Adaptive Beamforming as an Inverse Problem

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Abstract

In previous work, we developed a minimum mean square error beamformer (MMSE-BF). When compared to the minimum variance distortionless response beamformer (MVDR-BF), we concluded that it is specially suited for correlated returns. This improvement is at the cost of some frequency distortion in wideband applications. To circumvent this problem, we introduce the minimum mean square error distortionless response beamformer (MMSEDR-BF). Its behavior is compared with the MVDR-BF and the MMSE-BF. Adaptive beamforming is discussed as an inverse problem. Within this framework, we suggest the use of alternative norms, e.g., the L_1 norm. Preliminary results for L_1 adaptive beamforming are presented.

1. MMSE-BE and MVDR-BF

Consider a linear array of equispaced omnidirectional sensors and assume that the incoming wavefronts are planar. In matrix form, the array output in the frequency domain is

$$Z(\omega) = \sum_{p=0}^{P} a(\omega, \theta_p) x_p(\omega) + W(\omega), \tag{1}$$

where

$$Z(\omega) = [Z_1(\omega), \ldots, Z_N(\omega)]^T, \tag{2}$$

$$W(\omega) = [W_1(\omega), \ldots, W_N(\omega)]^T, \tag{3}$$

and $\{a(\omega,\theta_p)\}_{p=0}^{P}$ are the N-dimensional steering vectors.

The objective of a beamformer is to estimate the signal arriving from a desired direction θ_0 in the presence of noise (ambient and sensor noise) and interfering signals impinging on the array from directions $\{\theta_p\}_{p=1}^{P}$.

In this paper, we consider intermediate solutions between the MMSE-BF developed in [2] and the MVDR-BF studied in [1], [6]. The MMSE-BF is derived as the minimizer of

$$E\{|x_0(\omega) - \hat{x}_0(\omega)|^2\}, \tag{4}$$

given the observations $Z(\omega)$ defined in 1, where $\hat{x}_0(\omega)$ is the estimate of the signal at the desired direction θ_0. The MMSE-BF solution is

$$\hat{x}_0(\omega) = S_{0Z}(\omega) S_Z^{-1}(\omega) Z(\omega), \tag{5}$$

where $S_Z(\omega)$ is the power spectral matrix of the received wavefield which can be directly estimated from the data, if a sufficient number of snapshots are available, and $S_{0Z}(\omega)$ is a row vector of the cross-power between the desired signal and the observations.

We turn now our attention to the MVDR-BF. This is specified by the vector of complex weights $h(\omega)$ which minimizes the output noise power

$$E\{|h^+(\omega) Z(\omega)|^2\} \tag{6}$$

subject to the distortionless constraint along θ_0

$$h^+(\omega) a(\omega, \theta_0) = a^+(\omega, \theta_0) h(\omega) = 1 \tag{7}$$

The output of the MVDR-BF is then given by

$$r(\omega) = h^+(\omega) Z(\omega) \tag{8}$$

with

$$h(\omega) = \left[a^+(\omega,\theta_0) S_Z^{-1}(\omega) a(\omega,\theta_0)\right]^{-1} S_Z^{-1}(\omega) a(\omega,\theta_0) \tag{9}$$

In [2] the analysis of the MMSE-BF was carried out. The major characteristics are:

1. It uses in an optimal fashion the intercorrelation between the signal and the interferences (see Fig.1 for a typical beampattern, where the source and interference are perfectly correlated). This is important in applications of highly correlated "returns" along different directions as in severe multipath environments with no unique strong returns, or in weak signal to noise ratio (SNR) ambients.

2. It explores to the advantage of the task of optimal signal reconstruction the correlation between the look direction and the secondary returns, by placing local maxima in all directions of correlated arrivals (if the separation is larger than the main lobe beamwidth).
3. For uncorrelated paths, the MMSE-BF is essentially equivalent to the MVDR-BF placing nulls at the interfering directions
4. The optimal minimum mean square error reconstruction of the source signal is obtained at the penalty of introducing frequency distortion.

The main properties of the MVDR-BF are:

1. For uncorrelated signals, it performs as a rejector of interferences and ambient noise.
2. It reconstructs the signal with no frequency distortion if the source signal and the interferences are uncorrelated. With a correlated interferer the MVDR-BF minimizes the output power by processing the interference signal in such a way that cancels the desired signal [4].

To implement the MVDR-BF, (9), one only needs to know the power spectral matrix of the observation's vector $S_Z(\omega)$ which, as already pointed out, can be directly estimated from the data. This major advantage of the MVDR-BF fails to be true when the source signal and interferences are correlated. In contrast, the MMSE-BF requires either complete knowledge of the signal and background noise, including the cross-correlation between the source and the interferences, or knowledge of the directions of arrival of the desired signal and of the interferences, as we now show.

Implementation of the MMSE-BF

Consider the model (1) written in a more compact form

$$Z(\omega) = Y(\omega) + W(\omega) \quad (10)$$

where

$$Y(\omega) = A(\omega, \underline{\theta})x(\omega); \quad (11)$$

In this equation

$$x(\omega) = [x_0(\omega), \ldots, x_p(\omega)]^T \quad (12)$$

is the state vector of the incoming signals and

$$A(\omega, \underline{\theta}) = [a(\omega, \theta_0), \ldots, a(\omega, \theta_p)] \quad (13)$$

is a full rank matrix of the steering vectors which are assumed to be linearly independent. Under this assumption and using the linearity of MMSE estimates, from (11) we get

$$\hat{x}(\omega) = A^{\#}(\omega, \underline{\theta})\hat{Y}(\omega), \quad (14)$$

where $A^{\#}(\omega, \underline{\theta})$ is the pseudo-inverse and

$$\hat{Y}(\omega) = S_Y(\omega)S_Z^{-1}(\omega)Z(\omega) \quad (15)$$

The signal estimate at the desired direction θ_0 is given by

$$\hat{x}_0(\omega) = e_1^T A^{\#}(\omega, \underline{\theta})S_Y(\omega)S_Z^{-1}(\omega)Z(\omega) \quad (16)$$

where $e_1^T = [1, 0, \ldots, 0]$. If the statistics of the uncorrelated noise term $W(\omega)$ are known, up to a scalar factor, then $S_Z(\omega)$ and $S_Y(\omega)$ can be estimated directly from the data, by using well known techniques for estimation of structured covariance matrices [3]. In general the directions of arrival of the interferences $\{\theta_p\}_{p=1}^{P}$ are not known. Hence. the MMSE-BF must be preceded by a preprocessing that determines the directions $\{\theta_p\}_{p=1}^{P}$, see [7]. This is common practice in other approaches also, see [9] or [8].

In contradistinction with the adaptive procedure described above, more accurate models for the propagation channel provide prior knowledge about the cross-correlation between the desired signal and the observations, enabling, under some specific conditions, the characterization of coherent paths in terms of attenuation and direction of arrival, see [5]

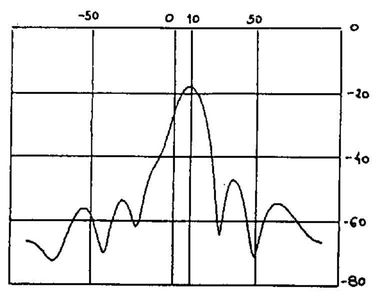

Figure 1: MMSE-BF

2. MMSE-BF with Distortionless Constraint

Although for narrowband (NB) problems the frequency distortion introduced by the MMSE-BF does not degrade seriously the reconstruction of the source signal, for wideband (WB) signals this may impair the performance of the beamformer. To counteract this effects but at the same time to preserve as much as possible the adaptation to the possibly existent cross-correlation between the source and interfering signals (as in multipath applications), we study the design of a MMSE-BF with the distortionless constraint (7). Using Lagrange multiplier tecnhiques, the task is that of minimizing the functional

$$\begin{aligned} J(h) &= E\{|x_0(\omega) - h^+(\omega)Z(\omega)|^2\} \quad (17) \\ &+\lambda[2 - a^+(\omega,\theta_0)h(\omega) - h^+(\omega)a(\omega,\theta_0)], \end{aligned}$$

where λ is the Lagrange multiplier and the beamformer output is specified by

$$\hat{x}_0(\omega) = h^+(\omega)Z(\omega). \quad (18)$$

By equating the gradient of $J(h)$ with respect to h to zero, it follows

$$h(\omega) = S_Z^{-1}(\omega)[S_{Z0}(\omega) + \lambda a(\omega,\theta_0)]; \quad (19)$$

using the restriction (7) in (19), we obtain the Lagrange multiplier

$$\lambda = \frac{1 - a^+(\omega,\theta_0)S_Z^{-1}(\omega)S_{Z0}(\omega)}{a^+(\omega,\theta_0)S_Z^{-1}(\omega)a(\omega,\theta_0)}, \quad (20)$$

which, when substituted in (19), gives

$$\begin{aligned} &h(\omega) = S_Z^{-1}(\omega)\times \\ &\left[S_{Z0}(\omega) + \frac{1-a^+(\omega,\theta_0)S_Z^{-1}(\omega)S_{Z0}(\omega)}{a^+(\omega,\theta_0)S_Z^{-1}(\omega)a(\omega,\theta_0)}a(\omega,\theta_0)\right]. \quad (21) \end{aligned}$$

By substitution of (21) in (18), we get

$$\begin{aligned} &\hat{x}_0(\omega) = \frac{a^+(\omega,\theta_0)S_Z^{-1}(\omega)}{a^+(\omega,\theta_0)S_Z^{-1}(\omega)a(\omega,\theta_0)}Z(\omega) \\ &+S_{Z0}(\omega)\times \\ &\left[S_Z^{-1}(\omega) - \frac{S_Z^{-1}(\omega)a(\omega,\theta_0)a^+(\omega,\theta_0)S_Z^{-1}(\omega)}{a^+(\omega,\theta_0)S_Z^{-1}(\omega)a(\omega,\theta_0)}\right] Z(\omega). \end{aligned} \quad (22)$$

Comparing the above formula with (5) and (9), terms corresponding to MMSE-BF and the MVDR-BF can be identified. It can be verified that, for the limit case of uncorrelated sources, this minimum mean square error distortionless response beamformer (MMSEDR-BF) is equivalent to the MVDR-BF and, except for a frequency dependent gain, to the MMSE-BF.

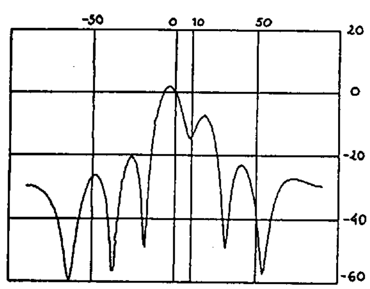

Figure 2: MVDR-BF

For completely correlated paths the three beamformers have remarkably different behaviors. In Figs. 1, 2, 3 typical assymptotic beampatterns are presented to exemplify the above sentence. Two coherent signals impinge on a seven elements array, the desired one from broadside and the interference along the direction $\theta = 10''$. The signal to noise ratio in the look direction is assumed to be 3dB and the interference is 6dB stronger than the desired signal. Notice the level of adaptation of the MMSE-BF to the environment conditions, searching for the stronger coherent signal, while combating ambient noise with secondary lobes exhibiting severe attenuations. Notice also the failure of the MVDR-BF neither combining the signals or rejecting the coherent interference. In contradistinction, the MMSEDR-BF can be classified as good interference rejector but sensitive to the ambient noise field. Insight on this behavior can be obtained by interpreting equation (22). Only the first term in (22) depends on the desired signal. In fact, the matrix in brackets, in the second term, is the projection matrix onto the orthogonal complement of the subspace spanned by $a(w,\theta_0)$, on the norm of $S_z(w)$. Hence, this term is responsible for the high sensitivity of this beamformer to the ambient noise field. However, the retained contributions from the correlated interferences are used to combat the poor performance of the first term (MVDR-BF) under these conditions, while getting the profit of its distortionless response property.

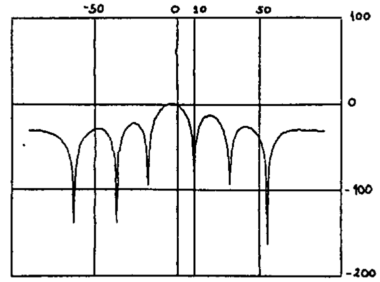

Figure 3: MMSEDR-BF

This shows that the adequated beamformer for multipath applications, whenever some prior information is available for the cross-correlation structure of the received signals, is MMSE-BF. Its performance is at the cost of possible frequency distortion. When this is a critical issue, the MMSEDR-BF is, under the same conditions, better than the MVDR-BF. The principal disadvantage of this beamformer is its high sensitivity to the ambient noise.

3. Beamforming as an Inverse Problem

In geophysics, or in image processing, like in beamforming, one is confronted with the task of field reconstruction from a set of possibly sparse spatial measurements. Let z collect the available spatial measurements and let x_0 represent the field to be reconstructed. The problem is that of finding a good estimate of x_0, in the sense of minimizing the cost function

$$J = d_1(z - A_0x_0; R) + \lambda d_2(x_0; Q) \qquad (23)$$

where Q and R are metrics constraining the solution and the residue, respectively, and λ is a regularizing parameter. By choosing appropriately the strength of the regularizing parameter λ, the designer adjusts the confidence placed on the prior knowledge versus reliance on the measurements. Said in other words, the first term in (23) tries to match the field to the measurements, while the second term attempts fitting the prior knowledge about the field.

We now recast both the MVDR-BF and the MMSE-BF as solutions of an inverse problem. With generality, the output of the beamformer is the signal x_0 that minimizes the functional (23). For an L_2 norm

$$J = \| z - a_0x_0 \|^2_{R^{-1}} + \lambda \| x_0 \|^2_{Q^{-1}}, \qquad (24)$$

and R and Q being the covariances of the residues and of the signal x_0. The MVDR-BF and the MMSE-BF can be recovered as particular solutions of the problem formulated above. The first one results when $\lambda = 0$ (high confidence on the measurements), the second when $\lambda = 1$ (reliable prior knowledge). Also, when no prior knowledge exists about the noise and signal statistics ($\lambda = 0, R = I$), the conventional delay and sum beamformer is recovered.

Matrices R and Q can have no statistical meaning, performing as penalty metrics on the functional to be minimized. This suggests use of iterative algorithms for searching the solution of the problem, adapting those metrics at each iteration. With the inverse framework, we can now consider alternative norms, such as a minimum entropy norm or a general L_p norm.

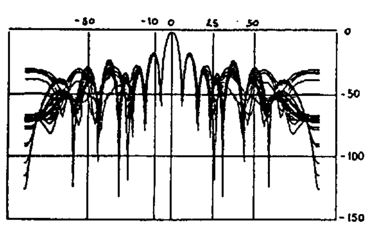

Figure 4: L_1 adaptive beamform

Fig. 4 shows an adaptive L_1 beamformer in the absence of prior knowledge. Each trace corresponds to a different snapshot. This was obtained by means of the iterative reweighted least squares (IRLS) algorithm. For each snapshot a maximum of 100 iterations was specified. Although this L_1 beamformer resembles the conventional beamformer, it may be particularly relevant when outliers are present on the data, as when faulty sensors arise. The robustness of this L_1 beamformer under these adverse conditions is presently under study.

References

[1] A. Baggeroer, "Space/Time Processes and Optimum Array Processing," Technical Rep. # 504, Naval Undersea Center, San Diego, CA, USA, 1976.

[2] V. A. Barroso, J. M. F. Moura, "Optimal Estimation and Beamforming," 1988 NATO-ASI on Underwater Acoustic Data Processing, Ed. Y. Chan, D.Reidel, to be published, 1989.

[3] Y. Bresler, "Maximum Likelihood Estimation of a Linearly Structured Covariance with Application to Antenna Array Processing," IEEE ASSP Workshop on Spectrum Estimation and Modeling, Aug. 1988.

[4] V. U. Reddy, A. Paulraj, T. Kailath, "Performance Analysis of the Optimum Beamformer in the Presence of Correlated Sources and its Behavior Under Spatial Smoothing," IEEE Tr. on ASSP, vol ASSP-35, pp. 927-936, Jul. 1987.

[5] J. M. F. Moura, M. J. Rendas, "Sensitivity of Range Localization in a Multipath Environment," IEEE Int. Conf. on ASSP, 1989.

[6] N. L. Owsley, "A Recent Trend in Adaptive Spatial Processing for Sensor Arrays: Constrained Adaptation," in Signal Processing, ed. J. Griffiths et al., Academic Press, New York, 1973.

[7] M. J. Rendas, J. M. F. Moura, "Resolving Narrow-Band Coherent Paths with Non-Uniform Arrays," IEEE Int. Conf. on ASSP, 1989.

[8] T. Shan, T. Kailath, "Adaptive Beamforming for Coherent Signals and Interference," IEEE Tr. on ASSP, vol ASSP-33, pp. 527-536, Jun. 1985.

[9] H. Wang, M. Kaveh, "Coherent Signal-Subspace Processing for the Detection and Estimation of Angles of Arrival of Multiple Wide-Band Sources," IEEE Tr. on ASSP, vol ASSP-33, pp. 823-831, Aug. 1985.