abstract*—One important goal of creating smart buildings is to offer highly comfortable services to the occupants at low cost. Real-time temperature measurement and monitoring is a critical task to facilitate high-quality service with low energy consumption and, hence, cost. In this paper, we propose a novel framework to accurately measure in-building temperature by using a small number of sensors. The key idea is to combine the prior knowledge on temperature statistics with a few sensor measurements and then predict the spatial temperature distribution by maximum-a-posteriori estimation. Our experimental results demonstrate that the average estimation error is less than 0.3 degree with very few sensors.**

***Keywords*—Smart building; Temperature estimation; Bayesian model fusion**

I. Introduction

Nowadays, smart buildings play an important role in everyday lives of people. One of the most important goals of creating smart buildings is to offer highly comfortable services to the occupants at the lowest cost (e.g., the energy consumption) and environmental impact. To increase the thermal comfort level and energy efficiency of smart buildings, it is essential to build an automatic temperature management system. As heating and cooling account for 30% to 50% of the total building energy consumption [1], efficiently implementing a smart temperature management system can significantly reduce the energy cost. The efficiency of such a system heavily relies on the monitored or measured in-building temperature. As a result, accurate temperature measurement and monitoring is a critical task for the aforementioned temperature management system.

Building simulation is widely used at the design stage to predict the thermal performance and energy consumption of buildings [2]. Through building simulation, it is possible to create temperature models to predict the in-building temperature. However, models created by building simulation are prior models which are based on the prior knowledge obtained at the design stage. It is difficult to consider real-time and real-world information (e.g., weather and environment variations) when creating models from building simulation. On the contrary, using temperature sensors is a practical way to obtain such information.

Using sensors to monitor temperature usually involves the Internet of Things (IoT) and wireless sensor network (WSN) techniques [3], [4]. In these approaches, all the sensory data are transmitted by a wireless network to an Internet server which processes the data and makes decisions based on the collected data. IoT/WSN-based approaches always monitor real-time temperature so the shortages of simulation-based approaches are overcome. However, creating a large wireless network with many sensors suffers from high cost and high complexity. It requires a careful design for a scalable solution for network architecture, network protocol, data processing algorithm, etc, especially when integrating a large number of sensors into the WSN.

In this paper, we propose a Bayesian model fusion (BMF) [5]–[7] framework to accurately measure in-building temperature. The key idea is to combine the prior knowledge of temperature statistics obtained by building simulation with a few sensor measurements and then predict the spatial temperature distribution by maximum-a-posteriori estimation. Consequently, the proposed approach takes advantages of both the simulation-based and IoT/WSN-based approaches, and thus, it is of low cost and high accuracy. We also propose to use a modified Latin hypercube sampling method [8] to make a good sensor placement to reduce the number of required sensors. Our experimental results demonstrate that the average estimation error can be less than 0.3 degree with very few sensors.

The rest of this paper is organized as follows. In Section II, we present the problem formulation for the proposed approach. The BMF method is explained in detail in Section III. We discuss how to place sensors to reduce measurement error in Section IV. The efficiency of the proposed method is demonstrated by a case study in Section V. Finally, we conclude in Section VI.

II. Problem Formulation

Given a building, its in-building temperature at different locations depends on many factors. We use a vector $\mathbf{X} = [x_0, x_1, x_2, \cdots, x_K]^T$ to denote the independent factors (e.g., power consumption of the appliance in a room) that may affect the in-building temperature, where $x_0 = 1$ and K is the number of independent factors. Let N be the number of locations in the building. We consider the following linear regression model [9] to approximate the temperature of each location:

$$t^{(n)} = \sum_{k=0}^{K} \alpha_k^{(n)} x_k = \alpha_0^{(n)} + \sum_{k=1}^{K} \alpha_k^{(n)} x_k, \tag{1}$$

where $t^{(n)}$ is the temperature of the nth location ($n = 1, 2, \cdots, N$), and $\{\alpha_k^{(n)}; k = 0, 1, \cdots, K\}$ are the model coefficients for the nth location. The linear regression model is widely used in the literature [10], [11]. Eq. (1) can be converted into a matrix form for all locations:

$$\mathbf{T} = \mathbf{AX}, \tag{2}$$

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where $\mathbf{A} \in \mathbb{R}^{N\times(K+1)}$ is a matrix containing all the model coefficients, i.e., $A_{n,k} = \alpha_k^{(n)}$, and $\mathbf{T} = [t^{(1)}, t^{(2)}, \cdots, t^{(N)}]^T$ is the temperature vector. The model coefficients $\mathbf{A}$ can be approximately solved by least-squares fitting [12] based on simulation data. For a practical building, we may not know the exact value of $\mathbf{X}$ and, therefore, it is difficult to accurately predict the in-building temperature by Eq. (2).

To address this challenge, we assume that M ($M < N$) locations are equipped temperature sensors so we can obtain accurate temperature measurements with a small error for these locations. Let $\mathscr{Y}$ and $\mathscr{N}$ be the index sets for those locations with and without sensors, respectively. For the locations with sensors, we have the following equation:

$$\mathbf{A}^{(\mathscr{Y})}\mathbf{X} = \mathbf{T}^{(\mathscr{Y})}. \tag{3}$$

If $\mathbf{X}$ can be solved from Eq. (3), the temperature of the locations without sensors can be simply predicted by a matrix vector multiplication:

$$\mathbf{T}^{(\mathscr{N})} = \mathbf{A}^{(\mathscr{N})}\mathbf{X}. \tag{4}$$

However, Eq. (3) is typically underdetermined, because M is usually much smaller than K in practice. Hence, Eq. (3) cannot be solved by a conventional linear solver. In this paper, we will apply the BMF method to solve $\mathbf{X}$ from Eq. (3) by maximum-a-posteriori estimation. As such, the temperature of the locations without sensors can be predicted by Eq. (4).

III. Bayesian Model Fusion

The proposed BMF framework contains two core steps: 1) statistically defining the prior knowledge learned from building simulation, and 2) applying maximum-a-posteriori estimation to predict temperature through Bayesian inference. In this section, we will describe the mathematical details of the BMF algorithm.

A. Prior Knowledge Definition

In the proposed BMF framework, we assume that the prior distribution of each factor x_k ($k = 1, 2, \cdots, K$) is known, which is denoted as $pdf(x_k)$, where $pdf(\cdot)$ means the probability density function. Unlike other conventional BMF methods [5]–[7] where the prior distribution is modeled as a Gaussian distribution, the prior distribution in this paper can also be non-Gaussian (e.g., uniform distribution), making the proposed BMF framework more practical.

Since all the factors that may affect the in-building temperature are defined to be independent as mentioned in Section II, the joint distribution of $\mathbf{X}$ is simply the product of all the individual density functions, i.e.,

$$pdf(\mathbf{X}) = \prod_{k=1}^{K} pdf(x_k). \tag{5}$$

After the prior distribution of $\mathbf{X}$ is defined, we can randomly generate S ($S > K + 1$) samples $\mathbf{X}^{(1)}, \mathbf{X}^{(2)}, \cdots, \mathbf{X}^{(S)}$ by Latin hypercube sampling [13] to perform building simulation. Then, the regression model coefficients of the nth location are solved by least-squares fitting [12] on the following over-determined equation:

$$\mathbf{Y}[\alpha_0^{(n)}, \alpha_1^{(n)}, \cdots, \alpha_K^{(n)}]^T = [f_n^{(1)}, f_n^{(2)}, \cdots, f_n^{(S)}]^T, \tag{6}$$

where $\mathbf{Y} = [\mathbf{X}^{(1)}, \mathbf{X}^{(2)}, \cdots, \mathbf{X}^{(S)}]^T \in \mathbb{R}^{S\times(K+1)}$, and $f_n^{(s)}$ is the simulated temperature of the nth location for the sample $\mathbf{X}^{(s)}$.

Consider the modeling error for the linear regression model Eq. (2), i.e.,

$$\mathbf{AX} = \mathbf{T} + \mathbf{e}, \tag{7}$$

where $\mathbf{e} = [e^{(1)}, e^{(2)}, \cdots, e^{(N)}]^T$ is the modeling error of all the locations. When the model coefficients are calculated, we can also obtain the distribution of the modeling error. The modeling error of the nth location of all the samples can be obtained as follows:

$$\mathbf{E}_n = \mathbf{Y}[\alpha_0^{(n)}, \alpha_1^{(n)}, \cdots, \alpha_K^{(n)}]^T - [f_n^{(1)}, f_n^{(2)}, \cdots, f_n^{(S)}]^T. \tag{8}$$

The components of $\mathbf{E}_n$ typically show a zero-mean Gaussian distribution. Hence, we use a zero-mean Gaussian distribution to model each $e^{(n)}$. The standard deviation of $e^{(n)}$ which is denoted as σ_n equals to the numerically calculated standard deviation of $\mathbf{E}_n$. The correlation coefficient between $e^{(i)}$ and $e^{(j)}$ which is denoted as $\rho_{i,j}$ equals to the numerically calculated correlation coefficient between $\mathbf{E}_i$ and $\mathbf{E}_j$.

B. Maximum-A-Posteriori Estimation

The key idea of BMF is to combine the prior knowledge obtained from building simulation with a few sensor measurements. For the locations with sensors, we can get the measured temperature $\mathbf{T}^{(\mathscr{Y})}$. The goal of maximum-a-posteriori is to solve $\mathbf{X}$ from Eq. (3) by maximizing the posterior distribution $pdf\left(\mathbf{X}|\mathbf{T}^{(\mathscr{Y})}\right)$.

Based on Bayes' theorem, $pdf\left(\mathbf{X}|\mathbf{T}^{(\mathscr{Y})}\right)$ is proportional to the product of the prior distribution $pdf(\mathbf{X})$ and the likelihood function $pdf\left(\mathbf{T}^{(\mathscr{Y})}|\mathbf{X}\right)$, i.e.,

$$pdf\left(\mathbf{X}|\mathbf{T}^{(\mathscr{Y})}\right) \propto pdf\left(\mathbf{T}^{(\mathscr{Y})}|\mathbf{X}\right) pdf(\mathbf{X}) \tag{9}$$

The prior distribution $pdf(\mathbf{X})$ is already given in Eq. (5). According to Eq. (7) and the prior knowledge of the modeling error defined in Section III-A, the likelihood function $pdf\left(\mathbf{T}^{(\mathscr{Y})}|\mathbf{X}\right)$ can be expressed as a multivariate Gaussian distribution which is related to the distribution of the modeling error, i.e.,

$$\begin{aligned} pdf\left(\mathbf{T}^{(\mathscr{Y})}|\mathbf{X}\right) &= \frac{1}{\left(\sqrt{2\pi}\right)^M \sqrt{|\boldsymbol{\Sigma}|}} \cdot \\ &\exp\left[-\frac{1}{2}\left(\mathbf{AX} - \mathbf{T}^{(\mathscr{Y})}\right)^T \boldsymbol{\Sigma}^{-1} \left(\mathbf{AX} - \mathbf{T}^{(\mathscr{Y})}\right)\right], \end{aligned} \tag{10}$$

where $\boldsymbol{\Sigma}$ is the covariance matrix of the modeling error for the locations with sensors, i.e., $\Sigma_{i,j} = \rho_{pi,pj}\sigma_{pi}\sigma_{pj}$, where pi and pj are the ith and jth elements in the index set $\mathscr{Y}$.

Combining Eq. (5), Eq. (9) and Eq. (10), the posterior distribution is expressed as

$$\begin{aligned} pdf\left(\mathbf{X}|\mathbf{T}^{(\mathscr{Y})}\right) &\propto \left[\prod_{k=1}^{K} pdf(x_k)\right] \cdot \\ &\exp\left[-\frac{1}{2}\left(\mathbf{AX} - \mathbf{T}^{(\mathscr{Y})}\right)^T \boldsymbol{\Sigma}^{-1} \left(\mathbf{AX} - \mathbf{T}^{(\mathscr{Y})}\right)\right]. \end{aligned} \tag{11}$$

The goal of maximum-a-posteriori estimation is to solve $\mathbf{X}$ from Eq. (11) such that $pdf\left(\mathbf{X}|\mathbf{T}^{(\mathscr{Y})}\right)$ is maximized. If $\mathbf{X}$

follows a multivariate Gaussian distribution, the right-hand-side of Eq. (11) is also a multivariate Gaussian distribution, and, therefore, $pdf\left(\mathbf{X}|\mathbf{T}^{(\mathscr{Y})}\right)$ reaches its maximum at the mean value. However, if factors in $\mathbf{X}$ are non-Gaussian (e.g., uniform distributions), it may not be possible to derive an analytical solution to maximize $pdf\left(\mathbf{X}|\mathbf{T}^{(\mathscr{Y})}\right)$. In this case, solving Eq. (11) is converted to an optimization problem and must be solved by a numerical solver.

C. Summary

Algorithm 1 summarizes the major steps of the BMF method for in-building temperature estimation. The efficacy of our proposed algorithm will be demonstrated by the case study in Section V.

Algorithm 1 BMF for in-building temperature estimation.

1: Define the prior distribution $pdf(x_k)$ and calculate Eq. (5).
2: Generate a set of samples and perform building simulation, and then calculate the model coefficients $\mathbf{A}$ and the distribution of the modeling error.
3: Collect M temperature values $\mathbf{T}^{(\mathscr{Y})}$ from M sensors.
4: Solve $\mathbf{X}$ from Eq. (11) such that $pdf\left(\mathbf{X}|\mathbf{T}^{(\mathscr{Y})}\right)$ is maximized.
5: Estimate the temperature of the locations without sensors by Eq. (4).

IV. Sensor Placement

The measured temperature values $\mathbf{T}^{(\mathscr{Y})}$ depend on the locations of the M temperature sensors. In other words, different sensor placements will provide different values of $\mathbf{T}^{(\mathscr{Y})}$ and, hence, different results of the BMF algorithm. It, in turn, motivates us to develop an efficient strategy to find a "good" sensor placement.

A building naturally has a number of rooms. Let's say there are R rooms in the building. A simple idea is to evenly distribute the M sensors over the R rooms, and in each room, sensors are evenly distributed over all the locations in the room. To achieve this goal, we use a modified Latin hypercube sampling (M-LHS) method [8] to generate well-controlled locations of temperature sensors. There are two main steps to implement M-LHS:

1) Evenly distribute the M sensors over the R rooms. If M is exactly divisible by R, then each room has M/R sensors. Otherwise $M\%R$ (% is the modulo operator) rooms have $\lfloor M/R \rfloor + 1$ sensors in each and the other rooms have $\lfloor M/R \rfloor$ sensors in each. Randomly select the $M\%R$ rooms that will be placed $\lfloor M/R \rfloor + 1$ sensors in each. Let S_r be the number of sensors in room r.
2) For each room r, evenly distribute the S_r sensors over all the locations in this room. Let L_r be the number of locations in room r. Evenly partition the L_r locations into S_r blocks. If L_r is exactly divisible by S_r, then each block has L_r/S_r locations. Otherwise $L_r\%S_r$ blocks have $\lfloor L_r/S_r \rfloor + 1$ locations in each and the other blocks have $\lfloor L_r/S_r \rfloor$ locations in each. Randomly select the $L_r\%S_r$ blocks that have $\lfloor L_r/S_r \rfloor + 1$ locations in each. Finally, randomly place one sensor in each block, resulting in M sensors in total.

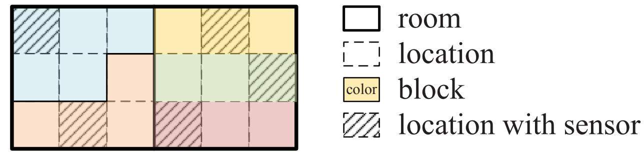

Fig. 1: A simple example is illustrated for the M-LHS method.

A simple example to illustrate the M-LHS method is shown in Fig. 1, where 5 sensors are placed in a building with 2 rooms and 18 locations in total. The efficacy of M-LHS will be further demonstrated by our case study in the next section.

V. Case Study

A. Simulation Setup

In this section, we will use a building example to demonstrate the efficiency of the proposed BMF method. A building with $R = 9$ rooms and $N = 144$ locations is created. This building has $K = 264$ independent factors associated with temperature/weather, properties of materials, etc. Among them, 94 factors follow Gaussian distributions and the others follow uniform distributions. We use the Latin hypercube sampling method [13] to generate 1000 random samples for training the temperature model (i.e., Eq. (2)) and 50 random samples for error estimation. EnergyPlus [14] is used to simulate these samples. For the 50 testing samples, the simulated results from EnergyPlus are treated as the "actual" temperature. The sensor noise is modeled by a Gaussian distribution with mean of zero and standard deviation of 0.3 degree. In other words, the measured temperature of a location with a sensor equals to the simulated temperature plus a randomly generated Gaussian noise.

In what follows, we will compare the estimation error of the BMF method with random placement and the M-LHS method proposed in Section IV. The estimation error is defined as the root-mean-square (RMS) error of the locations we are interested in (e.g., all locations or the locations without sensors). The reported RMS error is the average value of the 50 testing samples.

B. Simulation Results

Fig. 2 shows how the estimation error for locations without sensors varies with the number of sensors. For both the random placement and the M-LHS method, the estimation error of BMF decreases as the number of sensors increases. The estimation error saturates at about 0.12 degree when there are a sufficient number of sensors. It can be observed that given the same number of sensors, M-LHS is able to achieve higher accuracy than that of random placement, especially when only a few sensors are available. For example, when there are 12 sensors, the estimation error of M-LHS is 0.1 degree smaller than random placement. To achieve the same accuracy of M-LHS using 12 sensors, random placement requires 23 to 24 sensors, which is 2× of the number of sensors required by M-LHS. Fig. 3 shows the estimation error for all locations. It shows a similar trend as Fig. 2. In practice, by using 10 to 20 sensors, the proposed BMF method can obtain reasonable estimation accuracy (the estimation error is less than 0.3 degree) if the sensors are placed by the M-LHS method.

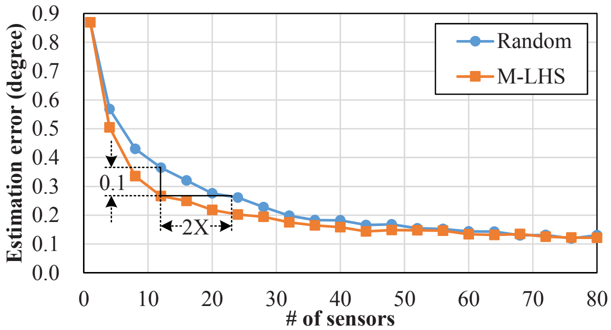

Fig. 2: The estimation error for locations without sensors is shown as a function of the number of sensors.

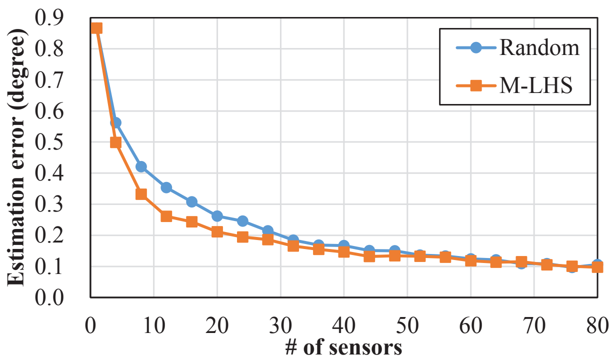

Fig. 3: The estimation error for all locations is shown as a function of the number of sensors.

It is worth explaining why the estimation error can be smaller than the sensor noise. Sensor noises are completely independent and random so they do not provide any useful information. When there are enough sensors, such measurement noises can be filtered when solving Eq. (3) by maximum-a-posteriori.

Fig. 4 compares the temperature profiles obtained by random placement and M-LHS with the "actual" temperature profile obtained by EnergyPlus simulation for one example. 16 sensors are used in this example. It provides an intuitive view of the distribution of the in-building temperature. It clearly shows that M-LHS obtains more accurate estimation than random placement for this example.

VI. Conclusion

Temperature estimation for smart buildings is important for offering comfortable services to the occupants and reducing energy consumption of buildings. In this paper, we have proposed a BMF framework for accurate in-building temperature estimation by using a small number of sensors. An M-LHS method is proposed to obtain a good sensor placement. Our experimental results demonstrate that the average estimation error of BMF with M-LHS is less than 0.3 degree with very few sensors.

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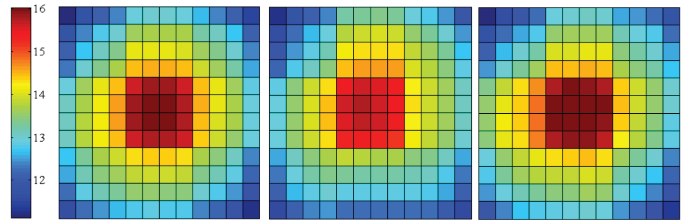

(a) Actual temperature (b) Estimation by random placement (c) Estimation by M-LHS

Fig. 4: The estimated temperature profiles are compared among "actual" temperature, random placement, and M-LHS. 16 sensors are used in this example.

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