STOCHASTIC CONTINGENCY ANALYSIS

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ABSTRACT

A probabilistic formulation for the steady state contingency analysis is presented. The formulation takes into account load and generation data uncertainties and considers the occurrences of contingencies as random variables. Results are obtained in a direct calculation and presented in a compact form in terms of expected values, standard deviations and confidence intervals.

Applications of the method in transmission system planning designs and on-line security calculations are envisaged. The method is developed using a linear model was tested on a 22 node system. Results of tests are included.

INTRODUCTION

Contingency analysis is a valuable tool for a reliable planning and secure operation of a power system. It is a study of the system under contingencies i.e. line outage, generator outage etc. As a result of the contingencies simulated, both transient and steady state responses occur, this paper deals only with steady state results.

The main objective of a deterministic contingency analysis is to determine every situation in which operating limits of system components have been violated because of prespecified contingencies. As the number of contingencies to be simulated in a real system is usually very large fast and efficient methods have been developed [2-3]. Due to the random ocurrence of outages it seems more natural and practical to assign a probability to every contingency and to obtain an overall uncertainty interval, for every variable of interest, due to the combined effect of all the contingencies "weighted" according to their probabilities. This approach is flexible enough to include uncertainties in load and generation data as in a stochastic load flow [4-5]. Therefore the results of a stochastic contingency analysis combine the uncertainties of nodal injections plus the probabilities of occurrence of events in a single calculation. It has been proved through tests, with linear models, that the results of a stochastic contingency analysis, probabilistically encompass the results of many load flows whose data have been perturbed by errors and contingencies.

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The method has been developed to be incorporated in a transmission system planning model. It considers a linear approximation for the network equations and uses compensation methods [1] to simulate contingencies. The results provided by the stochastic-contingency analysis are basically expected values and standard deviations of line flows.

In on-line applications the method proposed can be useful in the calculation of corrective actions required to maintain a system in a preventive state [6]. Deterministic approaches [7-10] to this problem consider all the possible insecure cases as constraints to the problem which highly penalizes the economic operation of the system or whatever the objective function maybe. This is because none of the prespecified contingencies may occur although the solution is found for the case in which any of them can occur. With the proposed method contingencies are included according to probabilities. A recent paper [16] formulates probabilistic algorithms which trade off power system operating cost with system security effects.

FORMULATION

Stochastic load flow

It has been shown in recent papers [4-5] that is possible to model statistically load and generation input data uncertainties in a load flow problem and to calculate the variances of all the system variables. Some of the important results and assumptions will be shown below.

The load flow problem can be described by a set of linear equations as

$$\underset{\sim}{y} = A \underset{\sim}{x}_t + \underset{\sim}{\mathcal{E}} \tag{1}$$

where

A is a constant matrix

$\underset{\sim}{y}$ is the vector of observed or forcasted quantities

$\underset{\sim}{x}_t$ is the vector of state variables (true values)

$\underset{\sim}{\mathcal{E}}$ is the noise vector associated with the observed or forcasted quantities

It is assumed that the errors will be randomly distributed around zero and that the covariance matrix of is known; hence

$$E\ (\underset{\sim}{\mathcal{E}}) = \underset{\sim}{0} \tag{2}$$

$$E\ (\underset{\sim}{\mathcal{E}}\underset{\sim}{\mathcal{E}}^t) = C_E \tag{3}$$

applying the least squares process to (1) the estimate obtained is [11].

$$\hat{\underset{\sim}{x}} = (A^t \; C_E^{-1} \; A)^{-1} A^t \; C_E^{-1} \; \underset{\sim}{y} \qquad (4)$$

The normal load flow formulation has zero degress of freedom [5] hence is not possible to obtain any error filtering. Therefore equation (4) becomes,

$$\hat{\underset{\sim}{x}} = A^{-1} \; \underset{\sim}{y} \qquad (5)$$

The covariance matrix of $\hat{\underset{\sim}{x}}$ is obtained as [11]

$$C_{\hat{\underset{\sim}{x}}} = (A^t \; C_E^{-1} \; A)^{-1} \qquad (6)$$

Let $\hat{\underset{\sim}{z}}$ be a linear function of $\hat{\underset{\sim}{x}}$

$$\hat{\underset{\sim}{z}} = H \; \hat{\underset{\sim}{x}} \qquad (7)$$

The covariance matrix of z can be written as

$$C_{\hat{\underset{\sim}{z}}} = H \; C_{\hat{x}} \; H^t \qquad (8)$$

This matrix contains all the information regarding variances and covariances of system variables i.e. line flows etc.

Compensation method

The compensation method is nothing more than an application of the superposition principle by which any change in the transmission system i.e. addition or removal of lines, can be simulated by suitable injections into the system, without the removal or addition of any lines. This method has proved [2-3] to be very efficient in the simulation of contingencies.

Compensating injections in a system like (1) can be obtained as [12]

$$\underset{\sim}{y_c} = D\,(D^t \; A^{-1} \; D + E^{-1})^{-1} \; D^t \; A^{-1} \; \underset{\sim}{y} \qquad (9)$$

where

D is a connection matrix that simulates the contingenci es, each colum has only two non-zero elements i.e. 1, -1. There is a column for every line outage.

E is a diagnonal matrix, each diagonal term equals the impedance of the line required to simulate the change.

It is important to note in (9) that the compensating injections are expressed as a linear function of the original injections and that y_c is also a random vector.

Probabilistic line outage simulation

Based on the compensation method principles a random vector ($\underset{\sim}{y_z}$) of nodal injections is used to simulate contingencies probabilistically. It is defined as

$$\underset{\sim}{y_z} = \underset{\sim}{0} \quad \text{with probability } (1-p_i)$$

$$\underset{\sim}{y_z} = \underset{\sim}{y_c} \quad \text{with probability } (p_i)$$

p_i is the probability of the event which involves the contingency simulated.

It can be said that there is a probability (p_i) that the compensating inyections, ($\underset{\sim}{y_c}$), will appear in the system and a probability ($1-p_i$) that any compensating injection is present.

It is assumed that the probability of line outage occurrence is known and that every outage is statistically independent. There fore the probability of an event which involves an outage can be obtained readily i.e. product of probabilities.

Estimated line flows

Real power line flows can be expressed as a linear function of voltage phase angles using an approximated model [13]. Estimated values for these flows that include the effects of contingencies and load and generation uncertainties can be obtained as follows.

Estimated line flows without any contingency considered can be written as

$$\hat{\underset{\sim}{F}} = Y_L \; \hat{\underset{\sim}{x}} \qquad (10)$$

where

Y_L is a matrix that relates real power line flows and phase angles, it is made up of elements of the admittance matrix

The estimated change in line flows for very contingency can be expressed as

$$\hat{\underset{\sim}{\Delta F_i}} = Y_L \; \hat{\underset{\sim}{\Delta x_i}} \qquad (11)$$

where

$$\hat{\underset{\sim}{\Delta x_i}} = A^{-1} \; \underset{\sim}{y_{z_i}} \qquad (12)$$

hence

$$\hat{\underset{\sim}{\Delta x_i}} = A^{-1} \; \underset{\sim}{y_{c_i}} \; p_i \qquad (13)$$

subscript i refers to the i^{th} contingency

Therefore total estimated line flows can be written as

$$\hat{\underset{\sim}{F_t}} = \hat{\underset{\sim}{F}} + \sum_i \hat{\underset{\sim}{\Delta F_i}} \qquad (14)$$

Line flow variances

Line flow variances can be obtained using the linear relationship between the total flow in a line F_{t_j}, and the line flow without outages F_j, and the change in line flow due to contingencies $\Delta F_{j/i}$

let us define

$$\underset{\sim}{g} = (1\ 1\ 1\ \ldots\ 1)$$

and

$$\Delta \underset{\sim}{F}_j = (F_j\ \Delta F_{j/1}\ \Delta F_{j/2}\ \cdots \Delta F_{j/n})$$

hence, F_{t_j} can be written as

$$F_{t_j} = \underset{\sim}{g} \Delta \underset{\sim}{F}_j^t \tag{15}$$

therefore the variance of F_{t_j} can be obtained as

$$\text{Var}\ (F_{t_j}) = \underset{\sim}{g}\, C_{\Delta \underset{\sim}{F}_j}\, \underset{\sim}{g}^t \tag{16}$$

where

$C_{\Delta \underset{\sim}{F}_j}$ is the covariance matrix of the random vector $\Delta \underset{\sim}{F}_j$

A line flow without any contingency considered can be expressed in terms of nodal injections as

$$F_j = \underset{\sim}{y}_L\ A^{-1}\ \underset{\sim}{y} = \underset{\sim}{y}_A\ \underset{\sim}{y} \tag{17}$$

where

$\underset{\sim}{y}_L$ is a row vector of Y_L in (10)

hence the variance of F_j can be obtained as

$$\text{Var}\ (F_j) = \underset{\sim}{y}_A\ C_E\ \underset{\sim}{y}_A^t \tag{18}$$

The change in line flow due to a contingency can be written, from (11) and (12), as shown below.

$$\Delta F_{j/i} = \underset{\sim}{y}_L\ A^{-1}\ \underset{\sim}{y}_{z_i} \tag{19}$$

hence

$\Delta F_{j/i} = \underset{\sim}{y}_L\ A^{-1}\ \underset{\sim}{y}_{c_i}$ with probability p_i

and

$\Delta F_{j/i} = 0$ with probability $1 - p_i$

let

$$\Delta F^c_{j/i} = \underset{\sim}{y}_L\ A^{-1}\ \underset{\sim}{y}_{c_i} \tag{20}$$

therefore the variance of $\Delta F_{j/i}$ can be obtained as

$$\text{Var}\ (\Delta F_{j/i}) = p_i\ \text{Var}\ (\Delta F^c_{j/i}) + p_i\ (1-p_i)\ (\Delta F^c_{j/i})^2 \tag{21}$$

The variance of $\Delta F^c_{j/i}$ can be written in terms of C_E as follows. Let us write (9) in a compact form as

$$\underset{\sim}{y}_{c_i} = G_i\ \underset{\sim}{y} \tag{22}$$

then (20) can be written as

$$\Delta F^c_{j/i} = \underset{\sim}{y}_L\ A^{-1}\ G_i\ \underset{\sim}{y} \tag{23}$$

let $\underset{\sim}{y}_b = \underset{\sim}{y}_L\, A^{-1}\, G_i$

hence

$$\text{Var}\ \Delta F^c_{j/i} = \underset{\sim}{y}_b\ C_E\ \underset{\sim}{y}_b^t \tag{24}$$

All the diagonal terms of $C_{\Delta F_j}$ have been defined in (18), (21) and (23). Most of the off-diagonal terms are zero or of a negligible value due to the line outage independence assumption and because of the multiple product of probabilities involved.

The covariance calculation between F_j and $\Delta F_{j/i}$ can be written as

$$\text{Cov}\ (F_j\ , \Delta F_{j/i}) = p_i\ \text{Cov}\ (F_j\ , \Delta F^c_{j/i}) \tag{25}$$

from (17) and (23)

$$\Delta F^c_{j/i} = \underset{\sim}{y}_A\ G_i\ \underset{\sim}{y}$$

therefore

$$\text{Cov}\ (F_j, \Delta F^c_{j/i}) = \underset{\sim}{y}_A\ C_E (\underset{\sim}{y}_A\ G_i)^t \tag{26}$$

It is important to note that although the number of contingencies can be very large, the probability of n simultaneous line outages is negligible for $n > 2$, so the number of events to consider is reduced. Appendix I shows simplified expressions for line flow estimated values and variances considering only single outages.

Line flow confidence limits

Although is possible, in special cases [14], to obtain the probability density function for line flows, in general this is not always possible nor convenient because of the large amount of work involved in the computations. However the variance of a linear combination of random variables whose covariance matrix is known can always be calculated regardless the type of distributions involved. The variance of a random variable gives an indication of the dispersion of the probability density function around the expected value. Even in cases where the actual shape of the density function is not available the Chebychev inequality [15] gives a bound on the probability that a random variable will be within k standard deviations of its mean. Hence the standard deviation can be considered as a rather natural unit for the probability law of a random variable. Line flow variability was obtained in terms of standard deviations for a probability based on the Chebychev's bound.

Tests

A 22 node system was used to test the proposed method. This system is a planning alternative for the Mexican 220/400 KV network. Appendix II contains all the test system data.

A Monte Carlo simulation program was developed in order to have a reference to compare the results of the stochastic contingency analysis program. It was considered in the simulation that line outages were statistically independent. Line outage probability was assumed proportional to the transmission line length. Standard deviations for load and generation input data values were assumed as 3.33% of the input data values. Table I presents some of the results obtained. It is important to point out that although the stochastic contingency analysis results were obtained including only single line outages, the standard deviations obtained by both methods are very close, this is because the contribution of simultaneous outages to the variance calculation has very small effect due to the product of probabilities. Line flow standard deviations for the case in which line outage probabilities are zero i.e. line flow variability due to the load and generation uncertainties, are shown in table II. A comparison between tables I and II show a small effect in the estimated flows and a substantial difference in the standard deviations due to the line outage occurrences. Table III presents results for the case in which line flow standard deviations are due only to line outages i.e. nodal injection variance is zero. Line flow variability is shown in all the tables for a 90% probability bound. It is important to note that some maximum line flow values will not be included within the variability shown, this is because the probability of occurrence is very small.

CPU computer time required to perform the stochastic contingency analysis was 8 seconds on a IBM 370/1145 computer. The Monte Carlo simulation took 36 minutes (10000 cases) to obtain comparable results.

Future developments

The stochastic contingency analysis presented will be incorporated into a transmission system planning model[17], it will replace a maximum flow algorithm and a Monte Carlo simulation program that were jointly used to obtain loss of load expected values.

The use of a non-linear model is under investigation and will be reported in the future.

TABLE I

Power flow estimated values and standard deviations from a stochastic contingency analysis (S.C.A) and Monte Carlo simulation program (M.C.S). Results include line outage probabilities and nodal injections uncertainties

Line	Estimated Flow (MW)		Standard Deviation (MW)		Line Flow Variability (MW)
	S.C.A	M.C.S	S.C.A	M.C.S	
13- 2	161.9	161.6	33.1	33.0	102.6
5- 4	35.8	35.8	6.1	6.4	18.9
6- 8	421.4	421.5	71.8	71.1	222.6
8-15	306.8	307.1	60.2	61.6	186.6
11-10	146.0	145.9	39.4	40.9	122.1
12- 1	278.6	278.1	47.3	50.1	146.6
1-11	129.1	128.6	33.1	33.8	102.6
16-17	229.0	229.1	36.3	37.8	112.5
18-16	64.7	64.9	20.6	21.6	63.9
20-19	477.9	477.6	74.5	76.1	230.9
19-18	322.6	322.2	65.6	66.0	203.4
18-17	260.9	260.1	41.5	43.0	128.6
21-17	502.5	502.3	72.0	75.6	223.2
15-21	583.9	583.5	125.6	125.1	389.4
1-14	47.7	47.5	8.2	8.5	25.4

TABLE II

Power flow estimated values and standard deviations from a S.C.A and M.C.S. Results include only nodal injections uncertainties

Line	Estimated Flow (MW)		Standard Deviation (MW)		Line Flow Variability (MW)
	S.C.A	M.C.S	S.C.A	M.C.S	
13- 2	159.5	159.8	14.6	14.9	43.8
5- 4	36.0	36.0	2.1	2.2	6.3
6- 8	418.5	418.6	21.8	22.4	65.4
8-15	303.5	303.4	28.1	25.9	84.3
11-10	145.2	145.1	17.2	16.4	51.6
12- 1	276.9	276.8	17.6	17.3	52.8
1-11	130.7	130.7	17.3	17.1	51.9
16-17	228.9	228.7	20.9	22.0	62.7
18-16	63.9	63.8	11.0	10.9	33.0
20-19	478.9	479.0	18.4	18.7	55.2
19-18	318.5	318.6	31.4	31.9	94.2
18-17	259.6	259.6	23.6	25.1	70.8
21-17	500.6	500.5	23.3	20.9	69.9
15-21	573.1	573.1	28.9	26.6	86.7
1-14	47.5	47.6	3.7	3.8	11.1

CONCLUSIONS

Load and generation uncertainties and probabilities of outage occurrence can be included in a single formulation that allows the calculation of estimated values, variances and confidence intervals for real power line flows.

The number of contingencies to consider in the study is limited, because simultaneous outages involve multiple products of probabilities which cause that many terms in the calculations are negligible. It was found in preliminary tests on a 22 node system that only single contingencies need to be considered to obtain results comparable to those obtained by Monte Carlo simulations.

Stochastic contingency analysis results can be efficiently used in Power System planning studies as they are expressed in probabilistic terms.

Considerable computer time savings can result from the use of the proposed method, compared to the time required by the simulations often used in planning studies.

TABLE III

Power flow estimated values and standard deviations from a S.C.A and M.C.S. Results include only line outage probabilities.

Line	Estimated Flow (MW) S.C.A	M.C.S.	Standard Deviation (MW) S.C.A	M.C.S	Line Flow Variability (MW)
13-2	161.9	161.7	29.5	28.8	88.6
5-4	35.8	35.8	5.8	5.4	17.3
6-8	421.4	421.6	68.3	69.0	204.8
3-15	306.8	305.9	53.0	52.8	158.9
11-10	146.0	146.1	35.4	35.7	106.2
12-1	278.6	278.5	43.9	43.2	131.5
1-11	129.1	129.1	28.0	27.8	84.2
16-17	229.0	229.2	29.6	29.5	88.9
18-16	64.7	64.5	17.3	17.3	52.0
19-20	477.9	477.8	72.1	70.9	216.3
19-18	322.6	322.5	57.3	56.6	172.0
18-17	260.9	260.8	34.0	34.2	101.9
21-17	502.5	502.6	68.0	69.1	204.2
15-21	583.9	583.7	122.0	120.4	366.2
1-14	47.7	47.8	7.3	7.2	21.9

APPENDIX I

Assume a system with 3 lines and consider the following notation.

Lines in service	Lines out of service	Event probability	Estimated flow
1, 2, 3	-	$p_a=(1-p_1)(1-p_2)(1-p_3)$	F_{sf}
2, 3	1	$p_b=p_1(1-p_2)(1-p_3)$	F_1
1, 3	2	$p_c=p_2(1-p_1)(1-p_3)$	F_2
1, 2	3	$p_d=p_3(1-p_1)(1-p_2)$	F_3
3	1, 2	$p_e=p_1p_2(1-p_3)$	$F_{1,2}$
2	1, 3	$p_f=p_1p_3(1-p_2)$	$F_{1,3}$
1	2, 3	$p_g=p_2p_3(1-p_1)$	$F_{2,3}$
-	1, 2, 3	$p_h=p_1p_2p_3$	$F_{1,2,3}$

The total estimated flow in line 1 can be written as

$$F_t = p_a F_{sf} + p_b F_1 + p_c F_2 + p_d F_3 + p_e F_{1,2} + p_f F_{1,3} + p_g F_{2,3} + p_h F_{1,2,3} \quad \text{(I-1)}$$

Equation (I-1) can be expressed in terms of the estimated flow without outages (F_{sf}) as

$$F_t = p_a F_{sf} + p_b(F_{sf} + \Delta F_1) + p_c(F_{sf} + \Delta F_2) + p_d(F_{sf} + \Delta F_3) + p_e(F_{sf} + \Delta F_{1,2}) + p_f(F_{sf} + \Delta F_{1,3}) + p_g(F_{sf} + \Delta F_{2,3}) + p_h(F_{sf} + \Delta F_{1,2,3}) \quad \text{(I-2)}$$

expressing F_t in terms of the line outage probabilities

$$F_t = F_{sf} - p_1 F_{sf} + (p_2 - p_2 p_1 + p_1 p_2 p_3)\ \Delta F_2 + (p_3 - p_3 p_1 - p_3 p_2 + p_1 p_2 p_3)\ \Delta F_3 + (p_2 p_3 - p_1 p_2 p_3)\ \Delta F_{2,3} \quad \text{(I-3)}$$

neglecting products of probabilities

$$F_t = (1 - p_1) F_{sf} + p_2\ \Delta F_2 + p_3\ \Delta F_3 \quad \text{(I-4)}$$

The variance of F_t can be expressed as

$$\mathrm{Var}(F_t) = p_a E(F_{sf})^2 + p_b E(F_1)^2 + p_c E(F_2)^2 + p_d E(F_3)^2 + p_e E(F_{1,2})^2 + p_f E(F_{1,3})^2 + p_g E(F_{2,3})^2 + p_h E(F_{1,2,3})^2 - (F_t)^2 \quad \text{(I-5)}$$

expanding each term in (I-5)

$$\mathrm{Var}(F_t) = p_a \left[\mathrm{Var}\, F_{sf} + (F_{sf})^2\right] + p_b \left[\mathrm{Var}\, F_1 + (F_1)^2\right] + p_c \left[\mathrm{Var}\, F_2 + (F_2)^2\right] + p_d \left[\mathrm{Var}\, F_3 + (F_3)^2\right] + p_e \left[\mathrm{Var}\, F_{1,2} + (F_{1,2})^2\right] + p_f \left[\mathrm{Var}\, F_{1,3} + (F_{1,3})^2\right] + p_g \left[\mathrm{Var}\, F_{2,3} + (F_{2,3})^2\right] + p_h \left[\mathrm{Var} F_{1,2,3} + (F_{1,2,3})^2\right] - (F_t)^2 \quad \text{(I-6)}$$

expressing (I-6) in terms of the variance of F_{sf} and simplifying terms

$$\mathrm{Var}(F_t) = \mathrm{Var}\, F_{sf} + (p_b + p_e + p_f + p_h) \left[(F_{sf})^2 - \mathrm{Var}\, F_{sf}\right] + p_c \left[\mathrm{Var}\ \Delta F_2 + 2\mathrm{Cov}(F_{sf}, \Delta F_2) + (\Delta F_2)^2\right] + p_d \left[\mathrm{Var}\ \Delta F_3 + 2\mathrm{Cov}(F_{sf}, \Delta F_3) + (\Delta F_3)^2\right] +$$

$$+ \; p_g \left[\mathrm{Var}\;\Delta F_{2,3} + 2\mathrm{Cov}(F_{sf}, \Delta F_{2,3}) + (\Delta F_{2,3})^2 \right] - (F_t - F_{sf})^2 \qquad \text{(I-7)}$$

neglecting products of probabilities and simplifying (I-7) becomes

$$\mathrm{Var}\;F_t = (1-p_1)\;\mathrm{Var}\;F_{sf} + p_1(F_{sf})^2 + p_2\left[\mathrm{Var}\;\Delta F_2 + 2\;\mathrm{Cov}(F_{sf}, \Delta F_2) + (\Delta F_2)^2\right] + p_3\left[\mathrm{Var}\;\Delta F_3 + 2\;\mathrm{Cov}(F_{sf}, \Delta F_3) + (\Delta F_3)^2\right] \qquad \text{(I-8)}$$

APPENDIX II

Test System Line Data

Line	Nodes Connected		Resistence p.u.	Reactance p.u.	Line Outage Probability
1	2	3	.0067	.0469	.010
2	2	3	.0067	.0469	.014
3	5	6	.0561	.3901	.010
4	5	6	.0561	.3901	.018
5	7	8	.0408	.2630	.010
6	7	8	.0408	.2630	.015
7	8	15	.0362	.0983	.010
8	10	11	.0137	.0916	.027
9	10	12	.0068	.0747	.010
10	12	1	.0060	.0755	.010
11	11	1	.0080	.0509	.025
12	16	17	.0039	.0514	.010
13	16	18	.0053	.0335	.022
14	19	20	.0026	.0367	.018
15	18	19	.0040	.0534	.010
16	17	18	.0041	.0540	.017
17	1	17	.0017	.0222	.010
18	1	17	.0016	.0131	.014
19	1	17	.0063	.0395	.026
20	21	17	.0044	.0362	.018
21	21	1	.0041	.0540	.010
22	21	15	.0095	.0645	.010
23	22	6	.0050	.0344	.010
24	22	6	.0050	.0344	.010
25	14	1	.0357	.2250	.010
26	13	2	.0075	.0619	.010
27	13	2	.0075	.0619	.025
28	4	5	.0197	.1642	.010
29	4	5	.0197	.1642	.016
30	6	8	.0081	.0557	.010
31	6	8	.0081	.0557	.017
32	8	9	.0240	.1633	.010
33	8	15	.0362	.0983	.021
34	10	11	.0045	.0606	.010
35	12	1	.0060	.0755	.019
36	11	1	.0038	.0539	.017
37	11	1	.0073	.0498	.010
38	16	18	.0053	.0335	.010
39	19	20	.0026	.0367	.016
40	19	20	.0027	.0348	.010
41	18	19	.0040	.0534	.023
42	17	18	.0041	.0540	.010
43	1	17	.0017	.0222	.022
44	1	17	.0040	.0322	.021
45	14	1	.0357	.2250	.014
46	21	17	.0044	.0362	.010
47	21	15	.0040	.0521	.023
48	22	3	.0050	.0337	.010
49	22	3	.0050	.0337	.019
50	8	9	.0240	.1633	.010

Test System Load and Generation Data

Node	Load (MW)	Generation (MW)	Node	Load (MW)	Generation (MW)
1	3545	SLACK	12	200	1230
2	175	230	13	439	758
3	60	634	14	195	100
4	115	43	15	267	953
5	251	200	16	448	549
6	363	443	17	623	102
7	267	79	18	148	158
8	716	712	19	851	26
9	145	107	20	209	1671
10	1108	392	21	198	360
11	1104	953	22	98	30

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