SPIRAL: Tuning DSP Transforms to Computing Platforms

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`http://www.ece.cmu.edu/~spiral`

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Sponsor

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Moore's Law and High(est) Performance Scientific Computing

(single processor, off-the-shelf)

Moore's Law:

- processor-memory bottleneck
- short life cycles of computers
- very complex architectures
  - vendor specific
  - special instructions (MMX, SSE, FMA, …)
  - undocumented features

Effects on software/algorithms:

- arithmetic cost model not accurate for predicting runtime (one cache miss = 10 floating point ops)
- better performance models hard to get
- best code is machine dependent (registers/caches size, structure)
- hand-tuned code becomes obsolete as fast as it is written
- compiler limitations
- full performance requires (in part) assembly coding

➡ **Portable performance requires automation**

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Automatic Performance Tuning: Research

Linear Algebra:

- ATLAS (J. Dongarra et al.)
- LAPACK
- PhiPACK (J. Demmel et al.)

Signal Processing:

- FFTW (M. Frigo and S. Johnson)
- SPIRAL

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Automates

Implementation
- cuts development costs
- code less error-prone

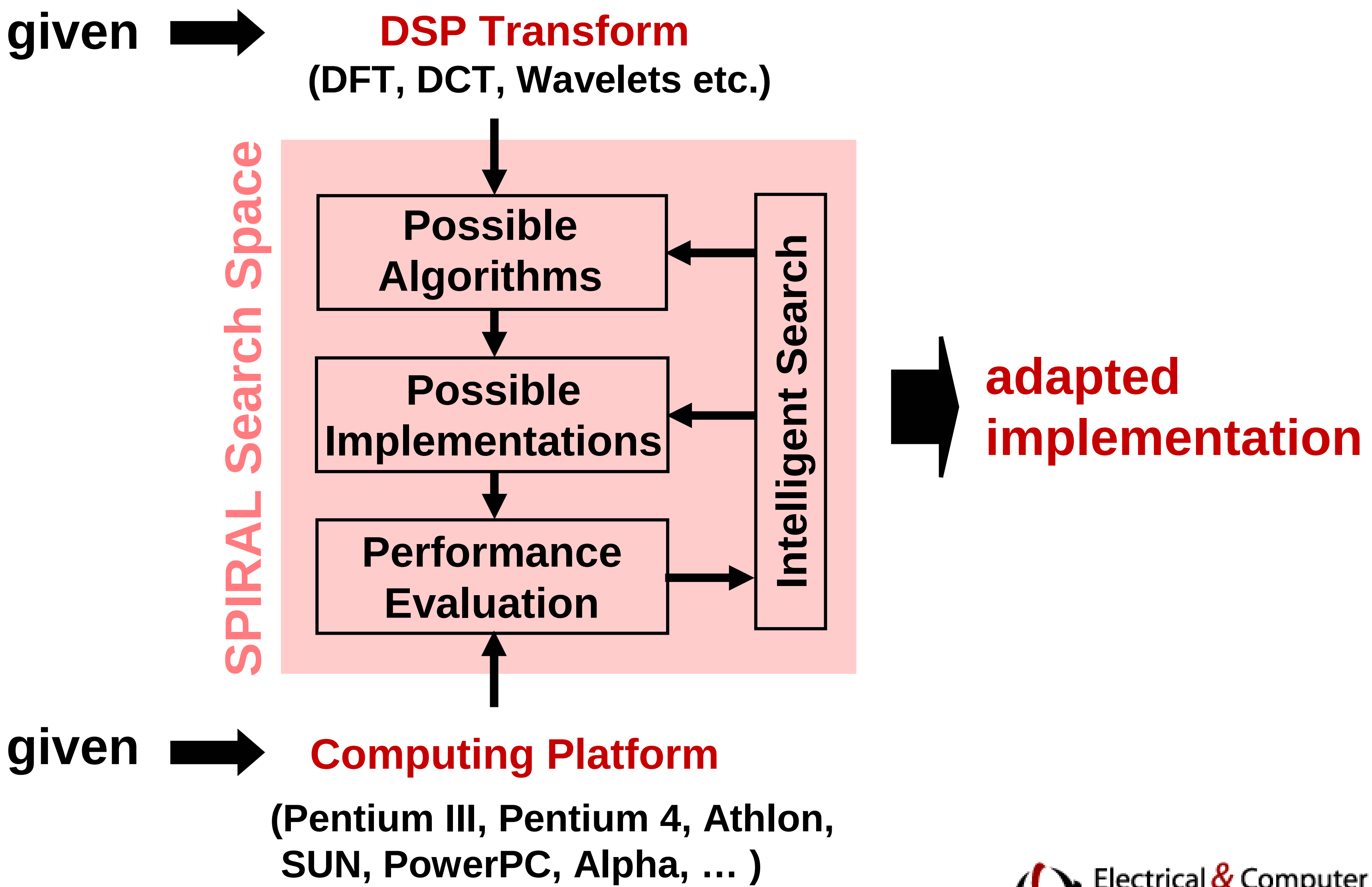
Optimization
- systematic exploration of alternatives both at algorithmic and code level

Platform-Adaptation
- takes advantage of architecture specific features
- porting without loss of performance

of DSP algorithms
- are performance critical

A library generator for highly optimized signal processing algorithms

SPIRAL Approach



Organization

- **Mathematical Framework**
  - Transforms, Rules, and Formulas
- Formula Generator
  - Transform $\rightarrow$ Algorithm
- SPL and SPL Compiler
  - Algorithm $\rightarrow$ Implementation
- Search Engine
  - How to find the best implementation
- SPIRAL system
  - Everything taken together
- Conclusions

DSP Algorithms: Example 4-point DFT

Cooley/Tukey FFT (size 4):

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fourier transform

Diagonal matrix (twiddles)

$$DFT_4 = (DFT_2 \otimes I_2) \cdot T_2^4 \cdot (I_2 \otimes DFT_2) \cdot L_2^4$$

Kronecker product

Identity

Permutation

- **product of structured sparse matrices**
- **mathematical notation**

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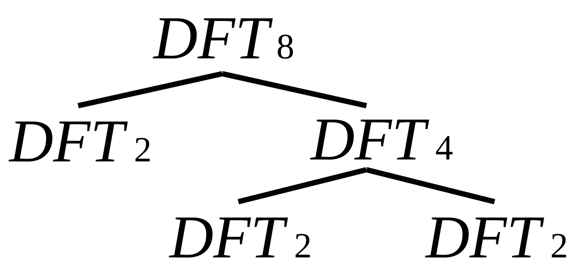
DSP Algorithms: Terminology

Transform DFT_n parameterized matrix

Rule $DFT_{nm} \rightarrow (DFT_n \otimes I_m) \cdot D \cdot (I_n \otimes DFT_m) \cdot P$

- a breakdown strategy
- product of sparse matrices

Ruletree



- recursive application of rules
- uniquely defines an algorithm
- efficient representation
- easy manipulation

Formula $DFT_8 = (F_2 \otimes I_4) \cdot D \cdot (I_2 \otimes (I_2 \otimes F_2 \cdots)) \cdot P$

- few constructs and primitives
- uniquely defines an algorithm
- can be translated into code

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More Cooley-Tukey Rules

- **DFT is symmetric $\Rightarrow$ transpose the rule:**

$$F_{RS} = L_S^{RS}\left(I_R \otimes F_S\right) T_S^{RS}\left(F_R \otimes I_S\right)$$

CT rule transposed

- **Commuting tensor product factors**

$$B \otimes A = L_n^{mn}\left(A \otimes B\right) L_m^{mn}$$

A* and *B* square size *m* and *n

- **Commutation property $\Rightarrow$ further variations**

$$F_N = L_S^{RS}\left(I_S \otimes F_R\right) L_R^{RS} T_S^{RS}\left(I_R \otimes F_S\right) L_R^{RS}$$

$$F_N = \left(F_R \otimes I_S\right) T_S^{RS} L_S^{RS}\left(F_S \otimes I_R\right)$$

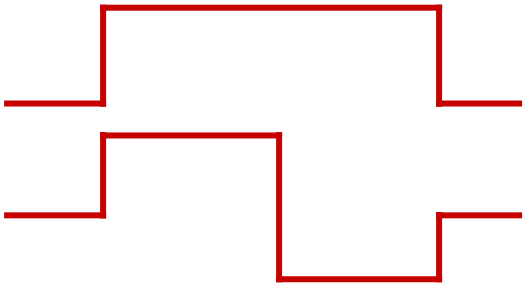
$$\left(F_2 \otimes I_2\right) x = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \left(I_2 \otimes F_2\right) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- **Different patterns for access, storage and flow of data**

Haar Wavelets – Example

- **Haar wavelets = square waves**

$$h = \frac{1}{\sqrt{2}}[\,1,\ 1],\ \ h^1 = \frac{1}{\sqrt{2}}[\,1, -1]$$



- **First stage: $V_2 \Rightarrow V_1 \oplus W_1$**

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} = L_2^4 \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} = L_2^4 \left(I_2 \otimes F_2 \right) \qquad \begin{bmatrix} \underline{c}_1 \\ \underline{d}_1 \end{bmatrix} = H\,\underline{c}_2$$

- **The process is repeated for the upper half of the output**

$$HT_{2^n} = \left(HT_{2^{n-1}} \oplus I_{2^{n-1}} \right) \underbrace{L_{2^{n-1}}^{2^n} \left(I_{2^{n-1}} \otimes F_2 \right)}_{H},\ \ HT_2 = F_2$$

Discrete-Time Wavelet Transform

- **Discrete-Time Wavelet Transform (DTWT) rule**

$$\mathrm{DTWT}_{2^n} = \left(\mathrm{DTWT}_{2^{n-1}} \oplus \mathrm{I}_{2^{n-1}}\right)\underbrace{\mathrm{L}^{2^n}_{2^{n-1}}\left(\mathrm{I}_{2^{n-1}} \otimes_{l-2} W\right)}_{H}$$

- **Scaling (lowpass) and wavelet (highpass) filter coefficients**

$$W = \begin{bmatrix} h_0 & h_1 & \cdots & h_{l-1} \\ h'_0 & h'_1 & \cdots & h'_{l-1} \end{bmatrix}$$

- **DTWT - convolution rule**

$$H = \Big(\big(\begin{bmatrix}1 & 1\end{bmatrix} \otimes \mathrm{I}_{m/2}\big) \cdot \big(C^{\mathrm{T}}_{m/2}(\underline{lo}) \oplus C^{\mathrm{T}}_{m/2}(\underline{le})\big) \cdot \mathrm{L}^n_2 \oplus$$

$$\big(\begin{bmatrix}1 & 1\end{bmatrix} \otimes \mathrm{I}_{m/2}\big) \cdot \big(C^{\mathrm{T}}_{m/2}(\underline{ho}) \oplus C^{\mathrm{T}}_{m/2}(\underline{he})\big) \cdot \mathrm{L}^n_2\Big) \cdot \left(\begin{bmatrix}1 \\ 1\end{bmatrix} \otimes \mathrm{I}_n\right)$$

$\underline{lo}$-lowpass odd coeffs., $\underline{le}$-lowpass even coeffs.

$\underline{ho}$-highpass odd coeffs., $\underline{he}$-highpass even coeffs.

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DSP Transforms

discrete Fourier transform $$DFT_n = [\exp(2kli\pi/n)]$$

Walsh-Hadamard transform $$WHT_{2^k} = DFT_2 \otimes \cdots \otimes DFT_2$$

discrete cosine and sine Transforms (16 types)

$$DCT^{(II)}_n = [\cos(k(l+1/2)\pi/n)]$$

$$DCT^{(IV)}_n = [\cos((k+1/2)(l+1/2)\pi/n)]$$

$$DST^{(I)}_n = [\sin(kl\pi/n)]$$

modified discrete cosine transform $$MDCT_{n\times 2n} = [\cos((k+(n+1)/2)(l+1/2)\pi/n)]$$

two-dimensional transform $$T \otimes T$$

discrete wavelet transform $$\mathrm{DTWT}_{2^n} = (\mathrm{DTWT}_{2^{n-1}} \oplus \mathrm{I}_{2^{n-1}})\underbrace{\mathrm{L}^{2^n}_{2^{n-1}}(\mathrm{I}_{2^{n-1}} \otimes_{l-2} W)}_{H}$$

Others: filtering, Haar, Hartley, …

Rules = Breakdown Strategies

$$DCT_2^{(II)} \to diag\left(1, 1/\sqrt{2}\right) \cdot F_2$$ base case

$$DCT_n^{(II)} \to P \cdot \left(DCT_{n/2}^{(II)} \oplus DCT_{n/2}^{(IV)}\right) \cdot \left(I_{n/2} \otimes F_2\right)^Q$$ recursive

$$DCT_n^{(IV)} \to S \cdot DCT_n^{(II)} \cdot D$$ translation

$$DCT_n^{(IV)} \to M_1 \cdots M_r$$ iterative

$$DFT_n \to CosDFT_n + j \cdot SinDFT_n$$ recursive

$$DFT_n \to B \cdot (DCT_{n/2}^{(I)} \oplus DST_{n/2}^{(I)}) \cdot C$$ recursive

$$DFT_{nm} \to \left(DFT_n \otimes I_m\right) \cdot D \cdot \left(I_n \otimes DFT_m\right) \cdot P$$ recursive

$$CosDFT_n \to \cdots CosDFT_{n/2} \cdots DCT_{n/4}^{(II)} \cdots$$ recursive

$$SinDFT_n \to \cdots SinDFT_{n/2} \cdots DCT_{n/4}^{(II)} \cdots$$ recursive

$$WHT_{2^n} \to \prod_{i=1}^{n} (I_{2^{n_1+\ldots+n_{i-1}}} \otimes WHT_{2^{n_i}} \otimes I_{2^{n_{i+1}+\ldots+n_t}})$$ iterative/ recursive

$$MDCT_{n \times 2n} \to S \cdot DCT_n^{(IV)} \cdot P$$ translation

$$\mathrm{DTWT}_{2^n} = \left(\mathrm{DTWT}_{2^{n-1}} \oplus \mathrm{I}_{2^{n-1}}\right) \underbrace{\mathrm{L}_{2^{n-1}}^{2^n}\left(\mathrm{I}_{2^{n-1}} \otimes_{l-2} W\right)}_{H}$$

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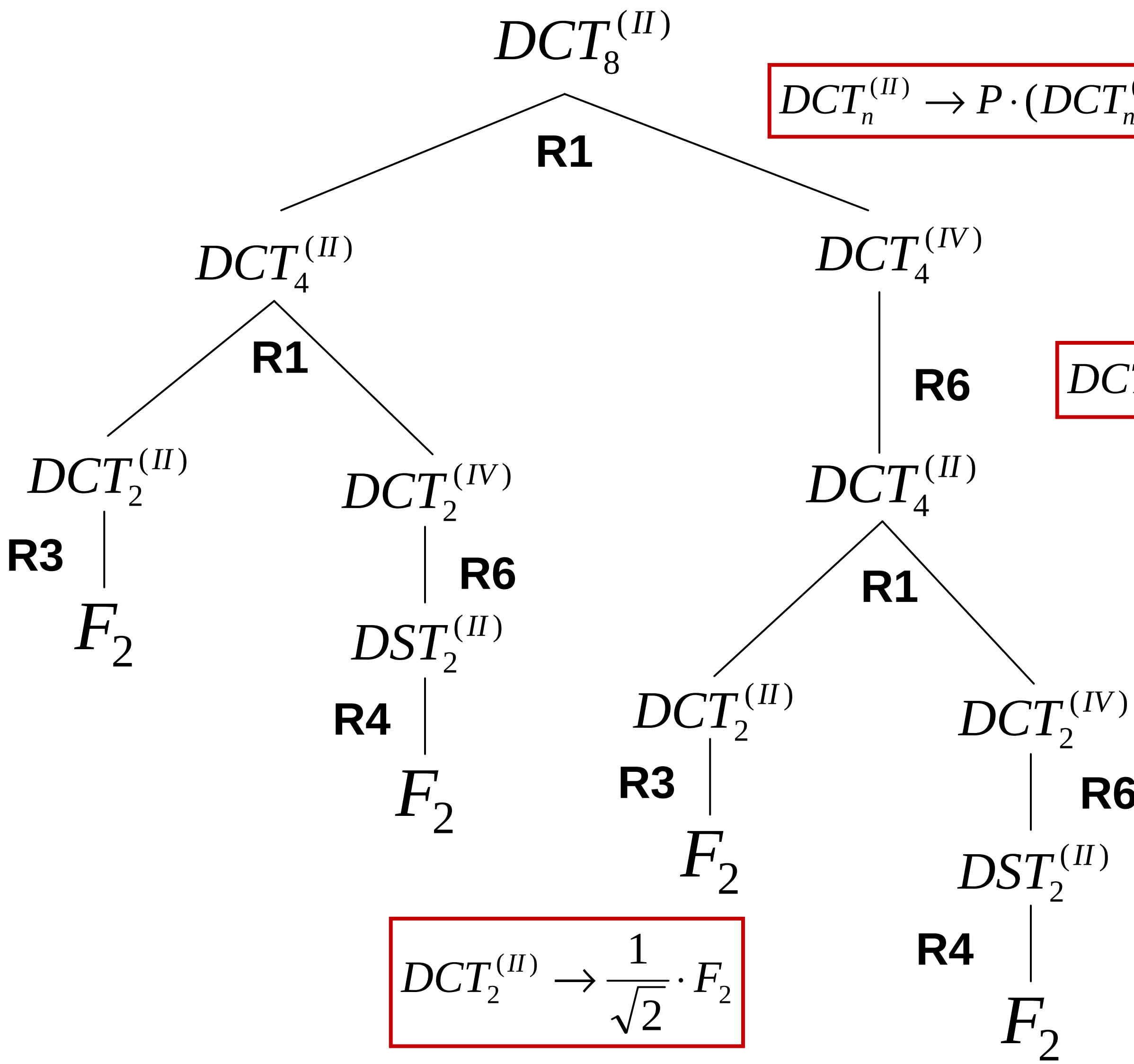
Algorithms = Ruletrees = Formulas

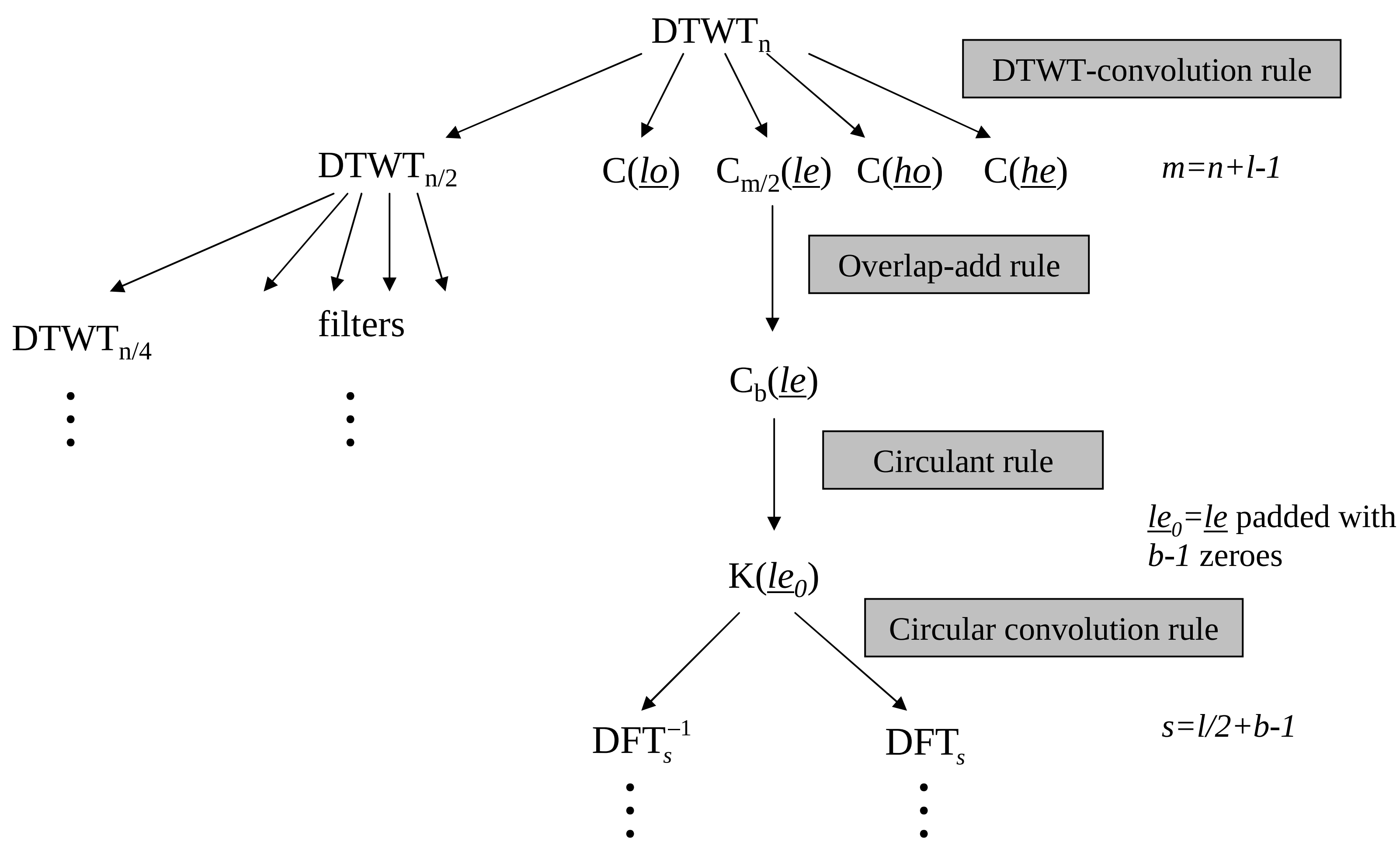


$$DCT_n^{(II)} \rightarrow P \cdot (DCT_{n/2}^{(II)} \oplus DCT_{n/2}^{(IV)}) \cdot (F_2 \otimes I_{n/2})$$

$$DCT_n^{(IV)} \rightarrow P \cdot DCT_n^{(II)} \cdot S$$

$$DCT_2^{(II)} \rightarrow \frac{1}{\sqrt{2}} \cdot F_2$$

DTWT Ruletree – Example



Formula for a DCT, size 16

$$[(2,16,9,5,3)(4,15,8,13,7)(6,14,10,12,11),16]\cdot$$
$$(([(2,8,5,3)(4,7),8]\cdot((([(2,4,3),4]\cdot((\mathrm{diag}(1,\sqrt{\tfrac{1}{2}})\cdot\mathrm{DFT}_2)\oplus([(1,2),2]\cdot\mathrm{R}_{\frac{13}{8}\pi})^{[(1,2),2]})\cdot$$
$$(\mathbf{1}_2\otimes\mathrm{DFT}_2)^{[(2,4,3),4]})\oplus(\mathrm{diag}(\tfrac{1}{2\cos(\frac{1}{16}\pi)},\tfrac{1}{2\cos(\frac{3}{16}\pi)},\tfrac{1}{2\cos(\frac{5}{16}\pi)},\tfrac{1}{2\cos(\frac{7}{16}\pi)})\cdot(\mathbf{1}_2\otimes\mathrm{DFT}_2)^{[(2,4,3),4]}\cdot$$
$$((\mathrm{DFT}_2\cdot\mathrm{diag}(1,\sqrt{\tfrac{1}{2}}))\oplus([(1,2),2]\cdot\mathrm{R}_{\frac{13}{8}\pi})^{[(1,2),2]})\cdot[(2,3,4),4]\cdot\begin{bmatrix}1&0&0&0\\1&1&0&0\\0&1&1&0\\0&0&1&1\end{bmatrix})^{[(1,4)(2,3),4]})\cdot$$
$$(\mathbf{1}_4\otimes\mathrm{DFT}_2)^{[(2,8,5,3)(4,7),8]})\oplus([(2,5,4,3,7,6,8),8]\cdot(\mathbf{1}_2\otimes(\mathbf{1}_2\oplus([(1,2),2]\cdot\mathrm{R}_{\frac{7}{4}\pi})))\cdot$$
$$(\mathbf{1}_2\otimes\mathrm{DFT}_2\otimes\mathbf{1}_2)\cdot(\mathbf{1}_4\oplus([(1,2),2]\cdot\mathrm{R}_{\frac{13}{8}\pi})\oplus([(1,2),2]\cdot\mathrm{R}_{\frac{1}{8}\pi}))\cdot(\mathbf{1}_1\otimes\mathrm{DFT}_2\otimes$$
$$\mathbf{1}_4)\cdot(([(1,2),2]\cdot\mathrm{R}_{\frac{49}{32}\pi})\oplus([(1,2),2]\cdot\mathrm{R}_{\frac{53}{32}\pi})\oplus([(1,2),2]\cdot\mathrm{R}_{\frac{57}{32}\pi})\oplus([(1,2),2]\cdot$$
$$\mathrm{R}_{\frac{61}{32}\pi}))\cdot[(2,8)(4,6),8])^{[(1,8)(2,7)(3,6)(4,5),8]})\cdot$$
$$(\mathbf{1}_8\otimes\mathrm{DFT}_2)^{[(2,16,9,5,3)(4,15,8,13,7)(6,14,10,12,11),16]}$$

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Helpful Concept

DSP Transforms		Formal Languages
DSP transform (of size)	⟷	Non-terminal symbol (with attribute)
Rule	⟷	Rule (production)
Formula/Algorithm	⟷	Element in Language (only terminals)

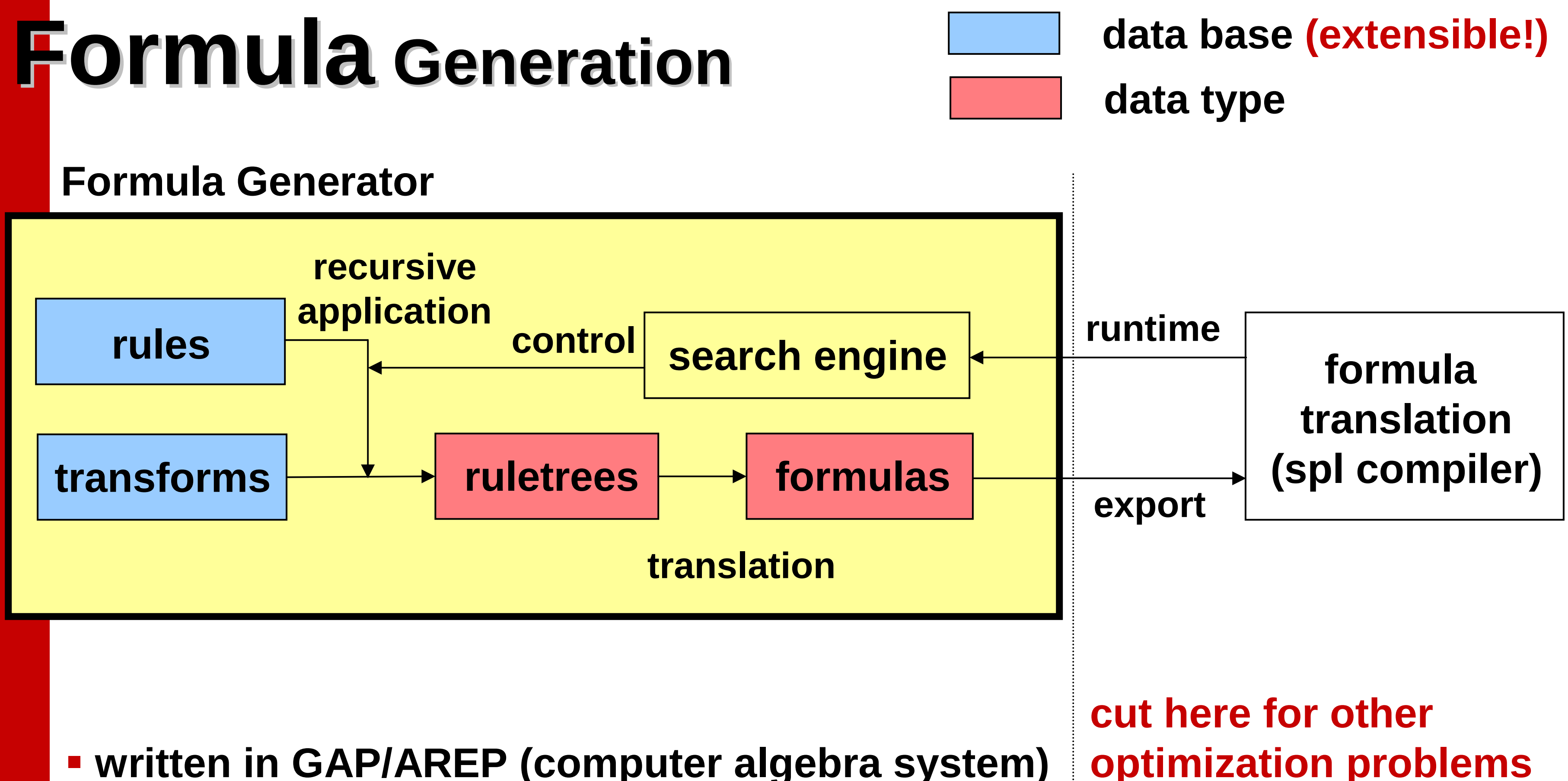
Mathematical Framework: Summary

- fast algorithms represented as ruletrees (easy generation/manipulation) and as formulas (can be translated into code)
- formulas built from few constructs and primitives
- many different algorithms/formulas generated from few rules (combinatorial explosion)
- these algorithms are (essentially) equal in arithmetic cost, but differ in data flow

Organization

- Mathematical Framework
  - **Transforms, Rules, and Formulas**
- **Formula Generator**
  - **Transform $\rightarrow$ Algorithm**
- SPL and SPL Compiler
  - **Algorithm $\rightarrow$ Implementation**
- Search Engine
  - **How to find the best implementation**
- SPIRAL system
  - **Everything taken together**
- Conclusions

Formula Generation



- written in GAP/AREP (computer algebra system)
- all computation/manipulation is symbolic
- exact arithmetic
- easy extensible rule and transform data base
- verification of rules and formulas

Organization

- Mathematical Framework

  Transforms, Rules, and Formulas

- Formula Generator

  Transform $\rightarrow$ Algorithm

- **SPL and SPL Compiler**

  Algorithm $\rightarrow$ Implementation

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Formulas in SPL

....

```
( compose
     ( diagonal ( 2*cos(1/16*pi) 2*cos(3/16*pi) 2*cos(5/16*pi) 2*cos(7/16*pi) ) )
     ( permutation ( 1 3 4 2 ) )
     ( tensor
       ( I 2 )
       ( F 2 )
     )
     ( permutation ( 1 4 2 3 ) )
     ( direct_sum
       ( compose
         ( F 2 )
         ( diagonal ( 1 sqrt(1/2) ) )
       )
       ( compose
         ( matrix
           ( 1 1 0 )
           ( 0 (-1) 1 )
         )
         ( diagonal ( cos(13/8*pi)-sin(13/8*pi) sin(13/8*pi) cos(13/8*pi)+sin(13/8*pi) ) )
         ( matrix
           ( 1 0 )
           ( 1 1 )
           ( 0 1 )
         )
         ( permutation ( 2 1 ) )
```

....

SPL Syntax (Subset)

- matrix operations:
```
(compose formula formula ...)
(tensor formula formula ...)
(direct_sum formula formula ...)
```
- direct matrix description:
```
(matrix (a11 a12 ...) (a21 a22 ...) ...)
(diagonal (d1 d2 ...))
(permutation (p1 p2 ...))
```
- parameterized matrices:
```
(I n)
(F n)
```
- scalars:
```
1.5, 2/7, cos(..), w(3), pi, 1.2e-04
```
- definition of new symbols: ← allows extension of SPL
```
(define name formula)
(template formula (i-code-list)
```
- directives for code generation
```
#codetype real/complex
#unroll on/off
```
← controls loop unrolling

SPL Compiler, 4-point FFT

```
(compose (tensor (F 2) (I 2)) (T 4 2)
  (tensor (I 2) (F 2)) (L 4 2))
```

fast algorithm
as
formula
as
SPL program

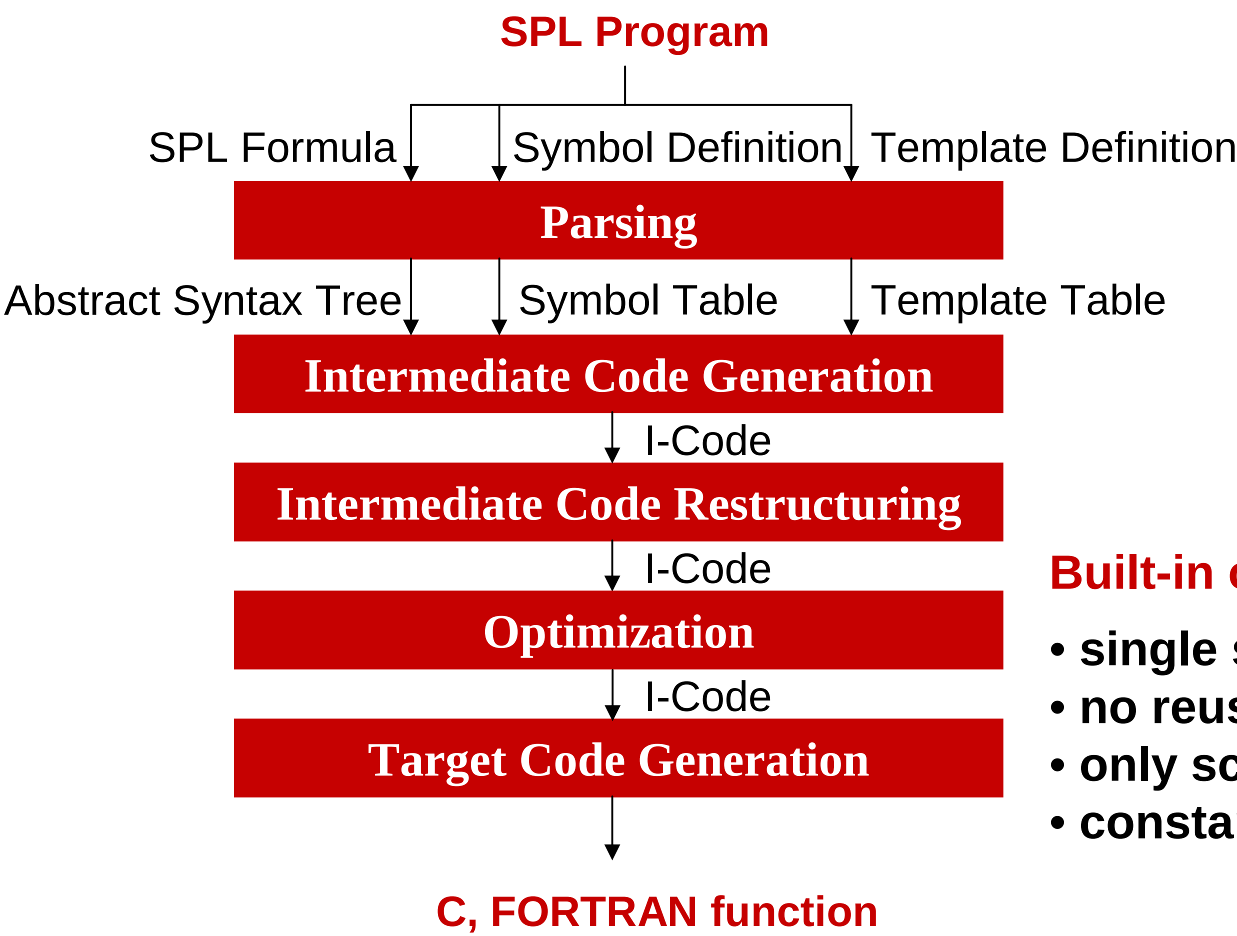
#codetype

complex | real

```
f0   = x(1) + x(3)
f1   = x(1) - x(3)
f2   = x(2) + x(4)
f3   = x(2) - x(4)
f4   = (0.00d0,-1.00d0)*f(3)
y(1) = f0 + f2
y(2) = f0 - f2
y(3) = f1 + f4
y(4) = f1 - f4
```

```
r0   = x(1) + x(5)
r1   = x(1) - x(5)
r2   = x(2) + x(6)
r3   = x(2) - x(6)
r4   = x(3) + x(7)
r5   = x(3) - x(7)
r6   = x(4) + x(8)
r7   = x(4) - x(8)
y(1) = r0 + r4
y(2) = r1 + r5
y(3) = r0 - r4
y(4) = r1 - r5
y(5) = r2 + r7
y(6) = r3 - r6
y(7) = r2 - r7
y(8) = r3 + r6
```

SPL Compiler: Summary



Built-in optimizations:

- **single static assignment code**
- **no reuse of temporary vars**
- **only scalar temporary vars**
- **constants precomputed**

Extensible through templates

SIMD Short Vector Extensions

vector length = 4
(4-way)

- Extension to instruction set architecture
- Available on most current architectures
  (SSE on Pentium, AltiVec on Motorola G4)
- Originally for multimedia (like MMX for integers)
- Requires fine grain parallelism
- Large potential speed-up

Problems:

- SIMD instructions are architecture specific
- No common API (usually assembly hand coding)
- Performance very sensitive to memory access
- Automatic vectorization very limited

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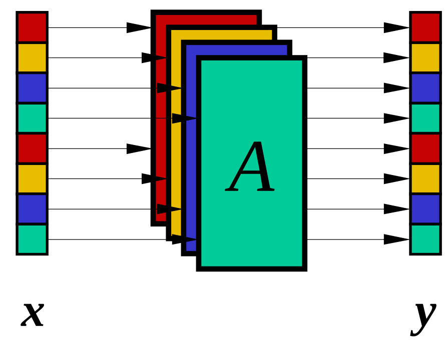
Vector Code Generation from SPL Formulas

Naturally vectorizable construct

$$A \otimes I_4$$

vector length



(Current) generic construct completely vectorizable:

$$\prod_{i=1}^{k} P_i D_i (A_i \otimes I_\nu) E_i Q_i$$

P_i, Q_i	**permutations**
D_i, E_i	**diagonals**
A_i	**arbitrary formulas**
ν	**SIMD vector length**

Vectorization in two steps:

1. **Formula manipulation using manipulation rules**
2. **Code generation (vector code + C code)**

Organization

- Mathematical Framework

  Transforms, Rules, and Formulas

- Formula Generator

  Transform $\rightarrow$ Algorithm

- SPL and SPL Compiler

  Algorithm $\rightarrow$ Implementation

- **Search Engine**

  How to find the best implementation

- SPIRAL system

  Everything taken together

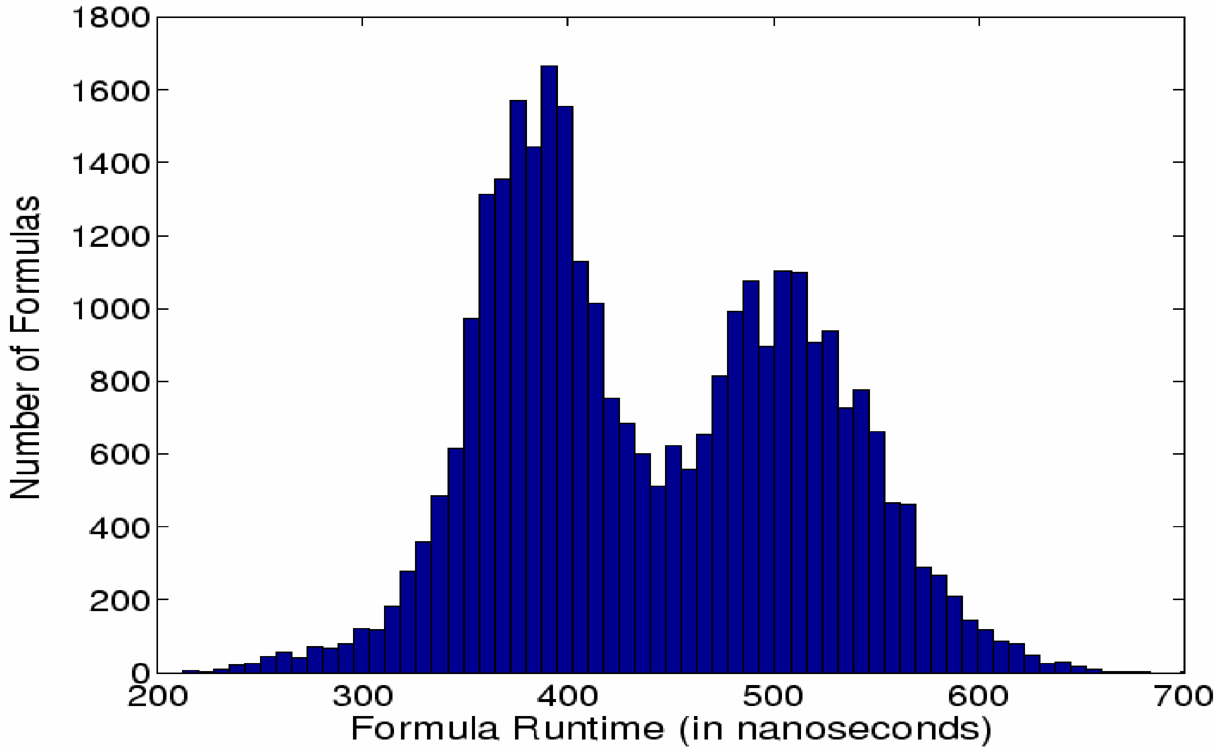
- Conclusions

Number of Formulas/Algorithms

k	# DFT, size 2^k	# DCT-IV, size 2^k
1	1	1
2	6	10
3	40	126
4	296	31242
5	27744	1924443362
6	162570361280	7343815121631354242
7	~1.01 • 10^27	~1.07 • 10^38
8	~2.31 • 10^61	~2.30 • 10^76
9	~2.86 • 10^133	~1.06 • 10^153

➡
- differ in data flow not in arithmetic cost
- exponential search space

Why Search?



DCT, type IV, size 16

~31000 formulas

- **maaaany different formulas**
- **large spread in runtimes, even for modest size**
- **not due to arithmetic cost**

Search Methods Available in SPIRAL

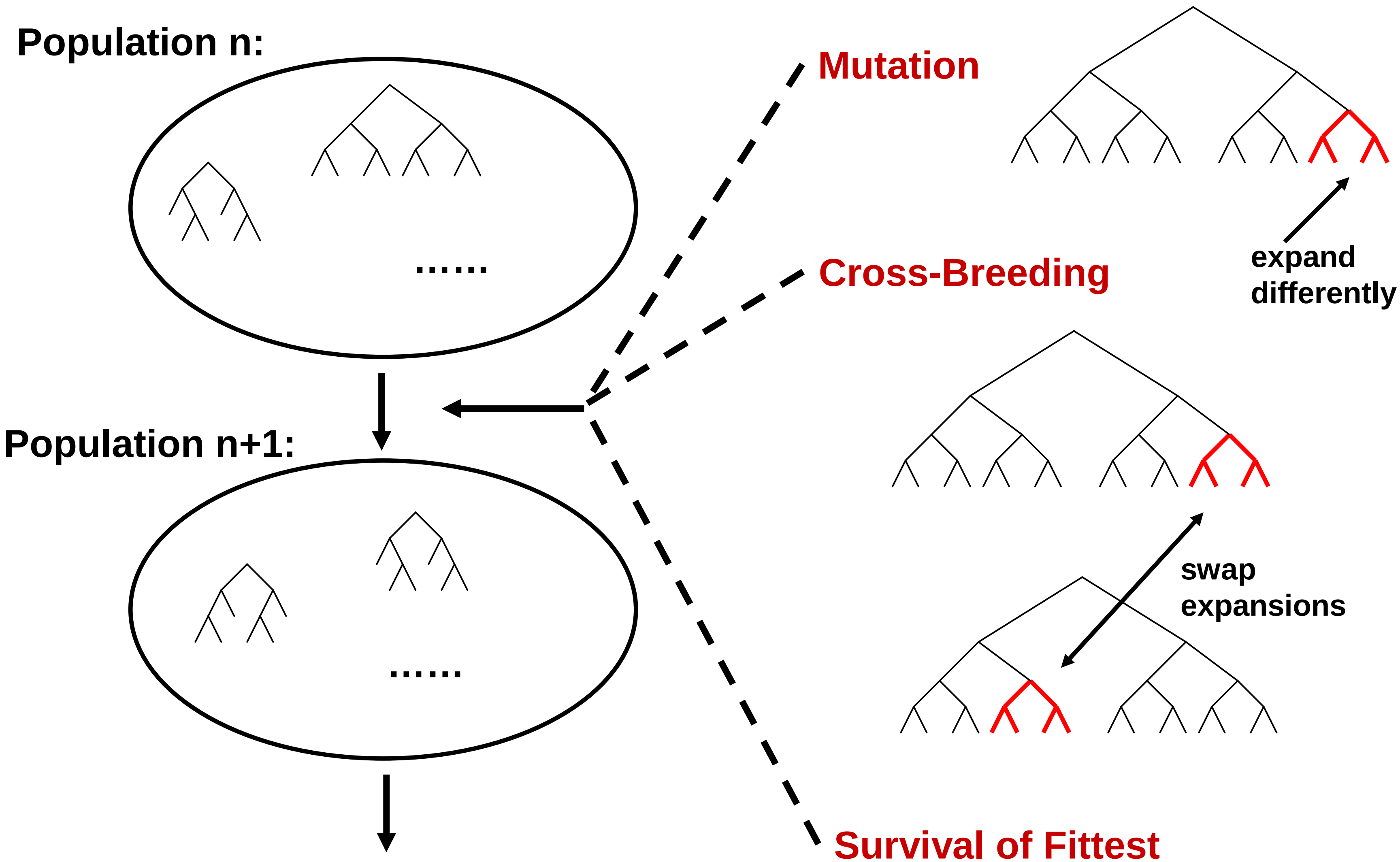
- Exhaustive Search
- Dynamic Programming (DP)
- Random Search
- Hill Climbing
- STEER (similar to a genetic algorithm)

	Possible Sizes	Formulas Timed	Results
Exhaust	Very small	All	Best
DP	All	10s-100s	(very) good
Random	All	User decided	fair/good
Hill Climbing	All	100s-1000s	Good
STEER	All	100s-1000s	(very) good

Search over

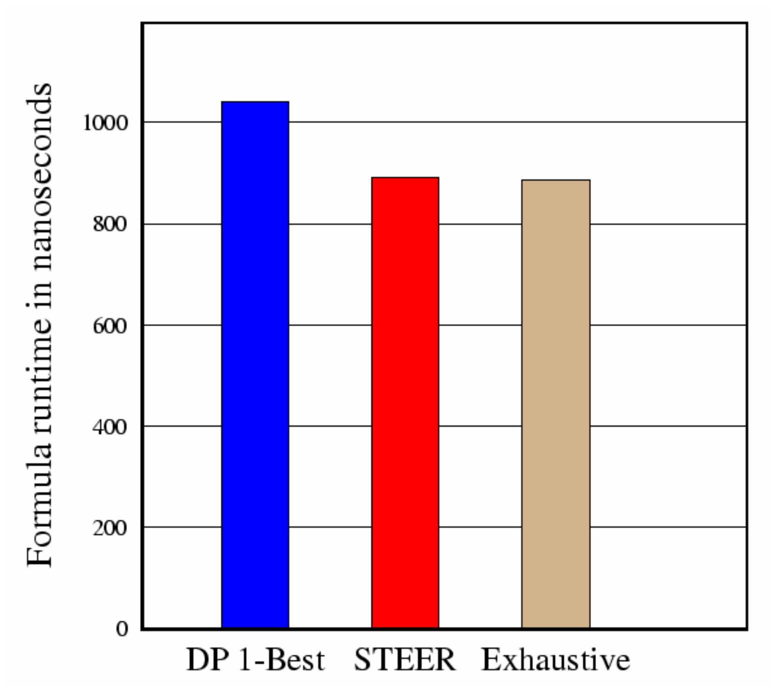
- algorithm space and
- implementation options (degree of unrolling)

STEER

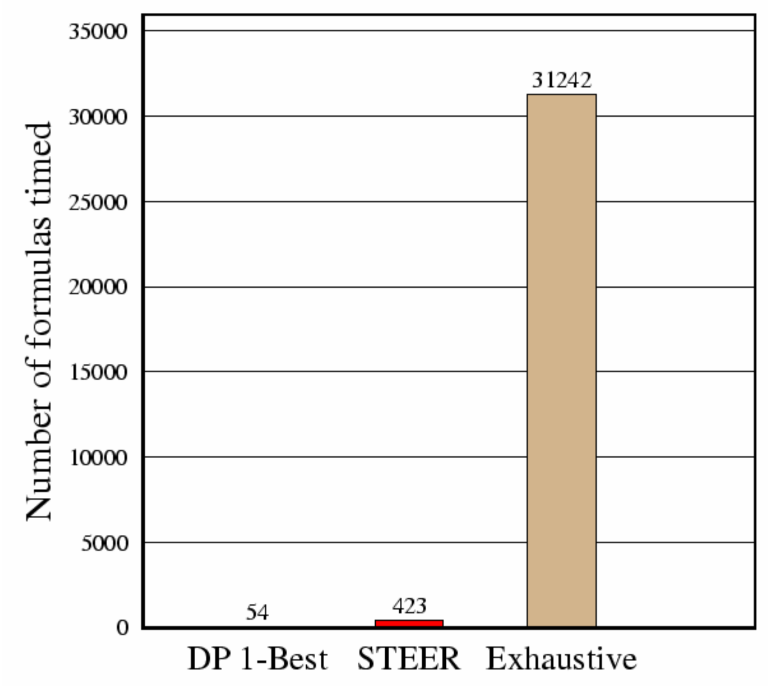

Population n:
......
Population n+1:
......
Mutation
expand
differently
Cross-Breeding
swap
expansions
Survival of Fittest

DCT Type IV Size 16

Fastest Found Formulas

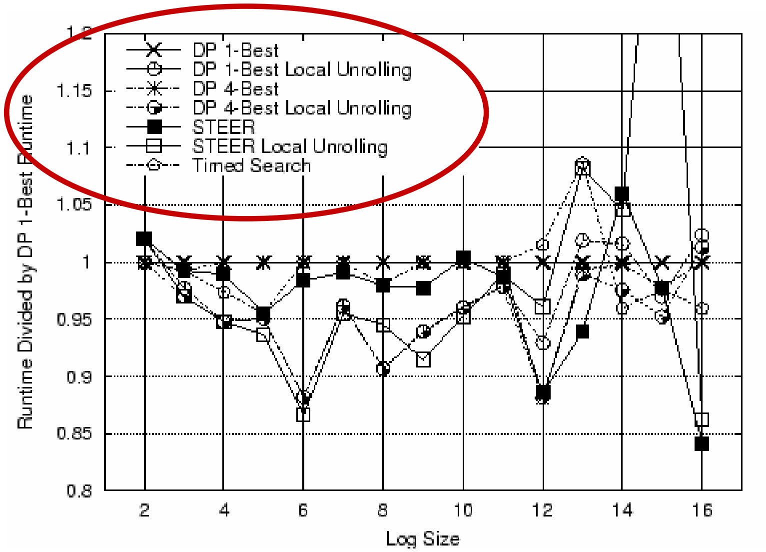


Number of Formulas Timed

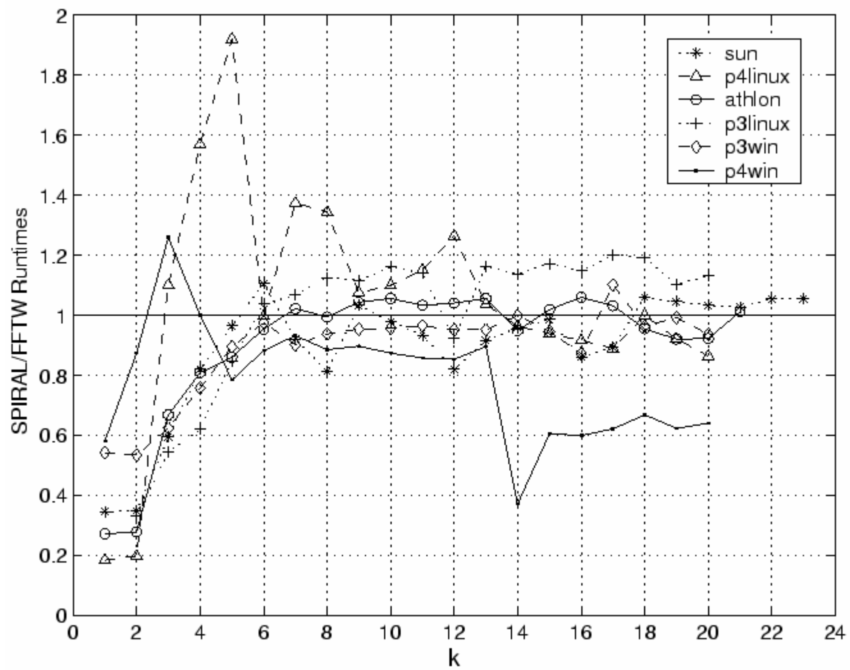


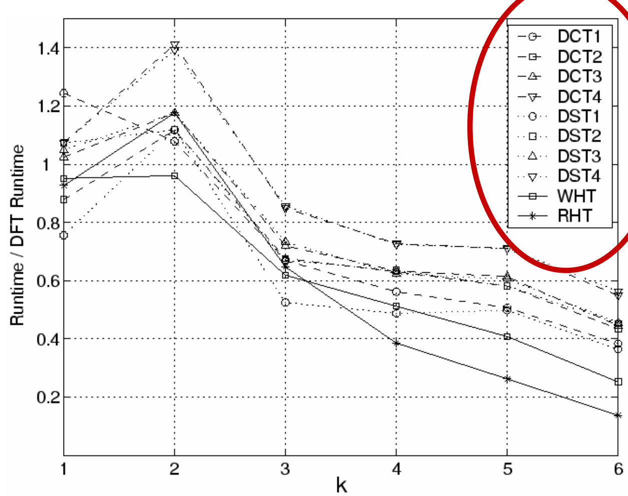
Experimental Results

search methods
(applicable to all transforms)



high performance code
(compared with FFTW)





different transforms

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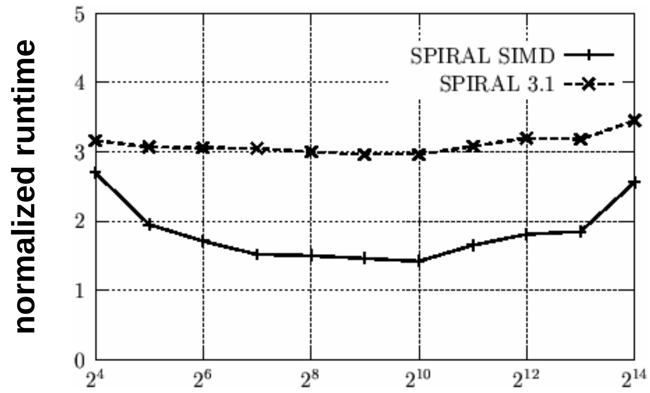
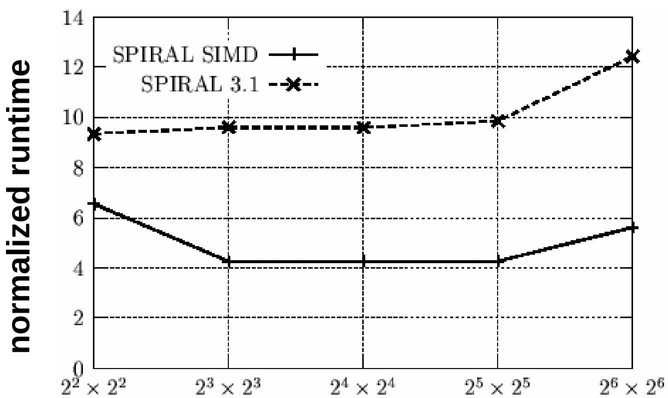
Vectorized Code

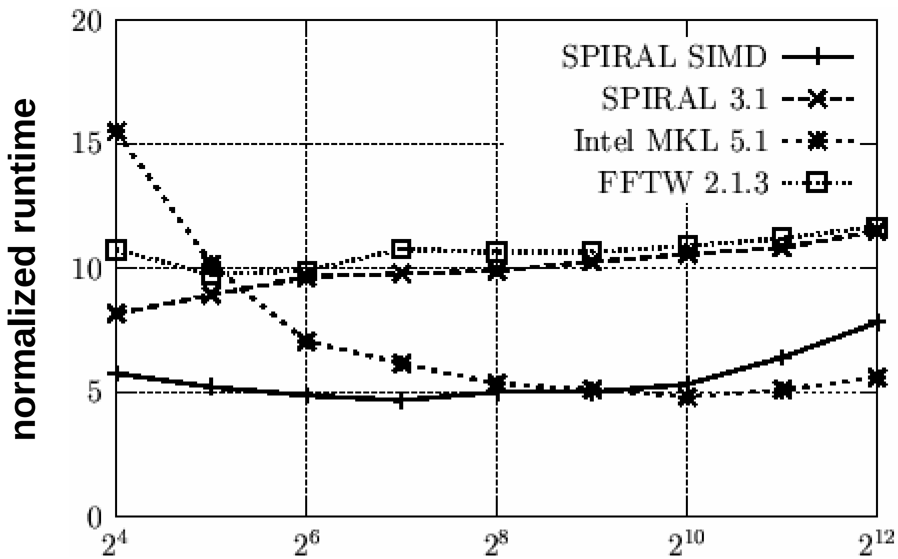
WHT



2-dim DCT



DFT



- speed-ups up to a factor of 2.5
- beats hand-tuned Intel MKL (< 1024)
- SIMD platforms supported

(Pentium III, SSE)

SPIRAL

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Learning to Generate Fast Algorithms

- Learns from given dataset (formulas+runtimes) how to design a fast algorithm (breakdown strategy)
- Learns from a transform of one size, generates the best algorithm for many sizes
- Tested for DFT and WHT

Fast Formula Generation Results

Size	Number of Formulas Generated	Generated Included the Fastest Known	Top N Fastest Known Formulas in Generated
2^{12}	101	yes	77
2^{13}	86	yes	4
2^{14}	101	yes	70
2^{15}	86	yes	11
2^{16}	101	yes	68
2^{17}	86	yes	15
2^{18}	101	yes	25
2^{19}	86	yes	16
2^{20}	101	yes	16

Organization

- Mathematical Framework
  - Transforms, Rules, and Formulas
- Formula Generator
  - Transform $\rightarrow$ Algorithm
- SPL and SPL Compiler
  - Algorithm $\rightarrow$ Implementation
- Search Engine
  - How to find the best implementation
- **SPIRAL system**
  - Everything taken together
- Conclusions

SPIRAL

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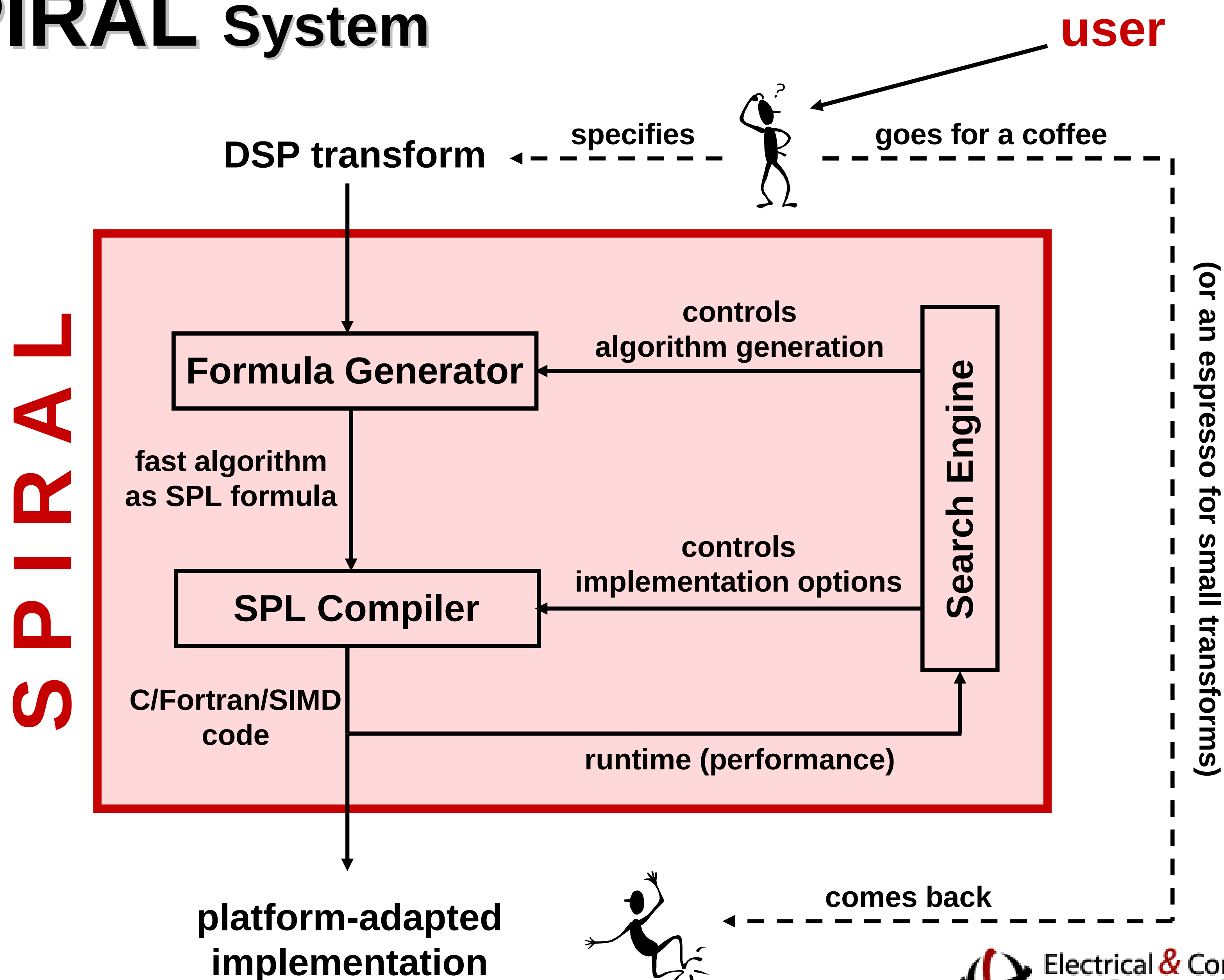
SPIRAL System



Extensibility of SPIRAL

New transforms are readily included on the high level

(easy, due to SPIRAL's framework)

New constructs and primitives (potentially required by radically different transforms) are readily included in SPL

(moderate effort, due to template mechanism)

New instructions sets available (e.g., SSE) are included by extending the SPL compiler

(doable one time effort)

SPIRAL System: Summary

- Available for download: `www.ece.cmu.edu/~spiral`
- Easy installation (Unix: configure/make; Windows: install shield)
- Unix/Linux and Windows 98/ME/NT/2000/XP
- Current transforms: DFT, DHT, WHT, DCT/DST type I – IV, MDCT, Filters, Wavelets, Toeplitz, Circulants
- Extensible

Organization

- Mathematical Framework

  Transforms, Rules, and Formulas
- Formula Generator

  Transform → Algorithm
- SPL and SPL Compiler

  Algorithm → Implementation
- Search Engine

  How to find the best implementation
- SPIRAL system

  Everything taken together
- **Conclusions**

Conclusions

Closing the gap between math domain (algorithms) and implementation domain (programs)

- **Mathematical computer representation of algorithms**
- **Automatic translation of algorithms into code**

Optimization as intelligent search/learning in the space of alternatives

- **High level: Mathematical manipulation of algorithms**
- **Low level: Coding degrees of freedom**