EFFICIENT 2D SHAPE ORIENTATION

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ABSTRACT

In this paper, we study the reorientation of 2D shapes. We describe an algorithm that removes orientational ambiguity from arbitrarily oriented 2D shapes. The algorithm is robust to error in pixel locations as well as in the presence of occluded or added pixels. After reorientation, the resulting shape is in a normalized orientation and can then be used effectively in post-processing stages of such applications as pattern detection, recognition, and registration. The algorithm combines a new measure of shape orientation, the variable-size window orientation indicator index (Δ–OII), and the point-based reorientation algorithm (PRA) that we presented before. We test the new algorithm against an extensive database of complex 2D shapes.

1. INTRODUCTION

Determining the orientation of 2D shapes is an important research topic. In application areas such as automatic target detection, recognition, and registration, it is essential that the patterns extracted from the image be brought consistently to the same orientation before any further processing can begin. Yet, the existing techniques do not provide an efficient and complete solution to the problem of 2D shape orientation. Existing methods solve the problem only for certain shapes while failing with others or they require extensive computational effort. We briefly describe the limitations of these methods below and contrast them with the new algorithm that we propose in this paper.

Review of the Literature The principal axis of a shape is the axis of the least moment of inertia [1]. It is defined as the straight line through the origin that makes an angle θ with the x–axis where θ is computed from the second order central moments of the shape. The principal axis cannot be used for certain shapes: for example, it is noted in [2] that the principal axis fails with shapes that are rotationally symmetric. Other techniques extending the principal axis approach include the generalized principal axis [2] and the fold principal axis [3]. These techniques are applicable only to rotationally symmetric shapes (RSS) when the fold number of the shape is known a priori. Techniques such as shape matrices [4], mirror-symmetry axes [5], shape specific points [6], fold-invariant shape-specific points [7], and modified Fourier descriptor [8], are short of being universal. That is, either they work only with certain classes of shapes (e.g., RSS, non-RSS, non-mirror symmetric) or fail with certain shapes [9], such as for example the shapes in Figure 1. The Shen-Ip symmetries detector [10] is concerned with the problem of detecting the reflectional and rotational symmetry axes of a shape. The algorithm requires the computation of all orders of the shape's generalized complex (GC) moments, not just a finite number of them. A practical implementation of this algorithm attempts to find at least three non-zero GC moments by computing up to 30th order GC moments of the shape, which is still inadequate for some of the shapes tested in their experiments.

PRA–Δ–OII In [11], we presented the point-based reorientation algorithm (PRA) to remove the orientational ambiguity of 2D shapes. The PRA determines the fold number of the shape and removes both the rotational and reflectional ambiguities. The algorithm is efficient with computational complexity $\mathcal{O}(N \log N)$, where N is the number of feature points in the shape. In practice, however, the exact locations of the feature points may be measured inaccurately due to the finite resolution of the input device and due to background noise. The PRA may be sensitive to such disturbances. To improve the robustness of the PRA to error and noise, we introduce here a moment-based measure of the orientation, referred to as the variable-size window orientation indicator index (Δ–OII). This measure is computed from the 3rd-order central moments of the shape and improves the orientation indicator index (OII) that we introduced in [12]. The coupled PRA–Δ–OII algorithm is robust to errors, such as round-off errors in pixel locations or missing/added feature points as shown by testing the algorithm

This work was partially supported by ONR grant # 14-0-1-0593.

0-7803-7750-8/03/$17.00 ©2003 IEEE

against 200 complex 2D shapes from the database [10].

Outline of the paper Section 2 introduces the variable-size window OII (Δ–OII), illustrating it with several difficult shapes from the database. Section 3 combines the Δ–OII with the PRA into a robust algorithm for shape orientation and demonstrates its good performance with several examples. Section 4 summarizes the paper.

2. VARIABLE-SIZE WINDOW OII (Δ–OII)

This section defines the variable-size window orientation indicator index (Δ–OII) that monitors the orientation of 2D shapes. We verify the properties of the Δ–OII with the shapes in Figure 1 that other methods have failed to work with. Although not presented here due to lack of space, we have successfully tested the Δ–OII against 200 symmetric and non-symmetric shapes provided by the database [10].

The shape is assumed to be centered with respect to its center of mass. The Δ–OII is defined from the third order central moments of the centered shape, but computed from the portion of the shape falling within a $\frac{2\pi}{\Delta}$–window where $\Delta = 4k$ for a positive integer k. The $\frac{2\pi}{\Delta}$–window is the region $\mathcal{W}$ of the coordinate plane enclosed between the positive x–axis and the straight line through the origin that makes the angle $\frac{2\pi}{\Delta}$ rads with the x–axis. This is the window that contains all points (x, y) where $x \geq 0$ and $y \leq x \tan(\frac{2\pi}{\Delta})$. For example, the $\frac{\pi}{2}$-window is the first quadrant of the coordinate plane.

The Δ–OII of the shape $\mathbf{X}$ consisting of N feature points is defined as

$$\begin{aligned} \Delta\text{–OII}_{\mathbf{X}} &= \sqrt{\mu_x^2 + \mu_y^2} \\ \mu_x &= \sum_{k \in \mathcal{W}} x_k^3 \quad \text{and} \quad \mu_y = \sum_{k \in \mathcal{W}} y_k^3 \\ \mathcal{W} &= \{(x_k, y_k) \in \mathbf{X}, \forall x_k, y_k \in \tfrac{2\pi}{\Delta}\text{–Window}\} \end{aligned}$$

The third order central moments μ_x and μ_y as computed for the Δ–OII do not vanish (except for the trivial shape of a single point at the origin). This is because every $\frac{2\pi}{\Delta}$–window is located within the first quadrant of the coordinate plane where the coordinate values of the points (x_k, y_k) are greater than zero. Moreover, whenever the shape rotates by an angle θ, the Δ–OII is computed from the different portion of the shape that currently falls within the window. The Δ–OII plot is therefore generated by plotting the value of Δ–OII at each position rotated by an angle $\theta = [0, 2\pi]$. We now list properties of the Δ–OII.

Property 1 If a shape is r-fold rotational symmetric, we can find a Δ such that the Δ–OII plot is periodic in the rotation angle θ with the period $T = 2\pi/r$.

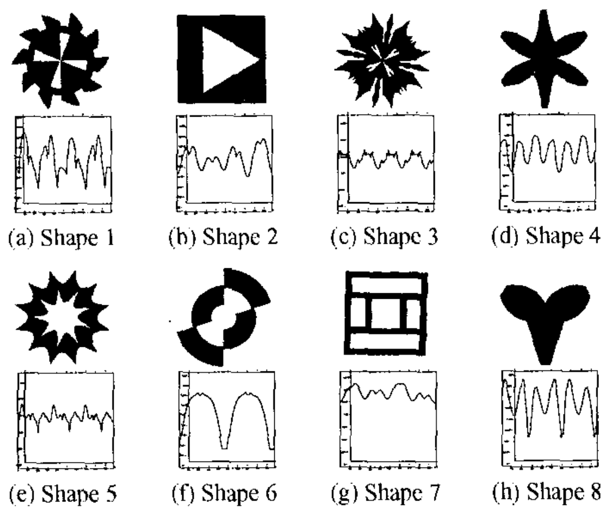
Property 2 If the Δ–OII plot of a shape is periodic with $T = 2\pi/k$, the shape is r-fold rotational symmetric where $1 \leq r \leq k$.

Property 3 If a shape is a circle or a union of rings, its Δ–OII plot is flat at a constant $K > 0$ over the entire rotation range $\theta = [0, 2\pi]$.

Property 4 The rotation of a shape circularly shifts its Δ–OII plot while the reflection of a shape reverses its Δ–OII plot.

These properties are easily verified and readers are referred to [13] for details. The next paragraph illustrates the above properties of the Δ–OII plot by testing the index against the two hundred complex 2D shapes in [10].

We show below each of the shapes in Figure 1 with the corresponding Δ–OII plot. As discussed before, many existing methods fail with with shapes. We easily observe that the Δ–OII plots of these shapes reflect the rotational symmetry of the shape through their periodicity, i.e., the fold number of the shape equals the periodicity of the plot as stated in Property 1.



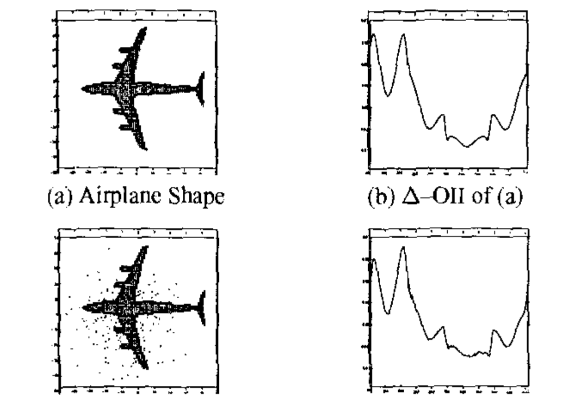
(a) Shape 1 (b) Shape 2 (c) Shape 3 (d) Shape 4

(e) Shape 5 (f) Shape 6 (g) Shape 7 (h) Shape 8

Fig. 1. Difficult Shapes and their Δ–OII Plots

We used $\Delta = \pi/2$ for all shapes in Figures 1, except for Shape 1 in Figure 1 (a) for which we used the $\pi/4$–window. We have also computed the Δ–OII for about 200 symmetric and non-symmetric shapes from the database [10]. The Δ–OII plots using the $\pi/2$-window (i.e., the first quadrant) exhibit the correct periodicity representing the fold number of the shape for all but nine of these shapes. The Δ–OII plots using the $\pi/4$–window resolved the problems with the $\pi/2$–window for these nine shapes.

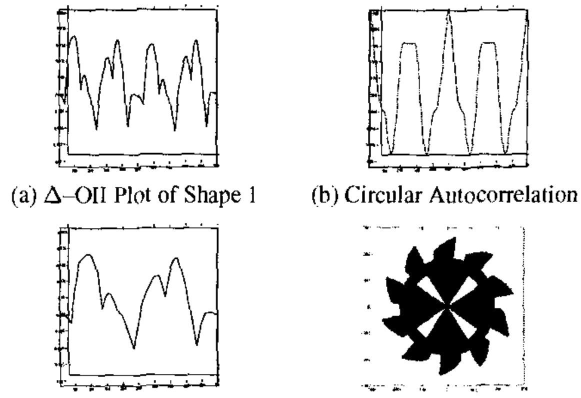
We also consider the robustness of the Δ–OII to errors, such as round-off errors in the coordinate values of the pixel points, missing or erroneously added pixel points. We illustrate this with the shape of the airplane shown in Figure 2 (a) defined by 1208 feature points and its Δ–OII plot shown in Figure 2 (b). We randomly add erroneous pixel points to



(a) Airplane Shape (b) Δ–OII of (a)

(c) 242 Added Points (20%) (d) Δ–OII of (c)

Fig. 2. Robustness to Error



(a) Δ–OII Plot of Shape 1 (b) Circular Autocorrelation

(c) One period of Δ–OII (d) Reoriented Shape 1

Fig. 3. Example: orienting Shape 1

this shape. Figure 2 (c) shows the resulting shape when 242 erroneous points are randomly added. This is 20 % of the total number of points in the original shape. Figure 2 (d) shows that the corresponding Δ–OII plot is essentially unaffected. The Δ–OII plot is still fairly unaffected even when the shape is corrupted by up to 50 % additional pixel points. More importantly, this distortion has little impact on the location of the peak of the Δ-OII plot, which is the significant feature needed to orient the shape.

3. SHAPE ORIENTATION: PRA–Δ–OII

We present an algorithm for shape orientation using the Δ–OII and the PRA. Readers are referred to [11, 13] for details on the PRA. The basic idea for the combined algorithm is as follows. Since the Δ–OII plot is a function of the rotation angle θ over the full range $\theta = [0, 2\pi]$, if we locate a unique peak point of the plot and orient (i.e., rotate and reflect) the shape until this unique point is brought to the origin at $\theta = 0$, the shape is uniquely normalized in its orientation. To achieve this, we first determine the fold number of the shape from its Δ–OII plot. Then, we select one period of the plot and denote this partial Δ–OII plot as $\mathbf{L}$. Next, we apply the PRA to $\mathbf{L}$ to find the unique peak in $\mathbf{L}$; the PRA requires that we compare the right-side and left-side neighbors of the maxima in $\mathbf{L}$ to locate the unique peak. Once this unique peak is found, we perform an appropriate rotation and reflection to bring the shape to its unique orientation.

Lemma 1 *The periodicity of a Δ–OII plot is obtained by computing the circular autocorrelation of the plot and counting the number of peaks with magnitude 1 in the resulting autocorrelation plot.*

A Δ–OII plot with periodicity d is periodic in the rotation angle θ with period $T = 2\pi/d$. That is, the plot is indistinguishable every time it is circularly shifted by the rotation $\theta = 2\pi/d$. This happens d times over the circular shift of the plot in the computation of the circular autocorrelation, resulting in d peaks with magnitude 1 in the autocorrelation plot. For example, consider a Δ–OII plot with periodicity $d = 2$ in Figure 3 (a). This is the plot obtained from Shape 1 in Figure 1 (a) using a $\pi/4$–window. Its circular autocorrelation is shown in Figure 3 (b). We see that there are two peaks in the autocorrelation plot with magnitude 1 as expected.

Conjecture 1 *The fold number of a shape is determined by the periodicity of that Δ–OII plot whose periodicity is unchanged across the window sizes $\Delta = 2\pi/4k$ and $2\pi/4(k+1)$ for an integer k.*

Although we have been unable to prove this conjecture yet, we have verified it for all the 200 shapes in the database [10]. In our experiments, we have observed that the window size of $\Delta = \pi/4$ solves the fold number problem for every shape tested.

Given the Δ–OII plot with periodicity d matched to the fold number of the shape, we designate the fundamental period of the plot in the domain $\theta \in [0, 2\pi/d]$ by $\mathbf{L}$. We apply to $\mathbf{L}$ the PRA, see [11], to locate in a unique way a specific value $\theta^* \in [0, 2\pi/d]$ that defines the so called *reorientation* point, and then rotate and reflect the shape until this unique *reorientation* point is brought to the origin at $\theta = 0$. Applying this for example to Shape 1, we take the first half of the Δ–OII plot to be the non-periodic portion $\mathbf{L}$ as shown in Figure 3 (c). There is a single peak in the list $\mathbf{L}$ located at $\theta = 3\pi/16$. We also observe that the left-side of this peak in $\mathbf{L}$ has larger values than the right-side of the peak. According to the PRA, this defines the reorientation point. We thus first rotate the shape by an angle $\theta = -3\pi/16$ and then reflect it about the x-axis to arrive at the unique orientation. This unique normalized orientation for Shape 1 is shown in Figure 3 (d).

The shape reorientation algorithm using the Δ-OII and the PRA is as follows. To start, we first set the counter $k = 1$. Then, we carry out the following steps.

Step 1 Given a shape, generate a pair of Δ-OII plots using $\frac{2\pi}{4k}$- and $\frac{2\pi}{4(k+1)}$-windows.
Step 2 Compute their periodicities d_1 and d_2.
Step 3 If $d_1 = d_2$, define $\mathbf{L}$ as the portion of the OII plot generated by the $\frac{2\pi}{4k}$-window for $\theta = [0, 2\pi/d_1]$. Otherwise, set $k = k + 1$ and go to Step 1.
Step 4 Locate and denote the unique peak in $\mathbf{L}$ using the PRA and denote it as the reorientation point.
Step 5 Rotate and reflect the shape until this unique reorientation point is brought to $\theta = 0$.

4. SUMMARY

The paper presents a novel algorithm to reorient *automatically* in a unique way arbitrarily oriented 2D shapes. This is an important problem in pattern recognition, shape detection, automatic target classification, and numerous other applications. The algorithm described is robust to distortions such as errors in pixel location, or missing or extraneous pixels. The algorithm combines the point reorientation algorithm (PRA) described in [11] with a moment based orientation indicator index (OII). The OII is defined for a variable window Δ. We presented the properties of the Δ-OII and related its periodicity to the r-fold symmmetry of the shape. The combined PRA–Δ–OII is shown to reorient in a unique way arbitrarily rotated shapes, including difficult shapes that have caused other existing algorithms to fail. We also show through examples that the algorithm is robust to significant shape distortions. In [11], we show that the reorientation step is one of four steps in finding what we term there the *intrinsic* shape of an object—the shape of the object that is robust to affine and permutation distortions. Affine distortions include translation, rotation, reflection, and also non-isotropic scaling and shearing. Permutation distortions arise when the actual scanning order is unknown, which is commonly the case when an object is for example rotated or translated. When combined with the remaining steps in [11], the PRA–Δ–OII algorithm provides a robust algorithm to find the *intrinsic* shape of an arbitrarily affine-permutation distorted object.

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