PROBABILISTIC LOAD FLOW

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ABSTRACT

The paper describes a method for evaluation of power flow which takes into consideration uncertainty of node data. The essence of the method is that the net loads are given as a set of values together with additional information on the frequency of its accuracy. The described mathematical model and the practical application are discussed and an example given.

List of principal symbols

N - number of nodes
B - number of branches
R - number of distribution functions of active power inputs or outputs
P - active power
T, A, A_o - transformation matrix
Z_B - matrix of moduli of branch impedances
C - connection matrix
E - unit matrix
ε - unit row vector
F - distribution function
f - density function
S - balance of power in network

Subscripts

k,n - node number
i - distribution number
j - branch number

1. Introduction

The paper deals with the problem which can be stated as follows: For networks with constant configuration and line parameters and given set of probable values of node loads, the problem is to find the set of corresponding values of branch flows. The necessity of dealing with such a problem stems from the uncertainty of load data. The uncertainty can be due, for example, to

(a) measurement error or forecast inaccuracy

(b) the load is known or assumed within certain limits

(c) unscheduled outage

For the above and other reasons the load is not known precisely but instead a range of values is given together with frequency of occurrence.

Orthodox methods of load flow solution require specific values for loads and any variation of values will require a new solution. For operational or planning problems, e.g. the assessment of reliability of system configuration or design of new transmssion networks, it is necessary to assess the line flows from the range of loads. For practical problems it is not feasible to carry out individual load flows for every change in loads for the following reasons:

(a) prohibitive amount of calculations. For networks with N nodes and K different load values at each node, K^N load flows is required (if N = 10, K = 2, the number of load flows is more than 1000)

(b) difficulty in analysing and synthesising the results of so many load flows.

A practical way to overcome the difficulties is by selection of a limited number of variations of loads. Often this is done arbitrarily, depending on the intuition and experience of engineers. The results are based on partial information and therefore they are inaccurate and they do not include measure of probability. The answer may be under or overestimated and lead to wrong decisions.

The paper proposes a method based on the application of the probability calculus to obtain more comprehensive results as stated in the introduction.

2. Formulation of the problem and derivation of the simplified assumptions

We can assume that the loads are static by considering the condition over a small time interval and therefore the loads are random variables. The branch flows in the network are a function of loads. Since the loads are random variables the branch flows are also random variables.

The probability model for load flow has the following advantages:

- all power inputs or outputs can be given as a set of values;
- it does not exclude the conventional load flow calculations;
- degrees of importance or frequency of occurrence of a given load data can be respected by associating a number (corresponding probability);
- the synthesis of all possible branch flows can be obtained in the form of distribution functions of branch flows.

However, this model causes great difficulties which are mainly due to:

- the nonlinear relation between the node loads and branch flows;
- since the generation has to meet the demand plus losses the mathematical model of the probabilistic load flow must take into account the control of the balance of power;
- the mathematical control of the balance is nonlinear complicated function of the power inputs and outputs in particular nodes;

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- the number of data processed is much greater than in the conventional load flow; so suitable numerical method is needed;

It is therefore desirable to make the simplified assumptions.

The more precise formulation of the object of this paper is as follows:

There are given:

- the graph of the N-nodes, B-branches, network and the parameters of the branches; the probability of this graph is equal to one.
- R distribution functions of the real power inputs and outputs; $(R \geq N)$
- the rule or procedure of balancing the power in the case of surplus or deficiency; the rule can be given analytically or in the form of an algorithm.

The problem to be solved is how to find the distribution functions of branch flows.

In order to solve the above problem the following assumptions are made:

(a) branch flows are linearly related to net nodal loads;
(b) active and reactive power flows are independent of each other;
(c) the balancing of power is the function of the sum of power inputs and outputs only, and is not dependent on the power inputs and outputs in particular nodes.

3. Mathematical model

P_n is a random vector of power inputs and outputs

$$P_n = \text{col}\,(P_{n1},\ldots P_{ni},\ldots P_{nR}) \tag{1}$$

Distribution functions F_{ni} of mutually independent (assumption c) random variables P_{ni} are known.

T is transformation matrix of order NxR. The element T_{ki} of the matrix T is zero or one according to:

$$T_{ki} = \begin{cases} 1 \text{ when } P_{ni} \varepsilon P_{Nk} \\ 0 \text{ when } P_{ni} \not\varepsilon P_{Nk} \end{cases} \tag{2}$$

Since P_{ni} belongs to one node only:

$$\sum_{k=1}^{N} T_{ki} = 1 \qquad i=1,2,\ldots R \tag{3}$$

P_N is a random vector of net nodal loads:

$$P_N = T\,P_n \tag{4}$$

Using assumption (a) and (b):

$$P_B = A\,P_W \tag{5}$$

where P_B - vector of branch real powers

P_W - vector of nodal real powers

A - transformation matrix of order BxN

$$A = [A_o|0] \tag{6}$$

A_o - transformation matrix of order Bx(N-1)

$$A_o = Z_B^{-1}\,C\,(C_t\,Z_B^{-1}\,C)^{-1} \tag{7}$$

Z_B - matrix of moduli of branch impedances

C - connection matrix

The sum of nodal real powers must be equal to zero since relation (5) is linear and therefore losses are neglected.

$$\varepsilon_N\,P_W = 0 \tag{8}$$

where ε is a row vector with all N-elements equal to 1.

Random vector P_N fulfils the relation (8) in the particular case only. The condition (8) is maintained generally by a dispatcher.

It may be written as

$$P_W = P_N - P_L \tag{9}$$

where P_L is a vector of changes of net nodal loads made by a dispatcher. Let S denote random variable

$$S = \varepsilon_N\,P_N = \varepsilon_R\,P_n \tag{10}$$

S is the balance of power in the network. Consider two possible values of the variable S:

$$S = \xi_1 \qquad\qquad S = \xi_2 \tag{11}$$

The first one denotes the deficiency and the second the surplus of power. In order to maintain the balance of power either inputs or outputs should be diminished. If $S=\xi_1$ the vector P_L is the vector of the deficiency distributed among the particular nodes and if $S = \xi_2$ the vector P_L is the vector of the reserve capacity distributed among the nodes.

Generally according to the assumption (d):

$$P_L = \phi(S) \tag{12}$$

Substituting (12) to (9) and then to (5)

$$P_B = A\,(P_N - \phi(S)) \tag{13}$$

In the above equation on the right hand side all data are known (see (10),(7), (6), (4)) and on the left hand side is the vector of unknown branch flows. The relation (13) provides the fundamental equation of the load flow using probability model. P_N is a random vector of net nodal loads, S is a random balance of power in the network, A is a function of constant configuration and parameters and ϕ is a function modelling the dispatcher's activity.

4. Solution of the problem

The solution of the model means the evaluation of the distribution functions F_{Bj} $(j = 1,2,\ldots B)$ of the branch flows P_{Bj}.

From the equation (13) for branch j:

$$P_{Bj} = G_R - \psi(S_R) \tag{14}$$

where $G_R = \sum_{i=1}^{R} \sum_{k=1}^{N} A_{jk} T_{ki} P_{ni}$ (15)

$$\psi(S_R) = \sum_{k=1}^{N} A_{jk} \phi_k(s) \quad (16)$$

The random variables G_R and S_R are dependent (see (10)). The distribution function F_{GSR} of the two dimensional random variable (G_R,S_R) can be evaluated. Using (15) and (10) the following can be written

$$G_i = G_{i-1} + W_i P_{ni} \quad (17)$$

for $i = 2,3,\ldots R$

$$S_i = S_{i-1} + P_{ni} \quad (18)$$

where

$$W_i = \sum_{k=1}^{N} A_{jk} T_{ki} \quad (19)$$

$$G_{i-1} = \sum_{r=1}^{i-1} W_r P_{nr} \quad (20)$$

$$S_{i-1} = \sum_{r=1}^{i-1} P_{nr} \quad (21)$$

When $i = 1$

$$G_1 = W_1 P_{n1} \quad (22)$$

$$S_1 = P_{n1} \quad (23)$$

thus the distribution function F_{GS1} of the variables (G_1,S_1):

$$F_{GS1}(W_1 \beta,\beta) = F_{n1}(\beta) \qquad \beta\epsilon<-\infty,+\infty> \quad (24)$$

Because (G_1,S_1) and P_{n2} are mutually independent then

$$F_{GS2}(g,s) = \int_{-\infty}^{+\infty} F_{GD1}(g-W_2\beta,s-\beta)\, dF_{n2}(\beta) \quad (25)$$

since

$$G_2 = G_1 + W_2 P_{n2} \quad (26)$$

$$S_2 = S_1 + P_{n2} \quad (27)$$

Generally using (17) and (18) and for $i=2,3,\ldots R$:

$$F_{GSi}(g,s) = \int_{-\infty}^{+\infty} F_{GSi-1}(g-W_i\beta ,s-\beta)\, dF_{ni}(\beta) \quad (28)$$

If the distribution function F_{GSR} is known then the density function f_{GS} of the random variables (G_R,S_R):

$$f_{GS}(g,s) = \frac{\partial^2 F_{GSR}(g,s)}{\partial g\ \partial s} \quad (29)$$

and the unknown distribution function F_{Bj} of the branch flow P_{Bj}:

$$F_{Bj}(g) = \int_{-\infty}^{g} \int_{-\infty}^{+\infty} f_{GS}(g-\psi(s),s)\, ds\, dg \quad (30)$$

The equations (22) to (30) make it possible to derive the algorithm of numerical calculations.

The evaluation of F_{GSR} is time consuming. The calculations are much more simple if the function (S) is linear

$$\phi(S) = L_o\ S \quad (31)$$

where L_o known vector ; L_{ok} defines the share of the node k in maintaining the balance of power S (S-scalar).

Then the equation (13) becomes

$$P_B = H\ P_n \quad (32)$$

where

$$H = A\ (E - L_o\varepsilon_N)\ T \quad (33)$$

For branch j:

$$P_{Bj} = \sum_{i=1}^{R} H_i P_{ni} \quad (34)$$

Similarly to (22) and (24) for $i = 1$:

$$P_{Bj1} = H_1 P_{n1} \quad (35)$$

$$F_{Bj}(H_1\beta) = F_{n1}(\beta) \quad (36)$$

and for $i=2,3,\ldots,R$

$$P_{Bji} = P_{Bji-1} + H_i P_{ni} \quad (37)$$

$$F_{Bji}(\gamma) = \int_{-\infty}^{+\infty} F_{Bji-1}(\gamma-H_i\beta)dF_{ni}(\beta) \quad (38)$$

The calculation of (38) is less time consuming.

Observe that if $L_{oN}=1$ (thus $L_{ok}=0$ for $k = 1,2,\ldots,N-1$) then equation (32) reduces to

$$P_B = A\ T\ P_n \quad (39)$$

The equation (39) enables the evaluation of load flow by assumption that dispatcher's activity is confined to one node N only. The node N is a slack node of the network. If P_n is not a random vector then (39) reduces to the conventional equation of d.c. load flow.

5. Computer program and numerical example

The computer program is written in Fortran. The program is capable of solving a network consisting of a maximum of 100 branches and 100 node loads.

The computer results contain:

- the expected values and standard deviations of

branch flows P_B,
- the distribution functions F_B and the density curves f_B of branch flows in all or any number of branches,
- the density curve of balance of power in network (or the load in the slack node),
- the expected values and standard deviations of net nodal loads P_N.

The practical example of calculations for the network from Fig. 1 is shown in Fig. 2. The calculation time including density curves for all branches was 4 min on CDC 3170.

6. Applications

The method presented enables the evaluation of the expected values, standard deviations and disribution functions of branch flows when the configuation and parameters of the network are constant and the power inputs and outputs are random variables. The given data may come from statistical records or can be a set of arbitrary values. In the first case results of calculations will give the probability of occurrence and the second case will give the synthesis of all possible branch flows corresponding to the given data.

The described method of calculation can be used for planning and operational purposes; the results of calculations provide much more information about the load condition of a network than the information obtained from conventional load flow. The density function, apart from the expected value and standard deviation, gives the answer to some questions which are important from the practical viewpoint such as:-

- what is the probability that the branch flow will exceed the capacity limit or will be greater or less than a certain value,
- what percentage of all possible values of branch load belong to the economically desirable range of branch load values,
- what is the practically possible range of branch load values
- what is the most probable load value etc.

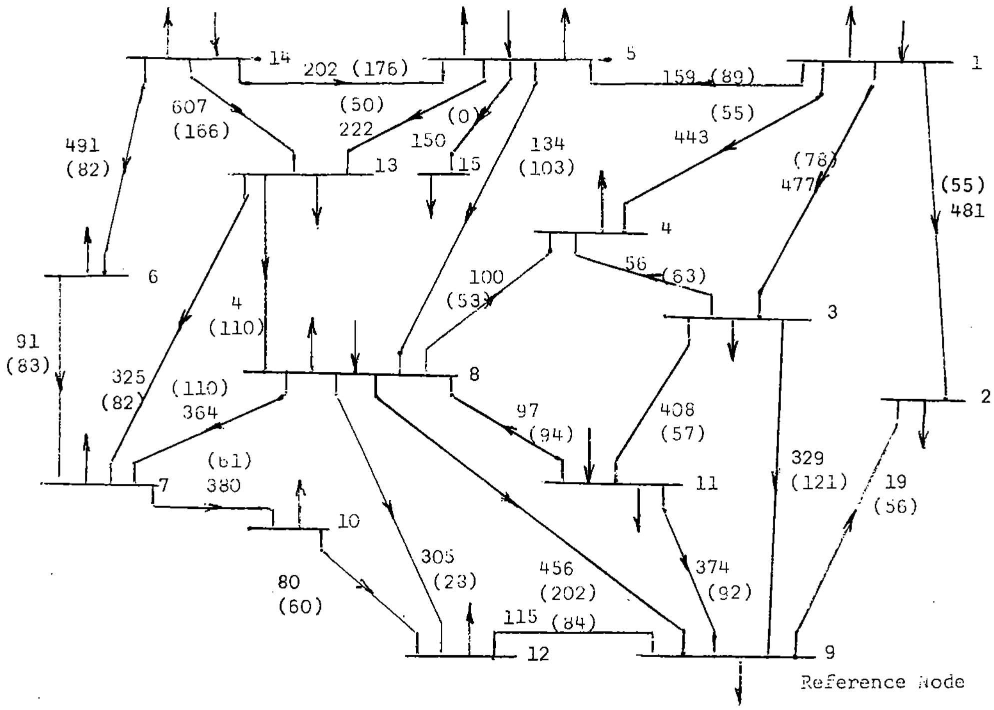
Generally, the method can be helpful in all problems in which the load conditions of a network should be analysed by uncertainty or variety of data available. In particular it can be applied to the solution of the following problems:-

- the proper choice of the number, capacity and the configuration of the branch in a network,
- the evaluation of operational cost and economical effectiveness of a network. The systematic error in calculation of network losses can be eliminated using probabilistic load flow,
- the forecasting of load inputs and outputs in the network planning and operation taking into account forecasting error,
- the assessment of reliability of power supply in the particular network substations.

ACKNOWLEDGEMENTS

The author would like to thank Prof.L.M.Wedepohl, Chairman of the Department of Electrical Engineering & Electronics, UMIST, Manchester, England, for giving her the opportunity to stay with the department, Dr. A. Brameller for help in writing the paper and Dr. H.H. Happ for presenting it.

LOAD DATA:	NODE-NUMBER	PROBABILITY FUNCTION	DATA	
	1	NORMAL	-200.00 MW	8.000
	1	BINOMIAL	10 200.00 MW	0.120
	2	NORMAL	-500.00 MW	6.000
	3	NORMAL	-500.00 MW	5.000
	4	ANY DISCRETE	LOAD MW	PROBABILITY
			-560.00	.200
			-600.00	.200
			-580.00	.200
			-620.00	.200
			-640.00	.200
	5	NORMAL	1500.00 MW	2.000
	5	BINOMIAL	8 200.00 MW	0.100
	5	ANY DISCRETE	LOAD MW	PROBABILITY
			-2500.00	.050
			-3000.00	.100
			-2900.00	.300
			-2800.00	.250
			-2700.00	.200
			-2600.00	.100
	6	NORMAL	-400.00 MW	7.000
	7	NORMAL	-400.00 MW	10.000
	8	ONE POINT	250.00 MW	
	8	ANY DISCRETE	LOAD MW	PROBABILITY
			600.00	.500
			1000.00	.200
			800.00	.300
	10	NORMAL	-300.00 MW	10.000
	11	NORMAL	-200.00 MW	5.000
	11	BINOMIAL	12 100.00 MW	0.100
	12	NORMAL	-500.00 MW	8.000
	13	NORMAL	-500.00 MW	3.000
	14	BINOMIAL	4 500.00 MW	0.200
	14	ONE POINT	-300.00 MW	
	15	ONE POINT	-150.00 MW	

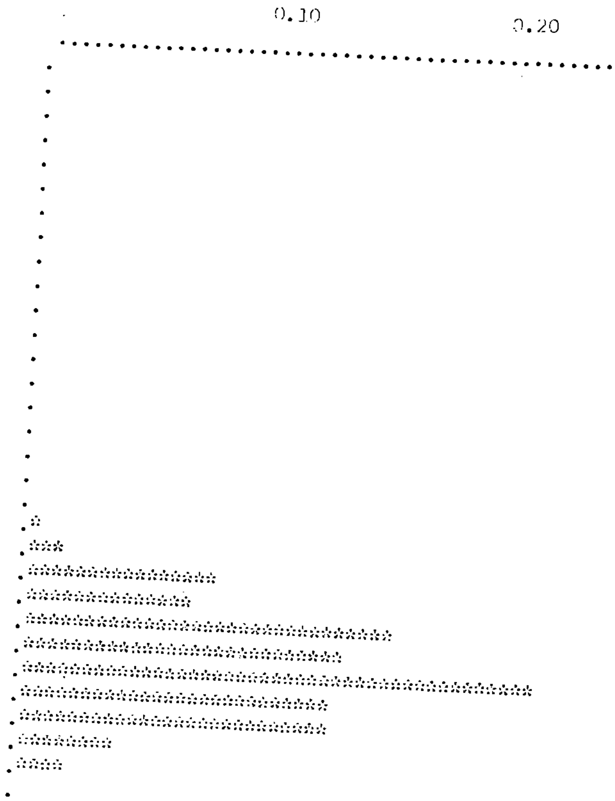


For every branch
expected value and
standard deviation
(in brackets) is given

Fig. 1: Example of probabilistic load flow

BALANCE OF POWER IN THE REFERENCE NODE (9)

DYS	PRO	POWER
0.000	0.000	5250.
0.000	0.000	5000.
0.000	0.000	4750.
0.000	0.000	4500.
0.000	0.000	4250.
0.000	0.000	4000.
0.000	0.000	3750.
0.000	0.000	3500.
0.000	0.000	3250.
0.000	0.000	3000.
0.000	0.000	2750.
0.000	0.000	2500.
0.000	0.000	2250.
0.000	0.000	2000.
0.000	0.000	1750.
0.000	0.000	1500.
0.000	0.000	1250.
0.000	0.000	1000.
0.002	0.002	750.
0.006	0.003	500.
0.019	0.014	250.
0.101	0.082	-0.
0.171	0.070	-250.
0.328	0.157	-500.
0.463	0.135	-750.
0.673	0.210	-1000.
0.807	0.126	-1250.
0.937	0.130	-1500.
0.976	0.041	-1750.
0.996	0.016	-2000.
0.998	0.002	-2250.



PROBABILITY CURVE IN THE BRANCH FROM NODE 13 TO NODE 14

DYS	PRO	POWER
0.001	0.001	-850.
0.060	0.060	-800.
0.240	0.180	-750.
0.370	0.131	-700.
0.406	0.035	-650.
0.469	0.064	-600.
0.643	0.180	-550.
0.760	0.131	-500.
0.815	0.035	-450.
0.841	0.027	-400.
0.909	0.068	-350.
0.958	0.049	-300.
0.971	0.013	-250.
0.976	0.005	-200.
0.987	0.011	-150.
0.995	0.008	-100.
0.997	0.002	-50.
0.999	0.001	0.
0.999	0.001	50.
0.999	0.000	100.
0.999	0.000	150.
0.999	0.000	200.
0.999	0.000	250.
0.999	0.000	300.
0.999	0.000	350.
0.999	0.000	400.

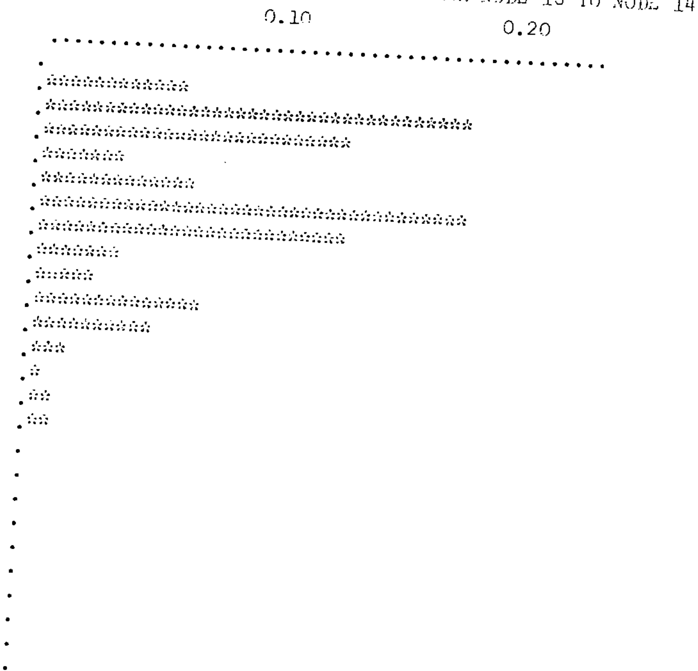


Fig. 2

Discussion

H. H. Happ, (General Electric, Schenectady, New York 12345): This paper describes a procedure for solving a power flow with probabilistic loads for a system whose configuration is fixed. It represents a novel and ingenious development, and it was my pleasure to have presented the paper for the author at the Summer Meeting (1973) in Vancouver.

The author's formulation does not make easy reading, however, and it would be helpful if a flowchart is presented in the closure showing the key calculations and their order including generators and loads. Perhaps the author can also describe the corresponding features of the present program and its relative speed to the deterministic. A few typographical errors appeared in the preprint which should be corrected in the final text or in the closure, as communicated to the author separately.

The author may wish to indicate what would be required in terms of additional calculations to extend the present method to the A. C. case.

The probabilistic treatment of loads in a load flow indeed has many applications, particularly when implemented in an A. C. load flow. It will not, in the opinion of this discussor, make the deterministic treatment obsolete; but it presents a unique option for handling loads to a planner or to an operator.

As with all new tools, the extent to which it will be used will depend upon the convenience of input and output, the relative speed and/or cost of execution, and the "feel" of the system from the output that a user can obtain. All factors except the last will probably evolve in programs in time. But whether or not users will get a similar feel for the system as they get with a deterministic load treatment, only the future will tell.

Perhaps the author can indicate how the program is or will be used in Poland, and experiences of users with this tool there.

Manuscript received July 24, 1973.

K. Clements, (Worcester Polytechnic Institute, Mass.); **R. J. Ringlee**, and **A. J. Wood**, (Power Technologies, Inc. Schenectady, New York 12301): The author is to be commended for developing a new and potentially quite useful method for dealing with the complex problem of calculating branch flow probability distributions.

There appear to be avenues opened by this method to permit the practical assessment of bulk power supply reliability recognizing the uncertainties of load and generation and the sensitivity of circuit outages on line loadings.

It should be noted that if one merely wishes to calculate the means and variances of the branch flows these can be obtained without recourse to the NxR convolution integrations (Equation 38 of the paper) that are required to obtain the branch flow probability distribution functions. If the linear balance of power function is assumed (Equation 31 of the paper) resulting in the linear relationship between the branch flows and P_n (Equation 32 of the paper)

$$P_B = HP_n ,$$

then one can calculate

$$\bar{P}_B = H\bar{P}_n \tag{2}$$

and

$$C_B = HC_nH^T \tag{3}$$

where

$\bar{P}_B$ – is a vector of means P_B

P_n – is a vector of means of P_n

C_B – is a BxB covariance matrix associated with P_B

C_n – is an RxR covariance matrix associated with P_n, and

T – denotes matrix transpose.

The diagonal elements of C_B are, of course, the variances of the branch flows. The assumption that the power inputs and outputs P_{ni} are mutually independent is not required for this calculation.

This assumption can also be removed for the calculation of the branch flow distribution functions provided that the dependent random variables are Gaussian. This is achieved by performing a transformation on P_n such that the transformed variables are mutually independent. In order to facilitate the discussion let us partition the vector P_n into two components

$$P_n = \mathrm{col}(P_a | P_c)$$

where

P_a has dimension n_a

and

P_c has dimension n_c

such that

$$n_a + n_c = R.$$

If the elements of P_a are assumed to be mutually independent and those of P_c are assumed to be correlated and Gaussian with covariance matrix C_c then Equation 32 of the paper can be written as:

$$P_B = H_aP_a + H_cP_c \tag{4}$$

where

H_a and H_c are partitioned on H corresponding to those on P_n.

Since C_c is a covariance matrix, it is symmetric and positive definite and, hence, it is always possible to find a square matrix L such that LL^T equals C_c. One method of doing this is by triangular decomposition in which case L is a lower (or upper) triangular matrix. We define a vector of mutually independent normal random variables x such that $P_c = Lx$ and $C_x = I$ the identity matrix where C_x is the covariance matrix of x. The equation (4) may be written as

$$P_B = H_aP_a + H_cLx \tag{5}$$

or

$$P_B = H^*P^*_n$$

where

$$H^* = [H_a | H_cL] \tag{6}$$

and

$$P^*_n = \mathrm{col}\ (P_a | x) \tag{7}$$

Equation 38 of the paper can then be used as before to compute the branch flow by replacing H_i with H^*_i and using the appropriate distribution functions for $F_{ni}(\beta)$.

Manuscript received August 6, 1973.

J. F. Dopazo, O. A. Klitin, and **A. M. Sasson**, (American Electric Power Serv. Corp., N. Y. 10004): The author deserves being congratulated for producing the first paper, to our knowledge, that deals with the propagation of the uncertainties in the input data to the results of the load flow problem. We concur with the author that this application is a potentially valuable tool for general real time and offline load flow applications. We at AEP, have recognized this potential and for some time have been working on the solution of the same problem. Our approach has been to perform this error analysis after the solution of a base case conventional load flow calculation by determining the variances of the output quantities considering a linear approximation around the solution point. The method is efficient and there is no need to simplify the model into a d.c. one. We can also handle a combination of different probability density functions for the input quantities, the variances of the output quantities belonging to a normal probability density function according to the Central Limit Theorem. In our approach we have included constraints on the total loads of given areas of the system corresponding to load forecasts which are known with greater accuracy, in some cases, than the individual loads themselves.

In closing, we again congratulate the author. We look forward for future publications of her work in this important area for power system analysis.

Manuscript received August 6, 1973.

W. O. Stadlin (Leeds & Northrup Company, North Wales, Pennsylvania 19454): The author's probabilistic analysis provides further insight to the relationship between network inputs (node injections) and outputs (branch flows). For operational purposes, the A transformation matrix provides a set of sensitivity coefficients relating generator (or load) shifts to transmission line flow changes. This information is useful for corrective strategies following a contingency. Equation (32) is also useful in the following form:

$$P_B = A\ (P_N - L_oS) = A\ (E - L_o\epsilon_N)P_N$$

Manuscript received August 2, 1973.

where we could consider the injections P_N as being independent random variables with normal probability distributions. Under this assumption we conclude that,

$$\sigma_B^2 = H_\sigma^2 \sigma_N^2$$

where each term of the matrix H_σ^2 is the square of the corresponding term in A $(E\text{-}L_{0}\epsilon_N)$ and σ is a vector of standard deviations. This relationship may be used to evaluate the performance of real time load flows. For "state estimation" applied to the author's linear model (assuming one measurement per branch) the following relationship may be derived,

$$\hat{P}_B = Z_B^{-1} C\,(C_t Z_B^{-1} Z_B^{-1} C)^{-1} C_t Z_B^{-1} P_B = D P_B$$

where $\hat{P}_B$ represents the best estimate of branch flows based on the measured set of branch flows P_B. As above we obtain,

$$\hat{\sigma}_B^2 = D_\sigma^2 \sigma_B^2$$

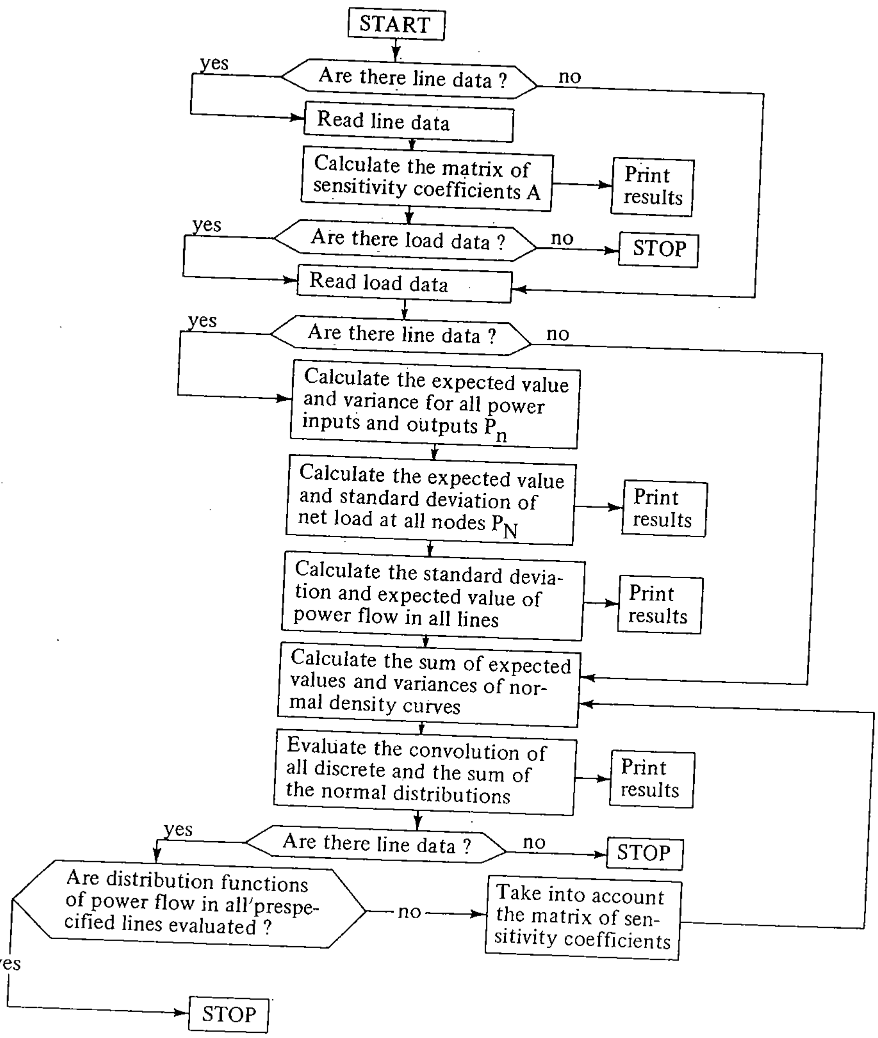
Barbara Borkowska: I am deeply indebted to Professor H. H. Happ who kindly agreed to present my paper for discussion at the IEEE Meeting. Pursuing his suggestion I enclose the simplyfied flowchart of Probabilistic Load Flow. The program which is now working enables calculations for networks up to 100 branches and 70 nodes. The nodal loads can be given by any of the following density curves: normal, binomial, any discrete or one point deterministic. For a node more than one density curve can be specified but not more than 100 curves for whole network can be given. The full amount of results is given in flow chart. Program is flexible and can be used for other purposes such as: availability of power capacity, probabilistic balance of power, sensitivity of load flow etc. The time of calculation if all nodal loads are one point density curves is similar to the time of the deterministic load flow. The time increases quickly with the number of discrete random variables.

Manuscript received October 23, 1973.

The experiences with the probabilistic load flow in Poland are not wide yet. However the program has been accepted by Polish National Dispatching Centre for off line data processing.

As far as a.c. load flow is concerned I see many difficulties. They are both of theoretical nature /nonlinearity, the correlations between variables and some constraints cannot be neglected / and of practical nature / the data needed and the time of calculations/. Personally I think that the problem of probabilistic treatment of network graph is more important and pressing.

I would like to thank K. Clements, R. J. Ringlee, A. J. Wood and W. O. Stadlin for their comments and for the additional insights they have contributed. I am pleased to learn from J. F. Dopazo, A. O. Kliting and A. M. Sasson that I am not alone on my way and I hope that all troubles with a.c. load flow will soon be solved by colleagues from AEP. It must be stressed however that the Central Limit Theorem should be applied with great caution. The calculations we have done show that if there are power stations with great generation units than distributions of many branch load flows are far from normal distribution.



The flow chart of Probabilistic Load Flow