Probabilistic load flow considering network outages

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Indexing terms: *Power transmission and distribution, Power system protection, Transformers, Mathematical techniques*

Abstract: Many techniques have been proposed to solve the load flow problem probabilistically. The great majority have only accounted for load and generation data uncertainties, and therefore, the network configuration has been considered fixed. So far, the effects of the configuration uncertainties due to the probabilistic nature of the network have not been deeply analysed. The paper presents a new method for obtaining a probabilistic load flow solution when network outages are modelled as a random variable. The proposed technique is applied to a typical power system and the results discussed.

List of principal symbols

c = network configuration
g, h = load flow functions
P = active power
Q = reactive power
V = voltage magnitude
X = state random vector (voltages and angles)
Y = input random vector (power injections)
Z = output random vector (power flows)
u = unavailability
θ = voltage angle
μ = expected value
σ = standard deviation
ρ = variance scaling factor
∂ = partial derivative

1 Introduction

Probabilistic load flow (PLF) is a subject of continuing interest [1–13]. The usefulness of the PLF technique is due to its ability to assess adequacy indices such as the probability of a line flow being greater than its thermal rating, and the probability of a busbar voltage being outside its operational constraints. These indices are obtained when the probabilistic nature of load, generation and network is analysed for power networks operating under steady-state conditions. This analysis can be carried out by Monte Carlo simulation (MCS) techniques, analytical methods, or by a combination of both.

Theoretically, there is no constraint for the MCS method. For instance, the exact nonlinear power flow equations can be used, statistical dependence between loads and generation can be easily considered, probability of different network configurations can be taken into account etc. On the other hand, the MCS method requires a large number of trials to ensure results of reasonable accuracy. This means that a huge amount of storage and computing time is necessary. To reduce the computational effort, linearised power flow equations can be used when the input uncertainty level is not very large and the degree of nonlinearity in the load flow functions is reasonably small.

Paper 3800C (P9), first received 19th September 1984 and in revised form 21st January 1985

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Whereas MCS methods only use the law of large numbers to justify their results, analytical methods use much more elaborate techniques and are based on probability theory. Owing to the inherent complexity of the analytical solution, the following assumptions have been considered by most PLF algorithms:

(*a*) linear load flow equations
(*b*) independence between input parameters
(*c*) constant network configuration.

Therefore the accuracy of the analytical methods is also limited because of the assumptions used to overcome the inherent difficulties. The main analytical methods can be summarised as follows.

The use of linearised power flow equations about an expected operating point [1–6] can be considered, in terms of accuracy, to be a reasonable assumption for a wide range of input uncertainties. As these uncertainties become very large, a more elaborate algorithm [7], or even an MCS technique using the exact load flow equations, should be used. Assumptions (*a*), (*b*) and (*c*) mean the PLF solution becomes a sum of independent random variables weighted by sensitivity coefficients. Consequently, the solution is obtained by a convolution process which can be efficiently carried out using fast Fourier transform algorithms [14]. However, there are various reasons for correlations to exist between nodal powers [8–10]. For example, correlation between generation exists because of the need to balance the active system power. This balance is achieved according to the utility's operating policy which includes economic dispatch, redispatch and load shedding. A direct analytical solution to this was not realisable owing to the complexity of the problem, because even a very simple criterion of economic dispatch means that an extra nonlinearity must be included in the PLF analysis. This solution was efficiently achieved by combining the convolution method with MCS techniques using the linear power flow equations [10].

The network configuration is assumed to be a fixed parameter in the great majority of PLF methods. Consequently, the probability of the basic configuration is assumed to be unity, and therefore the probability of losing any network element, such as transmission lines, transformers etc., is neglected. However, for a given operating point (load and generation strategy), there are several possible network configurations. Therefore assumption (*c*) may be considered unrealistic, particularly when the power uncertainties are small.

So far, only a few formulations [11–13] have considered the effects of network outages in the PLF analysis. Reference 13 did not assign a probability to every contingency, and so it is not possible to obtain an overall uncertainty interval, for every variable of interest, owing to the combined uncertainty effects from nodal powers and network. It will be shown in this paper that the probabilistic model used in References 11 and 12 have deficiencies. Also, these References have used a DC model for power flow equations, which is another restriction.

A new PLF algorithm is proposed in this paper that accounts for the probabilistic contingency effects of network elements.

2 Problem formulation

Any change in the network of power systems will alter the set of functions relating inputs and outputs. Therefore each probability density function (PDF) from the output random vector will change and so will the technical and economic decisions. Two basic sources of variation can be identified. The first is the variation in the actual parameters defining lines, transformers or other components. For example, the inductance of a line changes with temperature which is a random variable (RV). These changes are normally assumed to be negligible so that the parameter values of the network have a probability equal to unity. The second source of variation is associated with the availability or unavailability of components such as transmission lines, transformers, switchgear etc., as all are subject to outages due to faults and maintenance. In terms of planning, such outages may be satisfactorily modelled as RVs. Consequently each network configuration has an associated probability; i.e. it can be considered as an RV which follows a discrete distribution. Therefore, for a given configuration c, the load flow equations are

$$Y = g_c(X_c) \tag{1}$$

$$Z_c = h_c(X_c) \tag{2}$$

where

Y = input random vector
X_c = state random vector for configuration c
Z_c = output random vector for configuration c
g_c, h_c = load flow equations for configuration c

Linearising eqns. 1 and 2 around the expected value region [5, 7], and considering configuration c, gives

$$X_c = X'_c + A_c Y \tag{3}$$

$$Z_c = Z'_c + B_c Y \tag{4}$$

where

$$X'_c = X^0_c - A_c Y^0$$

$$Z'_c = Z^0_c - B_c Y^0$$

$$A_c = \left[\frac{\partial g_c}{\partial X_c} \bigg|_{X_c = X^0_c} \right]^{-1}$$

$$B_c = \left[\frac{\partial h_c}{\partial X_c} \bigg|_{X_c = X^0_c} \right] A_c$$

Vectors X'_c and Z'_c are deterministic linear conditions. A_c and B_c are sensitivity coefficient matrices for configuration c. Vectors X^0_c and Z^0_c are obtained from a conventional power flow using as input the expected value of vector Y, i.e. $E\{Y\} = Y^0$, and configuration c.

As the main objective of this work is to model probabilistically the network contingencies, the components of the input vector Y are assumed to be independent [assumption (*b*)]. Therefore, for a given configuration c, the PDFs $f^c_x(x)$ and $f^c_z(z)$ for components x and z of vectors X and Z, respectively, are evaluated by convolving the input PDFs, weighted by sensitivity coefficients obtained from matrices A_c and B_c. Thus

$$f^c_x(x) = N(X'_c, 0) * f_1(a^c_1 Y_1) * f_2(a^c_2 Y_2) * \cdots * f_m(a^c_m Y_m) \tag{5}$$

$$f^c_z(z) = N(Z'_c, 0) * f_1(b^c_1 Y_1) * f_2(b^c_2 Y_2) * \cdots * f_m(b^c_m Y_m) \tag{6}$$

The vector X'_c is interpreted as a normal random variable $N(\, , \,)$ with expected value X'_c and standard deviation equal to zero. The same applies to Z'_c.

Considering each possible network configuration to be associated with a probability of occurrence, the problem consists of combining statistically the PDFs obtained from eqns. 5 and 6.

3 Methods of solution

First, the formulations proposed in Reference 11 and 12 are briefly analysed to identify their main deficiencies. This analysis is followed by the proposed formulation.

3.1 *Formulation 1* [11]

This formulation uses a DC model for the power flow equations. The simulation of different contingencies is carried out through the compensation method. Thus, any change in the transmission system, i.e. addition or removal of lines, can be simulated by suitable injections into the system, without the removal or addition of any lines. The DC model is the first restriction of this formulation because only the variables related to the active power (voltage angles and active flows) are considered.

The major defect of this formulation is the probabilistic model used for the contingencies. Only first-order contingencies are considered and the final active power flow F in a given element e is a weighted summation of flows in this element for each contingency analysed; i.e.

$$F = \left(1 - \sum_{i=1}^{n} u_i\right) F_0 + \sum_{i=1}^{n} u_i F_i \tag{7}$$

where F_0 = flow in the element e with the basic network configuration (no outage case), F_i = flow in the element e when element i is on outage, u_i = unavailability of element i and n = number of elements (transmission lines and transformers).

The weights used in eqn. 7 are, in fact, approximations for the probabilities associated with the basic configuration p_0 and with the configuration considering the outage of element i, p_i, because

$$p_0 = \prod_{i=1}^{n} (1 - u_i) \simeq 1 - \sum_{i=1}^{n} u_i \tag{8}$$

$$p_i = u_i \prod_{j=1 \neq i}^{n} (1 - u_j) \simeq u_i \tag{9}$$

Using approximations 8 and 9, the space of all configurations considered by formulation 1 has a probability equal to unity which is very desirable. On the other hand, from the statistical point of view, the approximate values for p_0 and p_i used to define F in eqn. 7 can give incoherent results under some conditions. To illustrate this point, Table 1 shows the comparison between the exact and approximate probability values of the basic configuration, considering network elements with the same unavailability. Therefore, for a system with 1000 elements each having

Table 1: Comparison between p_0 (exact) and p_0 (formulation 1)

n	10		100		1000	
u	10^{-2}	10^{-3}	10^{-2}	10^{-3}	10^{-2}	10^{-3}
Exact	0.904	0.990	0.366	0.905	4.3×10^{-5}	0.368
Formulation 1	0.900	0.990	0	0.900	−9	0
$n \times u$	0.100	0.010	1	0.100	10	1

$u = 10^{-2}$, the probability of occurrence of the basic configuration, according to formulation 1, is equal to −9, which is not possible. Although not discussed in Reference 11, one condition to validate the above probabilistic model is that the product of the number of elements n and the unavailability u must be significantly smaller than 1.

Finally, formulation 1 also assumes that the PDFs for the power flows are normally distributed, which has been shown to be an unreliable assumption [5].

3.2 Formulation 2 [12]

This formulation also uses a DC model for the power flow equations. The contingency evaluation is carried out in a very efficient way, similar to formulation 1, although the efficiency is facilitated by the simplicity of the DC model.

The probabilistic contingency model has the following characteristics: (i) all first and selected second order outages are considered, and (ii) the exact probability value for each configuration is used, assuming independence between outages. Any type of distribution can be used to model load and generation uncertainties. The final PDFs for the components of vectors X and Z are computed through a weighted summation of the density functions obtained for each analysed configuration.

It should be noted that, because the exact probability values for the configurations are used, and because a limited number of contingencies are analysed, the subspace S' of these configuration states summates to a probability less than unity, if no compensatory action is taken. Table 2 shows the probability of S' for some pairs (u, n), assuming the same unavailability for all lines and considering first order outages only. Therefore, for a system with 1000 elements, each having $u = 0.01$, the area under the probability density curves, obtained through formulation 2, is equal to 4.79×10^{-4} due to the truncation of network states. Therefore, if only first and selected second order outages are analysed, a number of possible configurations will be neglected. This information should be included in the contingency probabilistic model to allow the statistical results obtained from this analysis to be easily interpreted. In fact, it was suggested in Reference 12 that all second order contingencies plus higher orders should be included to minimise the truncation effects. It was also suggested that common mode failures may have to be considered.

Table 2: Probability of $\{S'\}$ – formulation 2

u \ n	10	100	1000
0.1	0.73610	3.2169×10^{-4}	1.9596×10^{-44}
0.01	0.99573	0.73576	4.7924×10^{-4}
0.001	0.99996	0.99536	0.73576

3.3 Proposed formulation

The proposed formulation is an extension of formulation 2, in which the DC model is substituted by an AC linear model described by eqns. 3 and 4. Also, the probabilistic model of contingencies is structured in a way which compensates for the truncation effects.

Consider S to be the space of all possible system configurations c, and p_c the associated probability of each configuration S_c. Thus

$$S = S_1 U S_2 U \cdots U S_c U \cdots U S_t \tag{10}$$

It is not possible for two or more configurations to exist simultaneously; therefore the events S_1 to S_t are mutually exclusive:

$$p_s = probability\{S\} = \sum_{c=1}^{t} p_c = 1 \tag{11}$$

Eqns. 5 and 6 show how PDFs for components x and z from vectors X and Z, respectively, are evaluated conditioned by the network configuration c, i.e. by the event S_c. As all events S_c are mutually exclusive, then the final PDF for each variable x or z is given by

$$f_x(x) = \sum_{c=1}^{t} p_c f_x^c(x) \tag{12}$$

$$f_z(z) = \sum_{c=1}^{t} p_c f_z^c(z) \tag{13}$$

Eqns. 12 and 13 take the network changes into account and show that the final solution is obtained from a weighted sum of density curves. Appendix 8.1 shows how the expected value and variance of x and z can be evaluated.

The crucial point is the evaluation of the probability p_c associated with each configuration. As proposed in formulation 2, p_c can be calculated from the unavailability u_i of each element i of the network. This is done by enumerating all physical network states. Therefore, for a given network c, the probability p_c is given by

$$p_c = \prod_{i=1}^{n_a} (1 - u_i) \prod_{j=1}^{n_u} u_j \tag{14}$$

where

n_a = number of elements available
n_u = number of elements unavailable

Clearly, the analysis of all possible network configurations is impracticable for real power networks due to the huge amount of computing involved. Therefore, some criteria have to be used to reduce the number of analysed configurations. Usually, these criteria consider simultaneously those contingencies more likely to occur and their impact on the system operating point [15]. Although this procedure minimises the truncation effects, the adequacy indices obtained from this analysis have to be regarded as optimistic values. These indices will become more realistic by increasing a well selected list of contingencies.

As the number of networks to be considered is reduced, the probability $p_{s'}$ associated with subspace S' which contains all the analysed networks will be less than unity. Considering that S' is an adequate approximation for space S according to the state truncation criteria, the previous probability value p_c associated with each configuration c can be approximated to p_c' as follows:

$$p_c' = p_c / p_{s'} \tag{15}$$

This ensures, for the t' network configurations analysed, that

$$\sum_{c=1}^{t'} p_c' = 1 \tag{16}$$

Finally, the contributions of the proposed formulation in relation to formulations 1 and 2 are summarised as follows:

(i) The DC power flow model used in formulations 1 and 2 is extended to an AC linear power flow model, properly defined.

(ii) The approximation used for the probabilistic contingency model in the proposed formulation is more consistent from the statistical point of view than the one used in formulation 1.

(iii) Although the probabilistic contingency model used in the proposed formulation is as precise as that used in formulation 2, the statistical interpretation of the results obtained through the proposed formulation is more adequate.

4 Results and discussions

The probabilistic system data used to demonstrate the proposed method are shown in Appendix 8.2. Such data were simulated keeping the original characteristics of the IEEE 14 busbar system.

Two distinct analyses were carried out and the results are shown and discussed as follows. The probabilistic configuration data shown in Appendix 8.2 are only used in analysis 2.

4.1 Analysis 1

This is a sensitivity analysis for the random vectors X and Z in relation to load, generation and network uncertainties. The load uncertainties are increased by a multiplication factor ρ [5, 7] which takes values 0 (Y = deterministic), 1 (Appendix 8.2), 3 and 7. The unavailability u is assumed to be the same for all network elements and changes from 0 (fixed network) up to 10^{-1}. Only first order contingencies are considered, and consequently a correction factor is used to compensate for the state truncation. Table 3 shows the network probability for the basic configuration (no outage case) and for the first order contingencies.

Table 4 shows the results (μ = expected value and σ = standard deviation) for five typical variables, using different values of ρ and u.

An important conclusion from this analysis is that the configuration uncertainties are practically absorbed by the input random uncertainties, when these are large ($\rho = 7$). Therefore, the element outage effects of a network have to be carefully analysed, particularly in operational planning, because at this stage the load uncertainties are generally small. Although this is not a general conclusion, it should be valid for the great majority of existing power transmission networks.

4.2 Analysis 2

The network probabilistic data described in Appendix 8.2 were used. Each configuration has an associated probability. It can be seen that most contingencies are of first order, but some of second and third orders are also considered. A truncation factor has already been used. The criteria used to limit the number of network configurations to be analysed are not discussed. Although this contingency list is relatively small compared with all possible system configurations, it can be considered sufficient for illustrating some useful results and interesting effects.

Three cases are analysed and summarised in Table 5 and in Figs. 1, 2 and 3. In the first case (ΔY), only the uncertainties in the input vector are considered and the network is kept fixed with no outages. This is the conventional algorithm of probabilistic load flow. In the second

Table 3: Probability of configuration against *u*

	U				
Configuration	0	10^{-4}	10^{-3}	10^{-2}	10^{-1}
Basic	1	0.9980	0.9804	0.8320	0.3100
First order	0	1.0×10^{-4}	9.8×10^{-4}	8.4×10^{-3}	3.5×10^{-2}

Table 4: Sensitivity μ, σ against *u* against ρ

Variable	u	μ	σ			
			$\rho = 0$	$\rho = 1$	$\rho = 3$	$\rho = 7$
	0	−12.83	0	0.93	2.72	6.34
	10^{-4}	−12.83	0.34	1.02	2.77	6.37
θ_3, degrees	10^{-3}	−12.88	1.02	1.41	2.96	6.49
	10^{-2}	−13.28	2.94	3.12	4.17	7.44
	10^{-1}	−14.67	5.74	5.87	6.73	9.95
	0	1.01867	0	0.00078	0.00235	0.00548
	10^{-4}	1.01866	0.00037	0.00096	0.00241	0.00557
V_5, p.u.	10^{-3}	1.01860	0.00109	0.00141	0.00267	0.00578
	10^{-2}	1.01812	0.00312	0.00330	0.00422	0.00729
	10^{-1}	1.01641	0.00601	0.00619	0.00719	0.01091
	0	160.16	0	11.61	28.31	64.27
	10^{-4}	160.15	2.05	11.82	28.48	64.38
P_{1-2}, MW	10^{-3}	160.08	5.82	13.02	29.00	64.62
	10^{-2}	159.47	16.83	20.47	33.06	64.64
	10^{-1}	157.34	33.97	35.92	44.40	73.26
	0	−19.68	0	2.69	6.56	14.89
	10^{-4}	−19.68	0.31	2.72	6.58	14.91
Q_{1-2}, MVAR	10^{-3}	−19.68	0.86	2.84	6.63	14.93
	10^{-2}	−19.65	2.50	3.67	7.02	15.09
	10^{-1}	−19.55	5.05	5.72	8.26	15.66
	0	45.76	0	0.88	2.66	6.18
	10^{-4}	45.76	0.58	1.05	2.72	6.21
S_{5-6}, MVA	10^{-3}	45.76	1.70	1.91	3.16	6.43
	10^{-2}	45.78	4.94	5.02	5.64	8.04
	10^{-1}	45.85	10.00	10.05	10.42	12.11

Table 5: Network outage effects in the PLF solution

V_5, p.u.

Case	μ	σ	$p \leqslant 1.014$	$p \leqslant 1.017$	$p \leqslant 1.020$	$p \leqslant 1.022$
ΔY	1.01867	0.00078	0.0000	0.0186	0.9590	1.0000
ΔC	1.01797	0.00335	0.0590	0.0610	0.9970	0.9970
ΔYC	1.01796	0.00349	0.0573	0.0830	0.9636	0.9970

θ_9, degrees

Case	μ	σ	$p \leqslant -17$	$p \leqslant -15$	$p \leqslant -14$	$p \leqslant -13$
ΔY	−15.015	0.561	0.0003	0.5165	0.9699	1.0000
ΔC	−15.477	2.611	0.0540	0.9880	0.9970	1.0000
ΔYC	−15.478	2.690	0.0556	0.5669	0.9708	0.9992

P_{5-6}, MW

Case	μ	σ	$p \leqslant 42$	$p \leqslant 43$	$p \leqslant 45$	$p \leqslant 47$
ΔY	44.13	0.96	0.0018	0.1143	0.8081	0.9999
ΔC	44.32	4.59	0.0260	0.0480	0.9480	0.9610
ΔYC	44.32	4.69	0.0327	0.1625	0.7864	0.9621

P_{12-13}, MW

Case	μ	σ	$p \leqslant 1.4$	$p \leqslant 1.6$	$p \leqslant 1.8$	$p \leqslant 2.0$
ΔY	1.62	0.16	0.0439	0.4916	0.8126	0.9991
ΔC	1.64	0.53	0.0150	0.0370	0.9390	0.9700
ΔYC	1.64	0.56	0.0791	0.5073	0.7877	0.9667

Q_{5-6}, MVAR

Case	μ	σ	$p \leqslant 11.0$	$p \leqslant 11.7$	$p \leqslant 12.4$	$p \leqslant 13.0$
ΔY	12.07	0.32	0.0000	0.1251	0.8525	0.9993
ΔC	11.74	1.61	0.0590	0.0690	0.9830	0.9920
ΔYC	11.75	1.66	0.0585	0.1974	0.8606	0.9898

S_{2-4}, MVA

Case	μ	σ	$p \leqslant 48$	$p \leqslant 53$	$p \leqslant 58$	$p \leqslant 63$
ΔY	55.93	2.33	0.0001	0.1114	0.8195	0.9996
ΔC	55.55	9.43	0.0260	0.0260	0.9640	0.9690
ΔYC	55.55	9.72	0.0261	0.1333	0.7974	0.9675

S_{5-6}, MVA

Case	μ	σ	$p \leqslant 44$	$p \leqslant 46$	$p \leqslant 48$	$p \leqslant 49$
ΔY	45.76	0.88	0.0059	0.6615	0.9968	1.0000
ΔC	45.89	4.57	0.0480	0.9500	0.9700	0.9700
ΔYC	45.88	4.67	0.0512	0.6654	0.9614	0.9702

case (ΔC), only the configuration network uncertainties are considered and the expected value of the input random vector is used. This is an algorithm of contingency analysis where each configuration has an associated probability. Finally, in the third case (ΔYC), both input and network uncertainties are analysed simultaneously. Note that cases ΔY and ΔC are, in fact, particular cases for the proposed method.

Table 5 shows seven typical state/output random variables in terms of probabilistic parameters (μ, σ and probabilities). It can be seen that case ΔYC is extremely relevant in assessing X and Z uncertainties compared with the individual analysis ΔY or ΔC. For example, consider the random variable S_{5-6}. The probability of this flow being less or equal to 48 MVA is 0.9968 considering only the uncertainties ΔY, 0.9700 considering only the uncertainties ΔC and 0.9614 as both uncertainties ΔYC are considered at the same time.

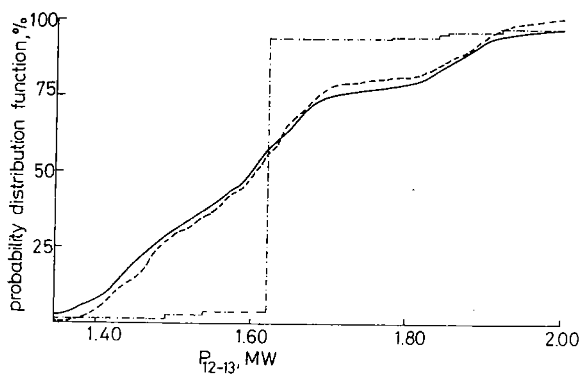

Fig. 1 MW *flow distribution* P_{12-13}

- - - - ΔY
-·-·-·- ΔC
—— ΔYC

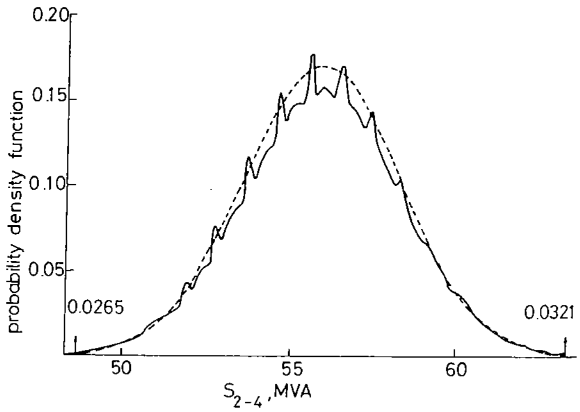

Fig. 2 MVA *flow density* S_{2-4}

- - - - ΔY
—— ΔYC

The probability distribution function for variables P_{12-13} and S_{5-6}, and the probability density function for variable S_{2-4} are shown in Figs. 1, 3 and 2, respectively.

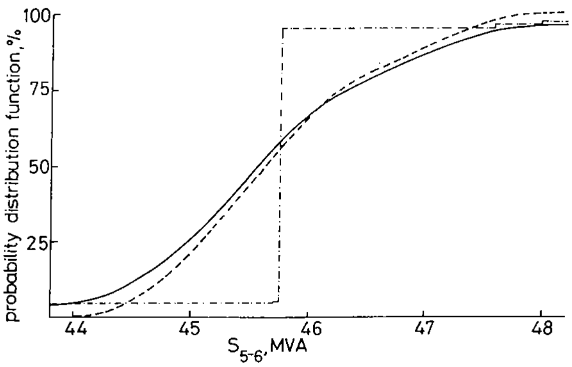

Fig. 3 MVA *flow distribution* S_{5-6}

- - - - ΔY
-·-·-·- ΔC
——— ΔYC

From these Figures, the differences between ΔY, ΔC and ΔYC are better visualised. Owing to scale limitations, the tails of some functions are not represented because they are very long. For the probability density function S_{2-4}, case ΔYC, two impulses of 0.0265 and 0.0321 are placed at points 48.64 MVA and 63.34 MVA, respectively, to compensate for the statistical information from the tails. As the level of uncertainties ΔY is not very large, the tails of the distribution functions are influenced more significantly by the uncertainties ΔC.

The average computer time using a CDC-CYBER 170/835 computer for evaluating each PDF, represented by 128 points, was 0.21 s for case ΔY, 1.40 s for case ΔC and 3.50 s for case ΔYC.

4.3 *Further comments*

In practice, the evaluation of state and output uncertainties, in terms of only expected value and variance parameters, is easily implemented in any conventional power flow algorithm. On the other hand, such probabilistic information is insufficient to determine which network elements (transmission lines, transformers, busbars etc.) are likely to operate inadequately. Therefore it is always necessary to evaluate the probability density or distribution functions to obtain system adequacy indices such as:

(*a*) the probability of a transmission line and transformer loading being greater than its thermal rating

(*b*) the probability of a busbar voltage going above or below its limits

(*c*) the probability of a generator violating its reactive power limits

(*d*) the probability of generation deficiency in a particular system.

For example, suppose that transformer 5–6 in analysis 2 has a maximum capacity of 48 MVA. It can be seen from Table 5 that there will be a probability of $(1 - 0.9614) = 0.0386$ (i.e. 3.86%) that an overload occurs with this transformer. If a risk of 1% is considered acceptable, then some measure has to be taken in relation to this equipment. This analysis can be extended for all network equipments.

The above overload risk of 3.86% would not be the same if practical operating policy criteria, including economic dispatch, load shedding and redispatch, were considered. Also, in the previous analysis, loads were assumed to be statistically independent. Therefore all these considerations have to be incorporated [10] into the proposed method to provide reliable adequacy indices.

Finally, the development of load and generation models [8] to be used for steady-state probabilistic analysis is a vital step to ensure that PLF algorithms will process realistic data.

5 Conclusions

This paper has presented a new probabilistic load flow method which considers the network configuration as a discrete random variable. The network uncertainties are modelled to account for the availability of components such as transmission lines, transformers, switchgear etc., which are all subject to outages due to faults and maintenance.

The proposed solution for the state and output probability density functions is obtained from a weighted sum of density functions evaluated for each possible network configuration. These weights are probabilities associated with the configurations. An AC linear power flow model is used.

It has been shown that the probabilistic nature of the network is extremely relevant in the probabilistic load flow solution. Moreover, the network uncertainties have more influence in the solution when the load uncertainty level is not very high, which usually occurs in operational planning.

Finally, although the proposed method requires more computing effort in relation to the conventional one, the evaluated adequacy indices contain, undoubtedly, more information.

6 Acknowledgments

The authors gratefully acknowledge the financial assistance granted by CNPq, CAPES and FINEP of Brazil in support of this project. Mr. Arienti is also indebted to the Federal University of Minas Gerais for granting leave of absence and financial support.

7 References

1 BORKOWSKA, B.: 'Probabilistic load flow', *IEEE Trans.*, 1974, **PAS-93**, pp. 752–759
2 ALLAN, R.N., BORKOWSKA, B., and GRIGG, C.H.: 'Probabilistic analysis of power flows', *Proc. IEE*, 1974, **121**, (12), pp. 1551–1556
3 DOPAZO, J.F., KLITIN, O.A., and SASSON, A.M.: 'Stochastic load flow method', *IEEE Trans.*, 1975, **PAS-94**, pp. 299–309
4 ALLAN, R.N., and AL-SHAKARCHI, M.R.G.: 'Probabilistic techniques in a.c. load-flow analysis', *Proc. IEE*, 1977, **124**, (2), pp. 154–160
5 ALLAN, R.N., LEITE DA SILVA, A.M., and BURCHETT, R.C.: 'Evaluation methods and accuracy in probabilistic load flow solutions', *IEEE Trans.*, 1981, **PAS-100**, pp. 2539–2546
6 SIRISENA, H.R., and BROWN, E.P.M.: 'Representation of non-Gaussian probability distribution in stochastic load-flow studies by the method of Gaussian sum approximations', *IEE Proc. C, Gen. Trans. & Distrib.*, 1983, **130**, (4), pp. 165–171
7 ALLAN, R.N., and LEITE DA SILVA, A.M.: 'Probabilistic load flow using multilinearisations', *ibid.*, 1981, **128**, (5), pp. 280–287
8 ALLAN, R.N., GRIGG, C.H., NEWEY, D.A., and SIMMONS, R.F.: 'Probabilistic power-flow techniques extended and applied to operational decision making', *Proc. IEE*, 1976, **123**, (12), pp. 1317–1324
9 EDWIN, K.W., and KÖNIG, D.: 'The effect of unit commitment in stochastic and probabilistic load flow'. Proceedings of 7th Power System Computation Conference, Lausanne, 1981, pp. 540–544
10 LEITE DA SILVA, A.M., ARIENTI, V.L., and ALLAN, R.N.: 'Probabilistic load flow considering dependence between input nodal powers', *IEEE Trans.*, 1984, **PAS-103**, pp. 1524–1530
11 ABOYTES, F.: 'Stochastic contingency analysis', *ibid.*, 1977, **PAS-97**, pp. 335–341
12 ALLAN, R.N., GRIGG, C.H., and PRATO-GARCIA, J.A.: 'Effect of network outages in probabilistic load flow analysis', IEEE PES winter meeting, Paper A79032-4, New York, February 1979

13 PRADA, R.B., and CORY, B.J.: 'Stochastic security assessment', Proc. 2nd Int. Symposium on Security of Power Systems Operation, Wroclaw, Poland, June 1981
14 ALLAN, R.N., LEITE DA SILVA, A.M., ABU-NASSER, A.A., and BURCHETT, R.C.: 'Discrete convolution in power system reliability', *IEEE Trans.*, 1981, **R-30**, pp. 452–456
15 MIKOLINAS, T.A., and WOLLENBERG, B.F.: 'An advanced contingency selection algorithm', *ibid.*, 1981, **PAS-100**, pp. 608–617

8 Appendixes

8.1 Expected value and variance evaluation

The expected value of component X_i and Z_k from vectors X and Z, respectively, are evaluated, conditioned by the network configuration c, i.e.

$$E\{X_i\} = \sum_{c=1}^{t} p_c E\{X_{c_i}\} = \sum_{c=1}^{t} p_c X^0_{c_i} \quad (17)$$

$$E\{Z_k\} = \sum_{c=1}^{t} p_c E\{Z_{c_k}\} = \sum_{c=1}^{t} p_c Z^0_{c_k} \quad (18)$$

where $E\{.\}$ represents the expected value operator, X_{c_i} and $X^0_{c_i}$ are the ith components of vectors X_c and X^0_c, respectively, Z_{c_k} and $Z^0_{c_k}$ are the kth components of vectors Z_c and Z^0_c, respectively.

Representing the variance operator by $VAR\{.\}$, then the variance of X_i and Z_k can be evaluated as follows:

$$VAR\{X_i\} = \sum_{c=1}^{t} p_c[VAR\{X_{c_i}\} + (X^0_{c_i})^2] - E^2\{X_i\} \quad (19)$$

$$VAR\{Z_k\} = \sum_{c=1}^{t} p_c[VAR\{Z_{c_k}\} + (Z^0_{c_k})^2] - E^2\{Z_k\} \quad (20)$$

where

$$VAR\{X_{c_i}\} = \sum_{j=1}^{m} A^2_{c_{ij}} VAR\{Y_j\}$$

$$VAR\{Z_{c_k}\} = \sum_{j=1}^{m} B^2_{c_{kj}} VAR\{Y_j\}$$

where $A_{c_{ij}}$ and $B_{c_{kj}}$ are elements of matrices A_c and B_c, respectively.

Note that, owing to the linearisation of eqns. 1 and 2, eqns. 17–20 are only approximations for the expected value and variance of the state and output random variables, the difference being related to the degree of uncertainty or dispersion of the input quantities and the system.

8.2 Data for 14 busbar system

The system studied is based on the IEEE/AEP 14 busbar system. The line and transformer data used in these studies are identical to those used in Reference 4. Table 6 shows the selected configurations and their associated probabilities after correction by the truncation factor. Table 7 shows the nodal probabilistic data, where σ in (ii) is expressed as a percentage of the expected value μ.

Table 6: Configuration probabilistic data

Elements on outage	Configuration probability	Elements on outage	Configuration probability
none	0.900	9–14	0.009
1–2	0.008	10–11	0.011
1–5	0.012	13–14	0.010
2–3	0.009	2–4/2–5	0.002
2–4	0.011	2–4/4–5	0.005
2–5	0.010	9–10/9–14	0.003
4–9	0.003	6–11/6–12/6–13	0.002
5–6	0.003	4–7/7–8/7–9	0.002

Table 7: Nodal probabilistic data

(i) *Binomial distribution*

Busbar Number	Busbar Type	Voltage, p.u.	Unit rating, MW	FOR	Number of units
2	PV	1.045	20.0	0.09	2

(ii) *Normal distribution*, $\rho = 1$

Busbar		Voltage,	Active power		Reactive power	
Number	Type	p.u.	μ, MW	σ, %	μ, MVAR	σ, %
2	PV	1.045	−21.74	9.00		
3	PV	1.010	−94.20	10.00		
4	PQ		−47.80	11.00	3.90	9.70
5	PQ		−7.60	5.00	−1.60	5.00
6	PV	1.070	−11.20	1.00		
7	PQ		0.00	0.00	0.00	0.00
8	PV	1.090	0.00	0.00	0.00	0.00
9	PQ		−29.50	1.00	−16.60	5.00
10	PQ		−9.00	10.00	−5.80	10.00
11	PQ		−3.50	9.50	−1.80	9.50
12	PQ		−6.10	1.00	−1.60	8.60
13	PQ		−13.50	1.00	−5.80	9.50
14	PQ				−5.00	8.60

(iii) *Discrete distribution*, $\rho = 1$

Busbar		Voltage,	Active	Power	Reactive power	
Number	Type	p.u.	MW	Probability	MVAR	Probability
14	PQ		−18.00	0.20		
			−15.00	0.45		
			−13.00	0.35		