Detection Performance of the L_1 Beamformer in the Presence of Underwater Burst Noise

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Abstract

In this paper, the problem of detecting a directional random signal in independent non Gaussian noise is addressed. In particular, we assume a Gauss-Gauss mixture model for the total noise field. The approach that we use consists on the substitution of the MMSE beamformer included in the optimum quadratic receiver for the Gaussian signal in independent Gaussian noise problem by a beamformer whose design is based on a least absolute value criterion. The presented analytical study and the reported simulation experiments show the robustness of the resulting suboptimum receiver in the presence of the unexpected impulsive noise field.

1. Introduction

It is known that the optimum detector of a Gaussian signal in independent Gaussian noise uses a sufficient statistic which can be obtained computing the cross-correlation between the observations process and the minimum mean square error (MMSE) estimate of the signal to be detected, [1]. In array signal processing, this estimate is the output of a MMSE beamformer which, as it is shown in [2], is the best quadratic (L_2 norm) field reconstructor.

In this paper, we study the behavior of the optimum array receiver when detecting a far field narrow-band Gaussian signal in independent Gaussian noise and in the presence of outliers, e.g., underwater burst noise. For this case, the total noise field is no longer Gaussian, but it can be represented by a Gauss-Gauss mixture model, [4]. It results from the superposition of a background Gaussian noise plus a strong impulsive noise field, also specified as Gaussian. The latter models equipment misfunction or ambient noise as, e.g., ice cracks. The presence of the unexpected impulsive noise field is responsible for the performance degradation of the optimum quadratic detector. In [3], we introduced an L_1 beamformer which is shown to be robust to the presence of outliers, namely the unexpected impulsive noise field. Recognizing this fact, we substitute, in the optimum array detector, the MMSE beamformer by the L_1 beamformer.

In section 2, we present these two implementations of the wavefront detector. In section 3, we compare the performance of the quadratic detector with that of the L_1 detector when the unexpected impulsive noise field is present. In section 4, we describe several simulations confirming the presented analytical results. Finally, in section 5 we discuss the main conclusions of this paper.

2. Implementation of the wavefront detector

Let $z(.)$ be the vector of complex envelopes of the narrow-band signals observed on a linear and uniform array of N sensors:

$$z(k+1) = a(\theta)x(k+1) + n(k+1), k = 0, 1, \dots \quad (1)$$

where $x(.)$ is a sequence of zero mean Gaussian complex random variables with covariance σ_x^2 and $n(.)$ is a sequence of zero mean Gaussian complex vectors with covariance matrix $R_n = \sigma_0^2 I$ and statistically independent of $x(.)$. In (1), $a(\theta)$ is the Vandermonde steering vector of the impinging signal. We assume that the signal complex envelope $x(.)$ is a first order Gauss-Markov discrete sequence

$$x(k+1) = fx(k) + u(k+1), k = 0, 1, \dots \quad (2)$$

with a zero mean initial condition $x(0) = \underline{x}_0$. The zero mean input sequence $u(.)$ has a covariance $\sigma_u^2 = (1 - |f|^2)\sigma_x^2$ and is statistically independent of $\underline{x}_0$.

Optimum quadratic detector.
The optimum quadratic detector for the Gaussian signal in independent Gaussian noise problem uses the sufficient statistic[1]

$$\ell = \frac{1}{K}\sum_{k=1}^{K} z^+(k)a(\theta)\hat{x}(k), \tag{3}$$

where K is the number of observed time samples (snapshots) and $\hat{x}(.)$ is the MMSE estimate of the impinging signal $x(.)$, [1]. Using the model described by (1) and (2), the MMSE estimate $\hat{x}(.)$ is provided by the L_2**-recursive beamformer**, [5]:

1. Initialization

$$\hat{x}_0 = 0 \qquad P(0) = 1 \tag{4}$$

2. Prediction

$$\hat{x}_k(k+1) = f\hat{x}_k(k) \tag{5}$$

$$P_k(k+1) = |f|^2 P_k(k) + \sigma_u^2 \tag{6}$$

3. Residues and Kalman gain

$$r(k+1) = z(k+1) - a(\theta)\hat{x}_k(k+1) \tag{7}$$

$$K(k+1) = \frac{\frac{1}{\sigma_0^2}P_k(k+1)a^+(\theta)}{\alpha + \frac{N}{\sigma_0^2}P_k(k+1)} \tag{8}$$

4. Filtering

$$\hat{x}_{k+1}(k+1) = \hat{x}_k(k+1) + K(k+1)r(k+1) \tag{9}$$

5. Updating

$$P_{k+1}(k+1) = [1 - K(k+1)a(\theta)]\,P_k(k+1) + (1-\alpha)\sigma_0^2 K(k+1)K^+(k+1) \tag{10}$$

In equations (8) and (10), α is a regularizing parameter adjusting the confidence placed on the prior knowledge, here represented by the state equation (2), versus the reliance on the observations (1). P denotes the output error covariance matrix.

L_1 detector
The L_1 detector is obtained substituting the MMSE signal estimate in (3) by the output of the L_1**-adaptive beamformer**, [5]:

1. Initialization

$$\hat{x}_0 = 0 \qquad P(0) = 1 \tag{11}$$

[1] $(.)^+$ denotes complex transpose

2. Prediction

$$\hat{x}_k(k+1) = f\hat{x}_k(k) \tag{12}$$

$$P_k(k+1) = |f|^2 P_k(k) + \sigma_u^2 \tag{13}$$

3. Residues and adaptive gain

$$r(k+1) = z(k+1) - a(\theta)\hat{x}_k(k+1) \tag{14}$$

$$W(k+1) = diag[|r_n(k+1)|] \tag{15}$$

$$K(k+1) = \frac{P_k(k+1)a^+(\theta)W^{-1}(k+1)}{\alpha + P_k(k+1)\sum_{n=1}^{N}|r_n(k+1)|^{-1}} \tag{16}$$

4. Filtering

$$\hat{x}_{k+1}(k+1) = \hat{x}_k(k+1) + K(k+1)r(k+1) \tag{17}$$

5. Updating

$$P_{k+1}(k+1) = |1 - K(k+1)a(\theta)|^2 P_k(k+1) + \sigma_0^2 K(k+1)K^+(k+1) \tag{18}$$

As with the L_2-recursive beamformer, the parameter α is prespecified and accounts for the smoothing of the solution $\hat{x}(.)$. Here P is the output error covariance matrix conditioned on the predicted residues r. Notice that, contrarily to what happens with the L_2- recursive beamformer, the adaptive gain $K(.)$ of the L_1-adaptive beamformer explicitly depends on the inverses of the absolute values of the predicted residues. This is an interesting property that is explored when solving the detection problem for the impulsive noise environment. In fact, those sensors where strong and unexpected noisy impulses occur are adaptively discarded from processing.

3. Performance analysis: burst noise

In the presence of unexpected impulsive noise, the actual input of both beamformers is

$$z_{in}(k+1) = z(k+1) + s(k+1), k = 0, 1, \ldots \tag{19}$$

where $z(.)$ represents the observations modeled by (1) and $s(.)$ denotes the impulsive noise field. The elements $s_n(.)$ of the vector $s(.)$ are modeled by identically distributed and statistically independent complex random variables with probability density function (pdf)[2]

$$p(s_n(k)) = \varepsilon\mathcal{N}(0,\sigma^2) + (1-\varepsilon)\delta(s_n(k)) \tag{20}$$

[2] $\delta(.)$ denotes a Dirac impulse

where σ^2 is the variance of the Gaussian impulses, $\varepsilon \leq 1$ is a generally unknown real parameter, and $\mathcal{N}$ denotes the normal pdf. Then, taking into account (1) and (19), and using (20), we obtain the pdf of the samples of the total noise field $n_c(.) = n(.)+s(.)$:

$$p(n_{c_n}(k)) = \varepsilon \mathcal{N}(0, \sigma^2 + \sigma_0^2) + (1-\varepsilon)\mathcal{N}(0, \sigma_0^2), \quad (21)$$

which describes the Gauss-Gauss mixture model. From (20), we have

$$E\{s_n(k)\} = 0 \quad (22)$$

and

$$E\{s_n(k)s_m^*(l)\} = \varepsilon\sigma^2\delta_{nm}\delta_{kl}. \quad (23)$$

For simplicity of the analysis, assume that the regularizing parameter is $\alpha = 0$. Then, the output errors of the L_1 and L_2 beamformers are

$$\varepsilon_{0_{L_1}} = \frac{\sum_{n=1}^{N} a_n^+(\theta)|r_n(.)|^{-1}[n_n(.) + s_n(.)]}{\sum_{n=1}^{N}|r_n(.)|^{-1}} \quad (24)$$

and

$$\varepsilon_{0_{L_2}} = \frac{a^+(\theta)[n(.) + s(.)]}{N}, \quad (25)$$

respectively. Notice that, under the assumed conditions, both beamformers are unbiased. The error power at the output of the L_2- recursive beamformer is

$$P_{L_2} = \frac{\sigma_0^2}{N} + \varepsilon\frac{\sigma^2}{N}. \quad (26)$$

If $\varepsilon\sigma^2 \gg \sigma_0^2$, then

$$P_{L_2} \simeq \varepsilon\frac{\sigma^2}{N}. \quad (27)$$

This means that the output of the optimum quadratic detector becomes completely masked by the impulsive noise. For the L_1-adaptive beamformer, we notice that the condition $\varepsilon\sigma^2 \gg \sigma_0^2$ is equivalent to assume that the probability of occurrence of strong impulses is very high. This implies strong values of the predicted residues and (24) can be approximated by

$$\varepsilon_{0_{L_1}} = \frac{\sum_{N-L} a_n^+(\theta)|r_n(.)|^{-1}n_n(.)}{\sum_{N-L}|r_n(.)|^{-1}}, \quad (28)$$

where L is the number of sensors where strong impulses are present. Hence, the error power conditioned on the residues is approximated by

$$P_{L_1} = \frac{\sigma_0^2\sum_{N-L}|r_n(.)|^{-2}}{\left(\sum_{N-L}|r_n(.)|^{-1}\right)^2}. \quad (29)$$

Notice that the summations in (29) only account for the residues in the sensors where impulses did not occur. Then, the dispersion of the absolute values of the residues is likely small and

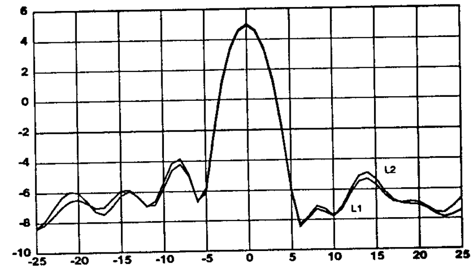

Figure 1: Broadside source

$$P_{L_1} \simeq \frac{\sigma_0^2}{N-L}. \quad (30)$$

Comparing (30) with (27) we conclude that, on the contrary to what happens with the L_2-recursive beamformer, the error power at the output of the L_1-adaptive beamformer is mainly determined by the variance of the assumed Gaussian noise field. Hence, the substitution of the quadratic beamformer by the least absolute value beamformer makes the sufficient statistic (3) robust to the presence of the unexpected strong impulsive noise field. This fact is confirmed by the simulations results that are reported in the following section.

4. Simulations

The simulations presented in this section were performed on data synthesized using the model described by equations (1) and (2). A linear and uniform array of $N = 20$ sensors spaced by one half wavelength was assumed. For both the L_1 and the L_2 receivers, figures 1 to 4 represent the sufficient statistic $\ell(\theta)$ for $\theta \in [-25°, 25°]$ and $K = 50$ snapshots. For the cases where the source is present, the signal to noise ratio at each sensor is $\sigma_x^2/\sigma_0^2 = 2$ (3dB).

Case 1: $\varepsilon\sigma^2 = 0$ and source at $\theta = 0°$

The results obtained for this case are depicted in figure 1 and show that, in the absence of the impulsive noise field, both receivers have similar behaviors. The resulting $\ell(\theta)$ is used as a calibration curve for the following experiments: assuming a threshold $\eta = 4$dB, we decide that a source is present if the global maximum of $\ell(\theta)$ is greater than η.

Case 2: $\varepsilon\sigma^2 = 36$ and source absent

Here we assume that, in the absence of any source, the mean power of the unexpected impulsive noise is $\varepsilon\sigma^2 = 36$. As predicted by the analysis developed

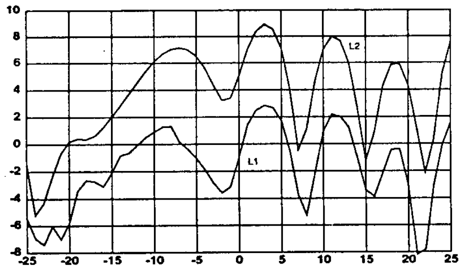

Figure 2: Absent source with impulsive noise

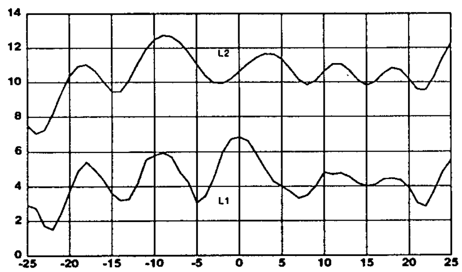

Figure 3: Broadside source with impulsive noise

in the previous section, the effect of the impulsive noise field, masking the output of the L_2 receiver, can be verified in figure 2. According to the specified decision criterion with threshold $\eta = 4$, the decisions provided by the two detectors would be: false alarm at $\theta = 3°$ for the L_2 receiver, and source absent for the L_1 receiver.

Case 3: $\varepsilon\sigma^2 = 36$ and source at $\theta = 0°$

In this case, see figure 3, the decisions would be: source present at $\theta = -9°$ for the L_2 receiver, and source present at $\theta = 0°$ for the L_1 receiver. In spite of detecting the presence of a source, the L_2 receiver misses the source at 0° and has a false alarm at −9°. On the contrary, the least absolute value receiver detects the source and simultaneously provides an unbiased estimate of its direction of arrival.

Case 4: $\varepsilon\sigma^2 = 18$ and source at $\theta = 0°$

Here the impulsive noise power is decreased to $\varepsilon\sigma^2 = 18$. However, the outputs of both receivers are identical to those of the previous case. This emphasizes the robustness of the L_1 receiver when compared with the L_2 receiver.

5. Conclusions

The reported simulations confirm the analytical results obtained in section 3. We conclude that the substitution of the MMSE beamformer by the least absolute beamformer in the optimum quadratic detector leads to a suboptimum receiver that adaptively detects and attenuates the strong impulsive noise samples. This fact is responsible for the robustness of the suboptimum receiver in the presence of an unexpected strong impulsive noise field.

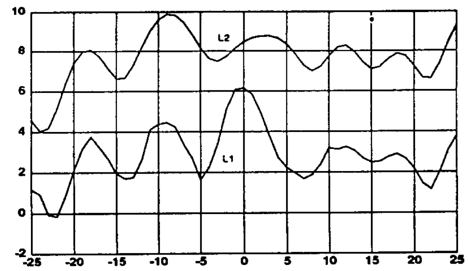

Figure 4: Broadside source with impulsive noise

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