Towards Grid Modernization through Enhanced Communications and Computing: Novel Performance Index and Information Structure for Monitoring Voltage Problems

Le Xie, *Student Member, IEEE*, Jovan Ilić, *Member, IEEE*, Marija D. Ilić, *Fellow, IEEE*

***Abstract*—In this paper we observe that recent voltage problems, including August 2003 blackout, are often caused by the lack of adequate and timely information. We study the type of information essential to avoiding such problems in the future. The needed information concerns (a) choice of a performance index characterizing problems of interest; (b) information structure necessary to monitor the performance index with acceptable accuracy; and, (c) the interdependence between (a) and (b). We propose a novel performance index and corresponding information structure. This information structure requires very little or no communications between decomposed subsystems. Illustrations of proposed concepts in multi control area systems are presented. These findings form a basis for grid modernization through communications and computing for improved voltage support in multi control area system.**

***Index Terms*—Voltage monitoring, grid modernization, communications in electric power system, performance index, system decomposition, multi-control area operations.**

I. Introduction

THIS paper is motivated by an observation that the lack of adequate information in support of power system monitoring and operation is becoming an increasingly critical problem. In particular, not many online monitoring and decision-making tools are available for predicting and controlling voltage problems [1]. During abnormal conditions, problems are two-fold: (a) the current logic design of individual controllers and the lack of information exchange for making these controllers adaptive over broad ranges of system conditions; and, (b) the lack of quasi-stationary system-wide voltage/reactive power monitoring for managing the system when it gradually begins to experience voltage deviations outside normal ranges.

One major observation is that (a) and (b) are interdependent. More advanced control logic requires less communication, and vice versa. As the country attempts a major grid modernization by means of smart control- and communication-equipped hardware, one should assess the interdependence of both (a) and (b) prior to choosing the specific technology. Planning system enhancements should take into consideration the effects of control logic and the supporting communications on the need for advanced hardware and for the type of new technologies. Most of candidate technologies for gird modernization, such as FACTS, distributed generation, demand-side response, and the communications for coordinating these, have embedded software-supported control and computing.

In this paper, in particular, we consider the potential of grid modernization for improved quasi-stationary monitoring of potential voltage problems, namely possible solutions to the problem (b) above. The problem of local feedback design for individual controllers (problem (a) above), has been studied elsewhere, and several adaptive voltage control schemes have been proposed [2], [6]. More recently, metrics for assessing economic benefits of controllers were proposed and used to illustrate the cost-benefit tradeoffs of various voltage controllers [3]-[5]. Although this paper does not concern a design of control logic itself, the work on this subject shows that it is generally not sufficient to entirely rely on local voltage control logic, no matter how advanced the control logic is. Namely, the implementation of these adaptive controllers requires information beyond local measurements. Typical information about the system needed is either the minimum eigenvalue of the system Jacobian [2, 4], or the direct and quadrature axes components of the generator armature current [6]. Moreover, problem (b) which is the main subject of this paper requires development of a performance index (PI) capable of detecting practical problems of interest. As such PI is defined, its evaluation and monitoring would generally require full state measurements.

Both problems (a) and (b) effectively lead to the same key question concerning the minimum information exchange and system decomposition (information structure) for identifying abnormal system conditions, voltage problems, in particular. Implied in this question is the need for a systematic approach to defining the communication of information, which guarantees adequate performance of voltage controllers over a wide range of operating conditions. Because of the communication cost limits to data gathering and processing, and the administrative

This work was supported in part by the U.S. National Science Foundation under award CNS-0428404, and, in part, by the U.S. Department of Energy, National Energy Technology Laboratory, under Research and Development Solutions, LLC contract number DE-AM26-04NT41817.305.01.21.002.The authors greatly appreciate this financial help.

Le Xie and Jovan Ilić are with the Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA 15213 (e-mail: lx@ece.cmu.edu, jilic@ece.cmu.edu).

Marija D. Ilić is a Professor in the Department of Electrical and Computer Engineering and Department of Engineering and Public Policy, Carnegie Mellon University, Pittsburgh, PA, 15213 (email: milic@ece.cmu.edu).

boundary barriers, it is most desired to minimize communication or decentralization as much as possible.

In this paper we basically assess what is essential to communicate. It is generally understood that controllers work well during normal operating conditions. Controllers are generally tuned around these conditions. Extending/enhancing the control logic and communications to automate controllers during more severe conditions is one of the main challenges underlying grid modernization. Moreover, assessing the severity of the condition requires communication beyond local.

II. The Need For Quasi-stationary Monitoring of Voltage Problems

The main contribution of this paper concerns potential of quasi-stationary communication for assessing and controlling voltage and reactive power problems for predictable performance[1]. To explain the type of online problems which could be controlled by using only quasi-stationary methods described in this paper, we consider changes on the system which gradually lead to degradation of relevant performance of the system. Such changes could be (a) real and/or reactive power loading change (b) loss of degree of controllability caused by the saturation of major voltage controllers. For example, saturation of the field voltage amplitude of an Automatic Voltage Regulator AVR), results in a voltage controlled generator behaving like load whose voltage varies as a result of system changes, i.e., it is no longer directly controlled. Similarly, voltage control limits can be reached on Static Var Compensator (SVC), shunt capacitor or On-Load Tap-Changing Transformers (OLTC)[2].

The basic idea of this paper is to suggest that it would be possible to detect if at the early stages of major blackouts a gradual voltage reduction occurs as loss of some of the voltage controllers prior to the ultimate voltage collapse. If one could have communication and control logic to make the remaining controllers more adaptive to these conditions, ultimate dynamic problems could often be avoided.

In this paper we show on a simple 7-bus and an IEEE RTS-1996 73-bus system [10] what type of information would be needed to: (a) monitor the severity of the changing conditions as the system load varies and/or voltage controllers reach their control limits; and, (b) adjust other controllers so that the critical performance index is improved.

The information question can be separated into three questions. The first question concerns the choice of a quantifiable performance index reflecting qualitative changes in system conditions. The second question concerns the information structure suitable for computing and monitoring the chosen performance index. The third question concerns the interdependence of the above two questions. In this paper we study these three questions. We propose a novel performance index which lands itself better to decentralization without losing the essential information about the practical problem. Moreover, for a multi-control area scenario, we propose a novel communications structure essential for monitoring voltage problems within the entire interconnection.[3]

Two information structures are proposed and compared. The first information structure is based on the system decomposition into administrative control areas. The second information structure uses so called "ε-decomposition" method to decompose a large interconnection into subsystems which require very little or no communication exchange. The reason for applying ε-decomposition is that there may exist a more natural decomposition than the one defined through the administrative boundaries of today's control areas. Or expressed otherwise, we consider the problem of "redefining control areas" so that voltage problems could be monitored and controlled with very little or no communication. In principle, the choice of system decomposition will determine the type of communication exchange required, and vice versa.

III. Proposed Performance Index For Monitoring Quasi-stationary Voltage Problems

A typical quasi-stationary voltage problem is reflected in significant changes of the system Jacobian J defined in Appendix (A.6). In particular, as the real and reactive power load vary away from nominal conditions for which the system was designed, the elements in J could change their numerical values. In some cases even the order of the Jacobian J changes as the voltage/reactive power limits are reached at buses where voltage is directly controlled. For generators, when Q limit is reached, a P-V bus becomes a P-Q bus, and this increases the order of J by 1.

Independent of the root causes of these changes, it is important to have an effective Performance Index (PI) indicating proximity to severe operating problems. While it is often difficult to relate real-life operating problems to the mathematical models, it is generally accepted that the closer J is to becoming singular, the more severe the operating problem becomes. So the closeness of matrix J to singularity would be a possible PI indicating voltage problems under today's hierarchical control.

There exists significant literature which uses the minimum eigenvalue of J as the measure of the proximity to the singularity of J. This measure is re-assessed in our paper in light of information structure needed to detect the singularity. In a typical multi control area power system, today's practice is for each control area (CA) to monitor its conditions in a decentralized way. This raises a fundamental question about how well does the information about the minimum eigenvalue in each CA reflect the possibility of having singular Jacobian in the interconnected system, i.e., the proximity of the system as a whole to the voltage collapse type of problem.

[1] Truly dynamic problems are unlikely to rely on control logic beyond local. The methods described in this paper and elsewhere require information and computation beyond what is doable before the system enters dynamic voltage collapse regime.

[2] In power flow calculations this is sometimes referred to as a PV bus becoming a PQ bus.

[3] The decomposition could be done within a single control area, and/or between the control areas of the interconnection.

A schematic picture representing typical relations between the minimum eigenvalues of J, minimum eigenvalues of J_i's for all i's is shown in Fig. 1 (J_i corresponds to a control area i):

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \quad (1)$$

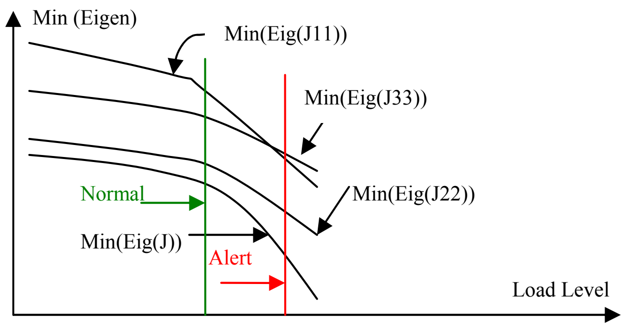

Fig. 1 Schematic change of minimum eigenvalue for subsystem matrix as load level changes

Therefore one could often have a situations in which individual CA does not anticipate system-wide voltage problem because their own Jacobian J_i's are still relatively far from having near-zero eigenvalue.

On the other hand it can be seen from this schematic picture that the rate of change of minimum eigenvalue of J_i's with respect to system-wide load level exhibits very similar slopes to the slope of the minimum eigenvalue J. Based on this observation, we propose in this paper a novel system-wide PI which to be defined as follows:

Let $\lambda_m = \min\{eig(J)\}$

Let $P_0 = \sum_i P_G(i)$, where $P_G(i)$ is the net input of real power from generator i into the network.

Let $Q_0 = \sum_i Q_G(i)$, where $Q_G(i)$ is the net input of reactive power from generator i into the network.

Let $S_0 = \sqrt{P_0^2 + Q_0^2}$, where S_0 represents the overall load level of the whole interconnection. Define PI as

$$PI_C = \frac{\partial \lambda_m}{\partial S_0} \quad (2)$$

If $PI_C \le Thershold$, the network is in a normal operating regime. Otherwise If $PI_C > Thershold$, the whole interconnection is running out of the normal range, alert will be broadcasted in the whole system and further control logic adjustment needs to be done. The value $Thershold$ varies in different systems, and needs to be obtained using different methods [11].

IV. INFORMATION STRUCTURE FOR MONITORING THE PROPOSED PERFORMANCE INDEX

The newly proposed PI defined in (2) above is for the entire interconnection. Its computation needs information from all buses within the interconnection, which is too much time consuming for online monitoring. Therefore, a real-time coordinated PI as suggested above to monitor the quasi-stationary voltage problem is impractical. In this section we present two possible decomposed information structures which have the potential of being incorporated seamlessly into today's industry practice.

A. Overlapping Decomposition Structure Based on Administrative Boundaries

The existing power system interconnection is horizontally structures into a number of control areas. These control areas form the administrative boundaries. Each control area monitors and controls the power network within its boundary, while the neighboring control areas are interconnected via tie-lines. Assume next that each control area can measure/estimate information (bus voltage magnitude, phase angle, real/reactive power flow) within its boundaries.

The overlapping decomposition structure is based on the information within the boundaries of each control area and the additional tie-line power flow information. This information structure can be described as follows:

Let

A={set of areas}= $\{a_1, a_2, \ldots, a_M\}$, where M is the total number of administrative areas;

B={set of buses}= $\{b_1, b_2, \ldots, b_N\}$, N is the total number of buses (nodes);

A_i ={set of buses in area a_i };

$A_1, A_2, \ldots, A_M$ are a disjoint partition of B, i.e., $A_i \cap A_j = \phi$ if $i \neq j$; $\bigcup_{i=1}^{M} A_i = B$;

T_i ={set of buses that are directly connected to area i but outside of area i};

Then the interconnection system Jacobian matrix model can be partitioned as

$$\begin{bmatrix} \Delta S_1 \\ \Delta S_2 \\ \vdots \\ \Delta S_M \end{bmatrix} = \begin{bmatrix} \frac{\partial \Delta S_1}{\partial \Delta t_1} & \frac{\partial \Delta S_1}{\partial \Delta t_2} & \frac{\partial \Delta S_1}{\partial \Delta t_3} & \cdots & \frac{\partial \Delta S_1}{\partial \Delta t_M} \\ \frac{\partial \Delta S_2}{\partial \Delta t_1} & \frac{\partial \Delta S_2}{\partial \Delta t_2} & \frac{\partial \Delta S_2}{\partial \Delta t_3} & & \frac{\partial \Delta S_2}{\partial \Delta t_M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \Delta S_M}{\partial \Delta t_1} & \frac{\partial \Delta S_M}{\partial \Delta t_2} & \frac{\partial \Delta S_M}{\partial \Delta t_3} & \cdots & \frac{\partial \Delta S_M}{\partial \Delta t_M} \end{bmatrix} \begin{bmatrix} \Delta t_1 \\ \Delta t_2 \\ \Delta t_3 \\ \vdots \\ \Delta t_M \end{bmatrix} \quad (3)$$

Here, ΔS_i stands for $\Delta P_i, \Delta Q_i$, Δt_i stands for $\Delta \delta_i, \Delta V_i$ (vector notation).

The overlapping decomposition partitions the entire system model into M subsystems. These M subsystems are overlapping via the tie lines that connect them.

$$\Delta S_i = \begin{bmatrix} \dfrac{\partial \Delta S_i}{\partial \Delta t_{Ai}} & \dfrac{\partial \Delta S_i}{\partial \Delta t_{T_i}} \\ \dfrac{\partial S_{T_i}}{\partial \Delta t_{Ai}} & \dfrac{\partial \Delta S_{T_i}}{\partial \Delta t_{T_i}} \end{bmatrix} \begin{bmatrix} \Delta t_{Ai} \\ \Delta t_{T_i} \end{bmatrix}, \text{ where i=1, 2,…,M} \qquad (4)$$

Δt_{Ai} stands for the $\Delta\delta_i, \Delta V_i$ for buses in set A_i, and Δt_{T_i} stands for the $\Delta\delta_i, \Delta V_i$ for buses in set T_i.

In words, the overlapping algorithm takes buses within an area and buses directly connected to this area as a subsystem's Jacobian matrix.

Centralized power flow Jacobian is composed of four sub-blocks (A.6). Under the assumption that the real-reactive power (P-Q) decoupling holds, $\dfrac{\partial P}{\partial \theta}$ and $\dfrac{\partial Q}{\partial V}$ can be individually partitioned using the overlapping decomposition.[4]

B. ε-decomposition Structure Based on Clustering

ε-decomposition method [12] is a clustering algorithm that decomposes a large interconnected system into several disjoint sub-groups. The sub-groups are disjoint because the interconnection among the grouped nodes is weaker than a pre-chosen ε. The basic principle of ε-decomposition can be illustrated by the following example:

$$\begin{bmatrix} 3.0 & 0.4 & 0.2 \\ 1.3 & 2.0 & 0.2 \\ 0.1 & 0.3 & 5.0 \end{bmatrix} \xrightarrow{\varepsilon = 0.5} \begin{bmatrix} \begin{bmatrix} 3.0 & 0.4 \\ 1.3 & 2.0 \end{bmatrix} & \\ & \begin{bmatrix} 5 \end{bmatrix} \end{bmatrix} \qquad (5)$$

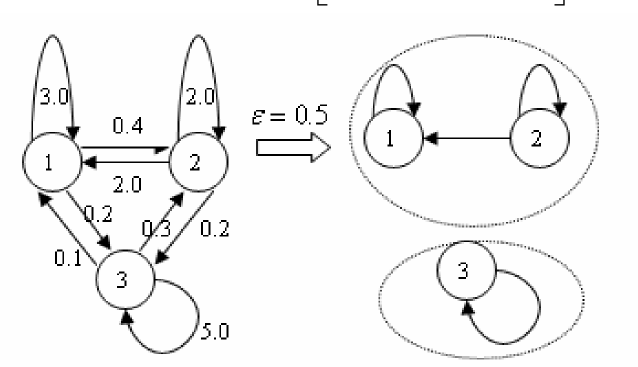

Fig. 2 Graphic representation of ε-decomposition method

Similar as the overlapping decomposition, here we assume that P-Q can be decoupled. Then we try to find appropriate value ε to decompose sub-matrix $\dfrac{\partial P}{\partial \theta}$ and $\dfrac{\partial Q}{\partial V}$. The choice of ε is experimental at this point of our research. We would like the disjoint decomposition after applying this method to $\dfrac{\partial P}{\partial \theta}$ and $\dfrac{\partial Q}{\partial V}$ be as balanced as possible.

[4] In principle, the information structures and performance index introduced in this paper can be applied without requiring the P-Q decoupling. However, some of the qualitative properties change significantly for the coupled P-Q model. We are currently pursuing research on this.

V. Illustration on Two Systems

Both 7 bus and IEEE RTS-96 system are used to illustrate the performance index and the corresponding communication mechanism to support the multi control area information structure.

A. 7-bus system

Fig. 3 shows a simple 7-bus system. There are three administrative areas as dashed.

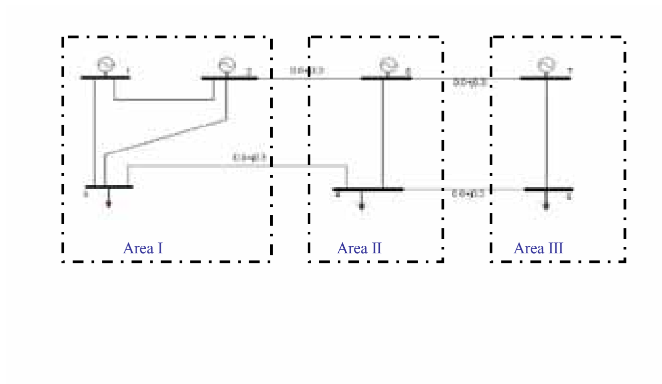

Fig. 3 One line diagram of a 7-bus system

1) Information structure based on overlapping decomposition

There are three administrative areas. Table I below shows the values of proposed PIs as the load level increases until the voltage collapse. Fig. 4 shows the minimum eigenvalues of decomposed sub-matrices using this information structure.

As we can see from Fig.4, Fig. 5 and Table I, when load level increases to the extent that alert condition is approaching, the proposed PIs in each control area increase significantly compared with normal load conditions, which show as a significant change of slope of minimum eigenvalues. The PIs change more significantly in the decomposed Q sub-matrices compared with the decomposed P sub-matrices.

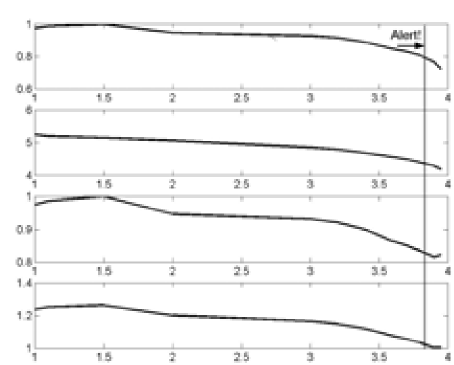

Fig. 4 Minimum eigenvalues for centralized Jacobian matrix J and overlapping decomposed sub-matrices P_{11}, P_{22}, P_{33} (from up to down).

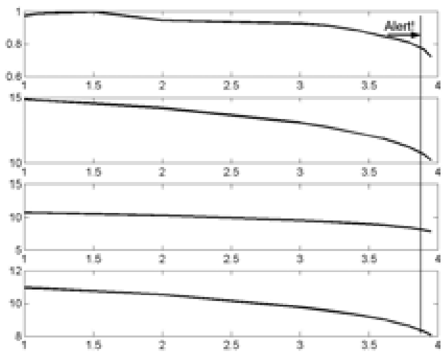

Fig. 5 Minimum eigenvalues for centralized J and the overlapping decomposed sub matrices Q_{11}, Q_{22}, Q_{33} (from up to down).

TABLE I
PIS UNDER DIFFERENT LOAD LEVELS (AFTER NORMALIZATION)

Load (S_0)	1.00	1.5	3.0	3.6	3.90 (Alert!)
PI_Central	-1.00	0.9138	0.5259	1.4569	7.2586
PI_P_1	1.00	0.4051	0.7448	1.5058	4.5688
PI_Q_1	1.00	0.8160	2.1150	4.2228	10.6707
PI_P_2	-1.00	0.8855	0.4573	1.0940	1.6410
PI_Q_2	1.00	1.5150	3.9780	7.9487	15.2601
PI_P_3	-1.00	0.8718	0.5810	1.2958	2.1197
PI_Q_3	1.00	1.7529	4.5798	9.2353	17.3361

Values in different rows of PIs are essentially the rate of change of minimum eigenvalue with respect to the change of system load level.

2) *Information structure based on ε-decomposition*

Two steps are needed to compute the PIs using this information structure. The first step is to choose an appropriate ε such that the decomposition results from this method are balanced across the subsystems. The second step is to compute the minimum eigenvalues of the partitioned sub-block matrices. For this particular example, when we choose $\varepsilon = 4$, the decomposed blocks are exactly the same as the administrative boundaries. In other words, there are three disjointly partitioned blocks corresponding to the control areas I, II and III, respectively.

As the value of ε becomes larger and larger, the partitioning of the system becomes more and more decentralized. It is worthwhile to understand how decentralized can the partitions be before the PIs lose their effectiveness. The authors have conducted a series of tests for ε ranging from 0.5 to 10 to investigate the effectiveness of partitioned PIs on both P and Q sub-matrices. For this small system, the next step of area-based decentralization is the bus-by-bus decentralization. Fig. 6 shows the most effective PI for different extent of decentralization. PIs are still useful for area-based decentralization. However, they can not give enough information to indicate the load level if the partition is extremely decentralized bus-by-bus.

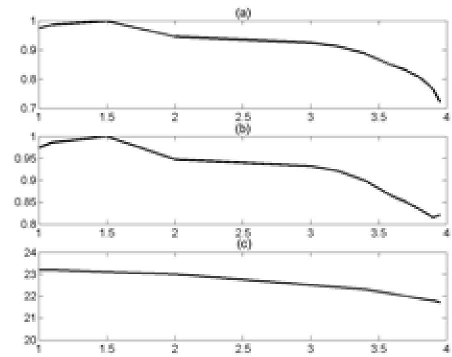

Fig. 6 (a) PI for centralized Jacobian (b) most effective PI in area-based decomposition (c) most effective PI in bus-by-bus decomposition

B. *IEEE RTS-1996 system*

The 1996 IEEE Reliability Test System (RTS-96) was developed to test and compare results from different power system reliability evaluation methodologies [10]. It contains three administrative control areas as shown in Fig. 7. Similar as in the previous example, two types of information structures have been tested concerning effectiveness of the corresponding PIs. However, unlike in the previous example, load level keeps increasing only in area III until voltage collapse occurs, while the power factor stays the same. Under this setup, voltage collapse problem occurs in area III, and the following results show the effectiveness of PIs to indicating proximity to the voltage collapse problem.

1) *Information structure based on overlapping decomposition*

Table II shows the values of PIs for both centralized and decomposed Jacobian matrices. Fig.8 and 9 are graphical representations of the minimum eigenvalues for corresponding matrices. As we can see from these results, the PI in area III increases substantially as the load in area III increases to a near-collapse level. PIs in area I and II, however, do not show significant increase. This illustration demonstrates that the proposed PI with overlapping decomposition structure can provide an effective indication of the potential local voltage problem.

Using this information structure, for a voltage problem cause by “local” load level increase, the local PI can effectively indicate this event. Upon receiving the over-threshold PI locally, the affected area will activate a broadcast mechanism to communicate this alert to the overall system and control logic will be adjusted accordingly. This shows a major advantage of decentralized PIs: Unless something extremely unusual happens, there will be very little information exchange between different administrative areas.

TABLE II
PIs UNDER DIFFERENT LOAD LEVELS (AFTER NORMALIZATION)
IEEE RTS 1996 SYSTEM

Load (S_0)	1.00	2.0	4.0	5.0	5.60 (Alert!)
PI_Central	1.00	3.00	43.5	105.0	662.5
PI_P_1	1.00	1.05	1.08	1.18	1.58
PI_Q_1	1.00	1.21	1.24	1.35	1.87
PI_P_2	1.00	1.01	1.09	1.27	1.39
PI_Q_2	1.00	1.44	1.36	1.68	1.90
PI_P_3	1.00	2.53	21.57	50.50	158.3
PI_Q_3	1.00	1.27	1.83	2.14	87.3

Values in different rows of PIs are essentially the rate of change of minimum eigenvalue with respect to the change of system load level.

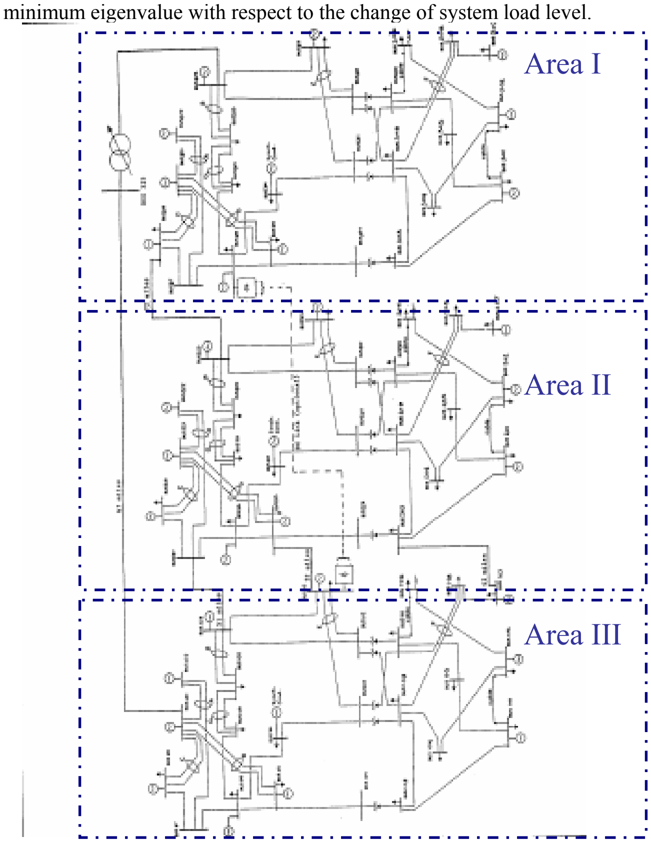

Fig. 7 IEEE RTS-1996 three-area system

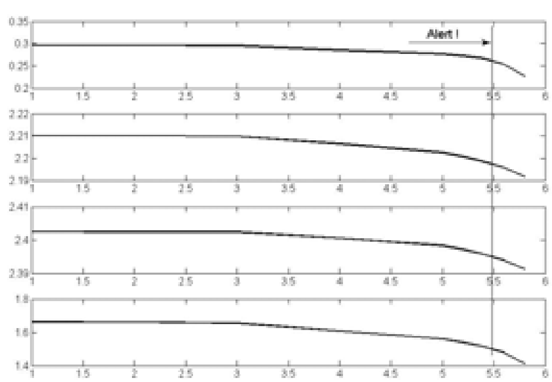

Fig.8 PIs for centralized Jacobian matrix J and overlapping decomposed sub-matrices P_{11}, P_{22}, P_{33} (from up to down.)

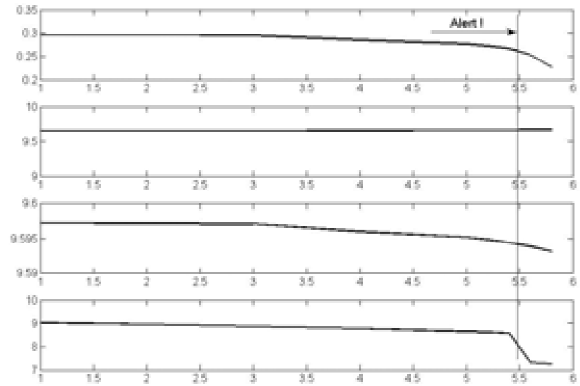

Fig.9 PI for centralized J matrix and overlapping decomposed sub-matrices Q_{11}, Q_{22}, Q_{33} (from up to down.)

2) Information structure based on ε-decomposition

Here we assume that the P-Q decoupling conditions hold. We choose ε among a wide range of value, until we get a relatively balanced clustering result. When $\varepsilon = 15$, the partitioning of P sub-matrix has the most balanced result. There exists three clustering of elements in the partitioning result, each of which corresponds to elements in one administrative area. When $\varepsilon = 3$, the partitioning of Q sub-matrix has the most balanced result. Similar clustering properties show in Q sub-matrix decomposition result. Table III describes the result of ε-decomposition.

The resulting partitions of both P and Q sub-matrices have three largest partitioned groups corresponding to strong correlation with the administrative boundaries. Because this ε-decomposition is based on the strength of connectivity among different buses, it supports the previous overlapping decomposition information structure, which is based on the administrative areas.

VI. CONCLUSIONS

A novel performance index for monitoring quasi-stationary voltage problem is proposed in this paper. The proposed PI reflects the rate of the minimum eigenvalue change in the corresponding Jacobian matrix and uses this as an indicator of the system voltage severity. Two decentralized information structures are proposed and compared to support this PI. Illustrations on both a small 7-bus-system and an IEEE RTS-1996 73-bus-system show the following observations:

(1) The proposed PI can effectively monitor both the system-wide and local voltage problems.

(2) Overlapping decomposition results are qualitatively consistent with ε-decomposition. Both decompositions are interpretable and useful for the horizontally structured administrative boundaries in today's interconnection. Overlapping decomposition information structure can take the tie line connection into consideration; while ε-decomposition is a disjoint partition of the system Jacobian.

Further work includes the theoretical proof for the general applicability of this proposed PI.

TABLE III
$\mathcal{E}$ -DECOMPOSITION RESULTS FOR RTS-1996 SYSTEM

Sub matrix	Number of partitioned groups	Partitions (bus numbers)
P	25	0 1
		2
		3
		4
		5 9
		6 7
		8
		10 12 13 14 15 16 17 18 19 20 21 22
		11
		23 24
		25
		26
		27
		28 32
		29 30
		31
		33 34 35 36 37 38 39 40 41 42 43 44 45 46
		47 48
		49
		50
		51
		52 56
		53 54
		55
		57 58 59 60 61 62 63 64 65 66 67 68 69 70 71
Q	8	**0 1 2 3 4 5 6 7 8 11 12 13 23**
		9 10
		14 15 16 17 18 19 20 21 22 26
		24 25
		27 28 29 30 31 32 33 34 35 39
		36
		37 38
		40

APPENDIX: MATHEMATICAL MODELING OF QUASI-STATIONARY VOLTAGE PROBLEMS

In order to understand the temporal and spatial interdependencies in large scale power system, it is necessary to understand the basic models and assumptions underlying the operation during normal conditions. In [8] a structure-based general model containing a set of coupled differential and algebraic equations (DAE) was presented to describe the current hierarchical control scheme. Without loss of generality, we consider an interconnected power system shown in Fig 9. This represents two administrative regions interconnected with each other. The loads are assumed not to have significant inertia, thus they are modeled as static sinks of pre-specified real and reactive power. We assume that there are automatic voltage regulator (AVR) and Governor-Turbine-Generator (G-T-G) controlling the generator dynamics. A generalized form of a closed-loop control generator is:

$$\dot{x}^P_{LC,j} = f^P_{LC,j}(x^P_{LC,j}, y^P_{LC,j}, y^{P,ref}_{LC,j}, p^P_{LC,j}) \tag{A.1}$$

where $x^P_{LC,j}$ are the state variables defining the system dynamics, $y^P_{LC,j}$ are the local coupling variables through which local dynamics of generator j interacts with the rest of the system, $y^{P,ref}_{LC,j}$ are the nominal set points for the coupling variables, and $p^P_{LC,j}$ are the parameters of a generator j.

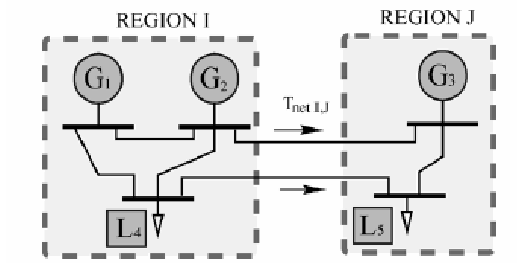

Fig. 9 One line diagram for interconnected 5-bus system

The network constraints during normal conditions real power and reactive power need to be balanced at each instant of time. In terms of nodal equations, these constraints require the complex-valued power into the network $\hat{S}^N$ to be equal to the complex valued power $\hat{S}$ injected into each node.

$$\hat{S}^N = diag(\hat{V})\hat{Y}^*_{bus}\hat{V}^* \tag{A.2}$$

Combining the closed-loop dynamics of all generators inside the control area (A.1) with the network flow constraints (A.2), generally results in a coupled set of nonlinear DAEs of the form:

$$\dot{x} = f(x, y, y^{ref}, p, d), \qquad x(0) = x_0 \tag{A.3}$$

$$0 = g(x, y, y^{ref}, p, d) \tag{A.4}$$

where x are state variables defining the system dynamics of all system components and y are coupling variables in the transmission lines. Vector p represents the system parameters such as forecast demand, damping of power plants, etc. The system disturbances are represented by vector d.

This model is a generalized description of the dynamics in the interconnected power system. It is of very high order for a typical control area or region. Because there is not much one could do to formalize control design for dynamic systems characterized as DAE models, current control designs first simplify the DAE model to a set of nonlinear ordinary differential equations (ODEs). One of the main assumption for this simplification is the nonsingularity of matrix J_2, where

$$J_2 = \begin{bmatrix} \frac{\partial P_L}{\partial \theta_L} & \frac{\partial P_L}{\partial V_L} \\ \frac{\partial Q_L}{\partial \theta_L} & \frac{\partial Q_L}{\partial V_L} \end{bmatrix} \tag{A.5}$$

A nonsingular matrix J_2 reflects that, specified real and reactive power load demand, one could compute the voltage magnitudes and angles at load buses. A nonsingular matrix J_2 is a necessary condition for the DAE model to be converted to ODE model, under which current hierarchical control is designed. Further more, the resulting ODE model of the form (A.3) is obtained by solving (A.4) for y in terms of x, and (A.3) takes on the form of ODE. System S is quasi-stationary when $\dot{x} \equiv 0$. It is known that under some modeling assumptions, the function $f(x, y, y^{ref}, p, d)$ has the same form analogous to the complex power flow solutions [7].

The sensitivity of the quasi-stationary power flow solution with respect to changes in system state is further defined as Jacobian matrix

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix} \tag{A.6}$$

where V is voltage magnitude whose voltage is not directly controlled (loads, in particular). θs are the phase angle of voltage at all but slack bus. There exists rich literature concerning the role and properties of Jacobian matrix J in relation to the possible potential quasi-stationary voltage problems [9]. This is the starting model for the analysis in this paper.

REFERENCES

[1] M.D. Ilić, E.H. Allen, J.W. Chapman, C.A. King, J.H. Lang, and E. Litvinov, "Preventing future blackouts by means of enhanced electric power systems control: from complexity to order," *Proceedings of the IEEE*, vol. 93, Nov. 2005, pp.1920–1941.

[2] A. Thatte, and M.D. Ilić, "An assessment of reactive power/voltage control devices in distribution networks," in *Proc. IEEE Power Engineering Society 2006 General Meeting*, Paper # 06GM1233, June 2006, Montreal, Canada.

[3] M.A. Elizondo, and M.D. Ilić, "Determining the cost of dynamic control capacity for improving system efficiency," in *Proc. IEEE Power Engineering Society 2006 General Meeting*, Paper # 06GM0839, June 2006, Montreal, Canada.

[4] T.Van Cutsem and R. Mailhot, "Validation of a fast voltage stability analysis method on the Hydro-Quebec system," *IEEE Trans. Power Systems*, vol. 12, pp. 282-292, Feb. 1997.

[5] L. Xie and M.D. Ilić, "Towards grid modernization through enhanced computation and control," *IEEE Trans. Power Systems.*, submitted for publication.

[6] J.W. Chapman, M.D. Ilić, C.A. King, L.Eng, and H. Kaufman, "Stabilizing a multi-machine power system via decentralized feedback linearizing excitation control," *IEEE Trans. Power Systems*, vol. 8, pp. 830-839, Aug. 1993.

[7] P.W. Sauer and M.A. Pai, "Power system steady-state stability and the load-flow Jacobian," *IEEE Trans. Power Systems*, vol. 5, pp. 1374-1383, Nov. 1990.

[8] M. D. Ilić and J. Zaborszky, *Dynamics and Control of Large Electric Power Systems.* New York: Wiley Interscience, 2000.

[9] P.-A. Lof, G. Andersson, and D.J. Hill, "Voltage stability indices for stressed power systems," *IEEE Trans. Power Systems*, vol. 8, pp. 326-335, Feb. 1993.

[10] C. Grigg, P. Wong, P. Albrecht, R. Alan, et. al, "The IEEE Reliability Test System-1996. A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee," *IEEE Trans. Power Systems*, vol. 14, pp. 1010-1020, Aug. 1999.

[11] J. Ilić, lL. Xie, M. Ilić, "Critical Voltage Monitoring Using Sensitivity and Optimal Information Machine Learning", North American Power Symposium, September 2006 (submitted).

[12] D. D. Siljak, *Decentralized Control of Complex Systems.* Academic Press, 1991.