EVALUATION METHODS AND ACCURACY IN PROBABILISTIC LOAD FLOW SOLUTIONS

R.N. Allan
Senior Member
A.M. Leite da Silva
Member
University of Manchester
Institute of Science and Technology
Manchester, England

R.C. Burchett
Member
General Electric Company
Schenectady, New York

Abstract

This paper presents a new method for obtaining a probabilistic load flow solution using a discrete frequency domain convolution technique. It is shown that this method has greater accuracy while providing a breakthrough in computational speed. A detailed example compares the numerical results with a Monte Carlo simulation. The effects of nonlinearity in the network equations are discussed.

INTRODUCTION

The Probabilistic Problem

The most widely used evaluation technique in power system analysis is the load flow algorithm used to assess the steady-state behavior and response of the system. These algorithms are deterministic in that they consider a fixed set of input parameters for each computer run. A complimentary method which has received considerable interest recently is probabilistic load flow or stochastic power flow. These techniques recognize the probabilistic nature of the generation and load with one solution. Essentially, a probabilistic load flow transforms these input random variables, defined in terms of probability density functions, into output random variables also defined in terms of density functions. It is not the purpose of this paper to discuss or describe the concepts and basic numerical techniques associated with probabilistic load flow as these have been detailed and documented previously [1-9]. Instead, however, the paper will address itself to various aspects concerning precision and computational efficiency. The same objectives of computational efficiency and accuracy apply to probabilistic load flow as they do to conventional power flow techniques.

Numerical and Analytical Considerations

One of the central problems associated with steady-state network analysis is the nonlinearity of the power flow equations. In the case of deterministic power flows the problem is approached using iterative techniques. The probabilistic approach offered here is based on a linearization of these equations at a particular operating state.

Another point that has received considerable discussion is the application of the Central Limit Theorem. If applicable, the output density functions could be assumed to be normal in distribution. It has been shown [1-3], however, that the assumption is not always valid. Furthermore, even if the input parameters are normally distributed, the output parameters will not be due to the nonlinearity of the power flow equations. The normality assumption is, at best, a rough approximation to the exact answer, and very often cannot be depended upon. It is the intent of this paper to offer a practical technique for computing a more exact and realistic solution for the probabilistic load flow problem.

The input parameters for most systems are not normally distributed and other more arbitrary distributions are necessary to describe generation and load uncertainty. Load forecast uncertainties which have small variance may be reasonably represented by the normal distribution (although truncated). It is therefore necessary to evaluate the actual output density function in order to see if the Central Limit Theorem is applicable.

In order to evaluate the output density function, the load flow equations [1-5] are linearized around an expected value region. After linearization, a convolution process is used to transform the input information (load and generation distributions) into the output information (voltages and line flows). This convolution process has been made [9] in the past by a "semi-conventional" technique based on Laplace Transformation. This method, although achieving the desired output information, is time consuming in order to achieve a reasonble level of precision. A completely different technique has been applied for this convolution process which is based on the Fast Fourier Transform (FFT). This new method has been tried with a wide range of systems and data and has been found to be quite reliable and gives greater precision with considerably less computational effort. The application of FFT and its performance in probabilistic load flow is described in this paper.

Since the load flow equations are linearized around the expected value region, input data with a high level of uncertainty will be transformed less accurately by the linear model used in probabilistic load flow, particularly in the tail regions as these are furthest from the point of linearization. To study this limitation, Monte Carlo simulation programs based on both the exact and the linear models have been developed. The results are discussed in a later section of this paper.

PROBABILISTIC LOAD FLOW SOLUTION

The standard load flow equations may be expressed as:

$$Y = f(X) \quad (1)$$
$$Z = g(X) \quad (2)$$

where, in the case of probabilistic load flow:

- Y = input random vector (real and reactive injections)
- X = state random vector (voltage and angle)
- Z = output random vector (power flows)
- f and g = load flow functions

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Let the vectors Y, X, and Z be the expected values of Y, X, and Z, respectively. Suppose that a deterministic load flow is calculated using Y as input. The solution will be given by the vectors X and Z such that:

$$\bar{Y} = f(\hat{X}) \tag{3}$$

$$\hat{Z} = g(\hat{X}) \tag{4}$$

It can be shown that $\hat{X}$ and $\hat{Z}$ are only approximations for $\bar{X}$ and $\bar{Z}$ due to the nonlinear load flow functions; the difference being related to the higher moments of the input quantities and the system.

Linearizing equations (1) and (2) around the points $(\hat{X}, \bar{Y})$ and $(\hat{X}, \hat{Z})$, gives:

$$X \cong \hat{X} + A\,\Delta Y \tag{5}$$

$$Z \cong \hat{Z} + B\,\Delta Y \tag{6}$$

where:

$$A = \left(\frac{\partial f}{\partial X}\Big|_{X = \hat{X}}\right)^{-1} \qquad \text{(Jacobian Matrix)}$$

$$B = \left(\frac{\partial g}{\partial X}\Big|_{X = \hat{X}}\right) A$$

$$\Delta Y = Y - \bar{Y}.$$

Equations (5) and (6) show that each random element of the vectors X and Z can be computed from a "weighted" sum of the random elements of vector ΔY. The weighting coefficients are defined [1,2,5,9] as sensitivity coefficients. This sum of independent (or in some cases linearly dependent) random variables can be made [9] using mathematical convolution techniques.

The assumption of either statistical independence or linear dependence is fundamental to this approach. This phenomenon is frequently observed in generation availability and demand forecast error. If the input information is to include time variation in demand, the mean value and all perfectly predictable parts should be filtered before constructing distributions. The remaining purely stochastic part will most likely demonstrate independence or linear dependence.

The convolution implied by equations (5) and (6) can be written as

$$f(X_i) = f(Y_1') * f(Y_2') * \ldots * f(Y_n') \tag{7}$$

where f represents density function,

Y_k' represents $(Y_k - \bar{Y}_k)a_{ik}$,

$*$ represents convolution,

a_{ik} represents an element of A.

There are many ways in which equation (7) can be evaluated. One is to use numerical methods based on Laplace transforms which is referred to as the conventional method. Another method, which is one subject of this paper, is to transform the equations into the frequency-domain using Fast Fourier Transform (FFT) techniques.

The random vector ΔY consists of normal and discrete distributions which represent the random variations or uncertainty of the loads and system generation. All normally distributed functions can be easily grouped in one unique normal equivalent since only the expected value and variance are required to define this function. Therefore, equation (7) contains discrete functions and this normal equivalent.

Conventional Method

After computing the normal equivalent, the conventional method [9] computes the equivalent discrete and finally convolves these two equivalent functions. The greatest problem is to compute the equivalent discrete function since a function represented by r impulses convolved with another represented by s impulses will have (rxs) impulses. This process becomes impractical when there are many functions. One approach is to initially determine the boundaries of the convolved function and divide this period into a fixed number of points. When a convolved impulse falls between two of these fixed points, it is shared between them using a straight line approximation of the distance of the convolved impulse from the two fixed points.

New FFT Technique

The proposed FFT algorithm takes advantage of some of the properties of exponential functions to give fast and precise representation of a function in the frequency domain. The process of convolving two functions using the FFT algorithm is shown in Appendix 1. Using this algorithm simplifies the complicated convolution problem. A variety of tests have been made using several systems in order to compare the precison and execution time of the previous conventional method and the FFT method. The results included in this paper are representative and typical of the comparisons that have been achieved. The two most important performance measures are execution time and precision and the following results and discussion center on these two aspects.

NUMERICAL COMPARISON

This section compares the execution time and accuracy of the FFT method with that of the conventional method. Two power systems are used for this comparison: a 14 bus system with 12 normal and 12 discrete distributions, and a 32 bus system with 56 normal and seven discrete distributions.

Figure 1 displays the difference in execution time requirements for the 12 bus system. The number of points, n, is the number of points used to represent the density functions in the discrete convolution, as explained in Appendix I.

At a small number of points, the conventional method is slightly faster than the FFT method but with a larger number of points, the time required by the conventional method is very much greater than the FFT method. It should be noted that in the region in which the conventional method is faster, it is not necessarily more accurate. The number of points which dictate the intersection of the two performance curves depends on the number of discrete functions, their type, the level of discretization, the computer, and the efficiency of programming.

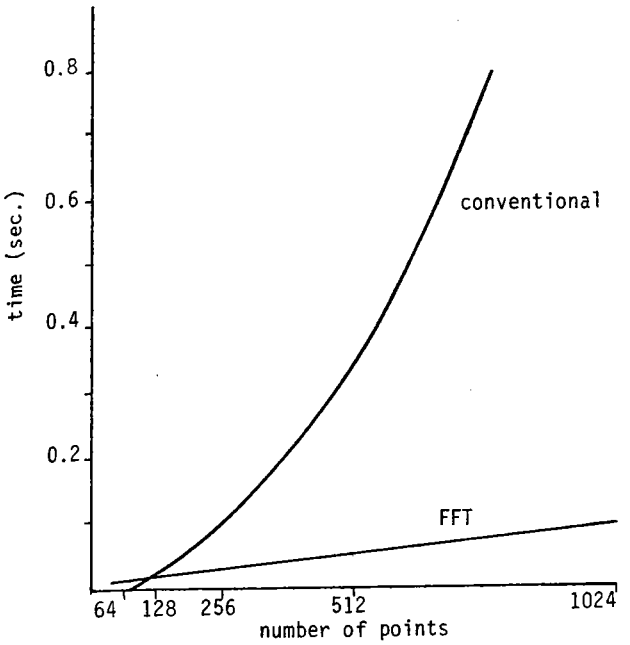

Figure 1 Performance: time vs. n

The results shown in Figure 2 indicate simultaneously the relationship between maximum relative error of the standard deviation, the average execution time for each curve and the number of points representing each curve for the two convolution methods. The relative error was evaluated from the standard of points representing each curve for the two convolution methods. The relative error was evaluated from the standard deviation derived numerically from equation (5) and that computed from the actual density function; this comparison being one of the best ways of measuring the precision of the convolution process. The error found in the expected value using the FFT method has been found to be consistently much better than that given by the conventional method and frequently found to be virtually zero.

The results shown in Figure 2 indicate that although there is a region in which the conventional method is faster than the FFT method for a given number of points, the FFT method gives results to a given precision in a shorter execution time and requires fewer points than the conventional method. Furthermore, there is a point in both of the performance curves that gives minimum error. Increasing the number of points beyond this minimum error point increases the error due to the large number of calculations required. It is evident from Figure 2 that the minimum error for the FFT method is much less than that in the conventional method; this point being consistently true for all studies performed.

It is clearly evident from the above discussion that the two methods cannot be compared directly by considering only execution time and number of points; the precison must also be included. When these three performance measures are considered simultaneously, the FFT method is shown to be consistently far superior to the conventional method.

CONSIDERATION OF CENTRAL LIMIT THEOREM

The Central Limit Theorem has been used as a basis for assuming that the output quantities will be normally distributed irrespective of the marginal distribution of the input quantities. In principle, as the problem involves a sum of random variables, it may be thought that the theorem could be applicable for large systems because of the great number of independent random variables involved. This is not true however, because this sum is weighted by sensitivity coefficients, therefore the output quantities are influenced by the input quantities in different ways. This fact is very important and will be shown in the following example.

Consider the 14 bus system as described in Appendix 2. It can be seen that only the active injected power of bus 14 is represented by a discrete function and all others are represented by normal functions. It can be seen from an analysis of the system that the sensitivity coefficients for the active power flow in line 9-14 are dominated by the

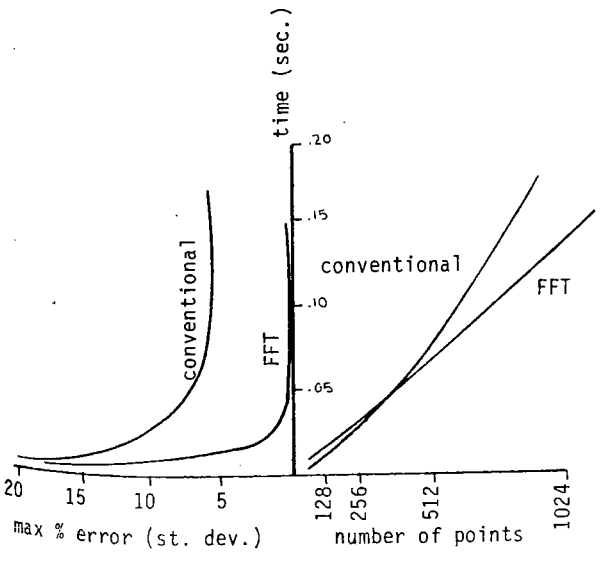

Figure 2 Error , Time vs. n

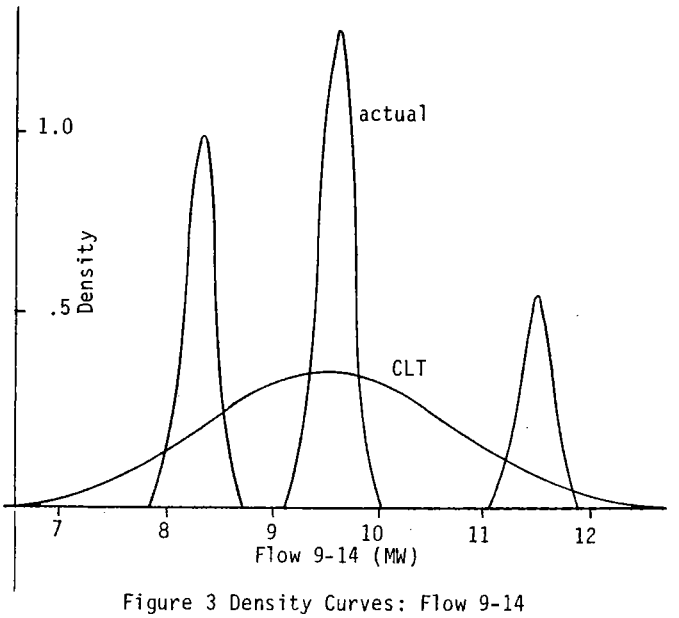

Figure 3 Density Curves: Flow 9-14

unique discrete function. This means that, although the input data contains only one discrete function, this will be the dominant factor defining the shape of the density curve of the active flow in line 9-14. This is illustrated in Figure 3 which shows two density functions for this random variable; one assuming normality and the other using the probabilistic load flow formulation. The two shapes are completely different and the effective range of the normal curve is much larger than the actual range. In constrast to the active power flow in line 9-14, the shape of most of the other output variables followed the normal distribution.

This simple example shows that irrespective of the system size, the theorem cannot be assumed because this ignores the sensitivity coefficients. Furthermore, it will be shown in the next section that, even when all inputs have a normal distribution, the output shapes will not follow a normal because of the nonlinearity of the load flow equations.

MONTE CARLO SIMULATION

The probabilistic load flow solution described previously may be interpreted as a very large number of deterministic linear solutions combined probabilistically by convolution. The effects of nonlinearity on this combination is now considered.

Since the load flow equations are linearized around the expected value of the input quantities, any movement away from this region produces an error. Consequently, the error is a function of the region of uncertainty of the input quantities which can be measured by the variance. The effects of the bounds of the region is determined by the nonlinear behavior of the system power flows and voltages.

One practical way of comparing the linear model and the exact one is through a Monte Carlo simulation which consists of running and probabilistically processing several cases of individual load flows where the data are generated by psuedo-random numbers. A linearized Monte Carlo solution and an exact model Monte Carlo solution are computed for comparison. Using the strong law of large numbers, it can be shown that the exact probabilistic solution R_E will converge in probability to the Monte Carlo solution using the exact model R_e within the accuracy of the random number generator. Also the linear solution using the probabilistic load flow approach R_L will converge in probability to the Monte Carlo solution using the linear model R_ℓ. This can be expressed as follows:

$$\lim_{N\to\infty} \Pr\{ R_E - R_e < \epsilon \} = 1$$

$$\lim_{N\to\infty} \Pr\{ R_L - R_\ell < \epsilon \} = 1$$

where N is the number of trials used to evaluate R_e and R_ℓ, and ϵ is any positive real number. It should be noted that R is some measurable result, such as mean, variance, higher moments, etc.

In practical terms, it is impossible to evaluate the difference between R_L and R_E because of the computation time and the finite length of the psuedo-random series. It is therefore necessary to establish the largest possible number N that is realistic and practical to use.

The system studied is a 14 bus system shown in Appendix 2. Several tests were performed, four of which are provided here. The number of simulation trials is 5000 unless otherwise specified. It is also noted that the same sequence of random numbers is used for both the linear and exact simulations.

The example comparison cases for the Monte Carlo simulation are as follows:

1. Case 1: For a particular line flow density, study the effects of increasing the number of Monte Carlo trials. Only the linear model is used. The solution should converge to the linear FFT solution.

2. Case 2: For a particular line flow, compare the linear and nonlinear Monte Carlo results with 5000 trials.

3. Case 3: Repeat case 2 for a different line flow that has a multi-modal distribution.

4. Case 4: For a particular line flow, study the effects of increasing the input (load and generation) variance levels. The linear and nonlinear Monte Carlo results are compared using 5000 trials.

Case 1 - Linear Monte Carlo (Figure 4)

Three density curves are shown in Figure 4; one for the linearized probabilistic load flow and the other two for simulations using the linear model with 2000 and 10000 trials, respectively. It can be seen that the probabilistic load flow curve follows a smooth normal distribution as expected from a knowledge of the input data and relevant sensitivity coefficients. The two simulation curves, however, are distorted, although they follow the same trend as the probabilistic load flow curve. The amount of distortion decreases with an increasing number of simulation trials. These distortions are not a function of the system but are due to the restricted number of trials and will diminish as the number of trials approaches infinity. In this particular case, a sample size of even 20000 trials proved insufficient to reproduce precisely the probabilistic load flow curve. In terms of computational effort, the time required to deduce each curve on a CDC7600 computer was 0.36 sec for the probabilistic load flow, 1.51 sec., 3.25 sec., 6.14 sec., and 11.93 sec. for 2000, 5000, 10000, and 20000 simulation trials, respectively. These times indicate that simulation is inappropriate for use in probabilistic load flow analysis whereas convolution methods are more efficient.

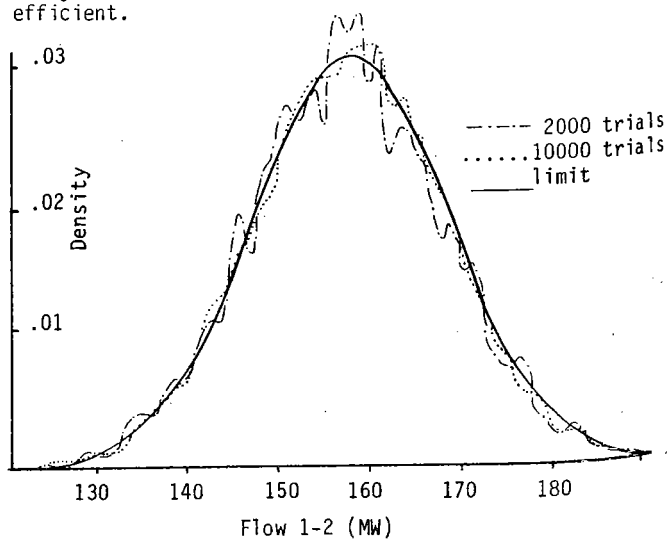

Figure 4 Monte Carlo Density Curves: Flow 1-2

Case 2 - Linear vs. Nonlinear Monte Carlo (Figure 5)

The density curves for the active power flow in line 1-2 for both the exact (nonlinear) and the linear simulation models are shown in Figure 5. The distortions exhibited by these curves are due to the limited number of trials (5000 in this case) as discussed previously and both would tend to a smooth curve as the number of trials is increased. The important feature of these curves is the very close relationship between the linear model and nonlinear (exact) model. This shows that for this system the

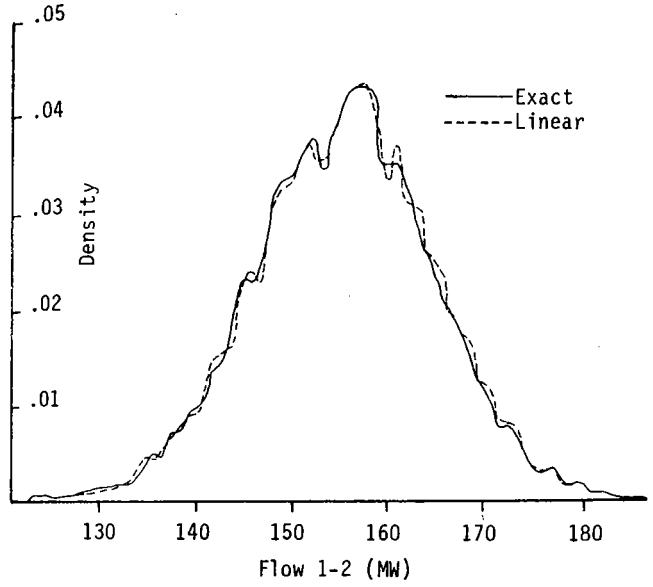

Fig. 5 Comparison of Linear & Nonlinear (5000 trials)

linear model is a very satisfactory approximation for solving the probabilistic load flow problem. This is a very important conclusion when it is observed that the time taken to produce 12 curves on a CDC7600 computer was 103 sec. for 5000 trials using the exact Monte Carlo model and only 8.4 sec. using the linear Monte Carlo model. The same curves were obtained using a convolution based probabilistic load flow in only 0.75 sec.

Case 3 - Effect of Multi-Modal Densities (Figure 6)

The density curve shown in Figure 6 represents the active power flow in line 9-14 using the exact model. An almost identical density curve was obtained using the linear model, again confirming a good performance by this linear model.

This case is the same as the study discussed in the previous section and a comparison between Figures 3 and 6 confirms that the Central Limit Theorem is not applicable here. Because of the shape of this particular density curve, the distortion caused by the restricted number of trials could not be detected.

Case 4 - Effect of input variance (Figure 7)

The density curves shown in Figure 7 were obtained for the same line and same number of trials as in Case 1, the difference being that the standard deviation of each input density curve was increased by a multiplication factor of 15. This is considered to be an extreme case because it represents a very wide degree of uncertainty or random variation in the input quantities.

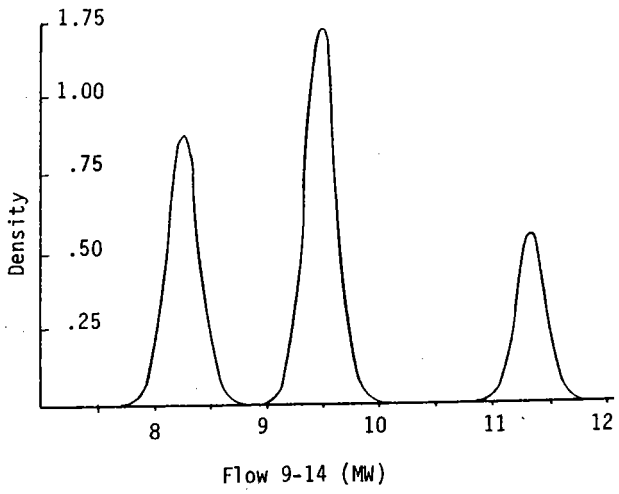

Figure 6 Monte Carlo Density (Exact)

It is seen from Figure 7 that, although the two curves for the exact and linear model are very similar over most of the range, the tails exhibit some differences. The density function for the linear model follows the trend of a normal distribution because the normally distributed input is linearly transformed into the output. The function for the exact model is skewed slightly due to the nonlinearity of the load flow equations. This supports the statements made previously and suggests that the error associated with the linear model will increase when the input standard deviations increase. On the other hand, since this case was the worst found after many studies, the results also suggest that the error is not likely to be significant for most sets of realistic data.

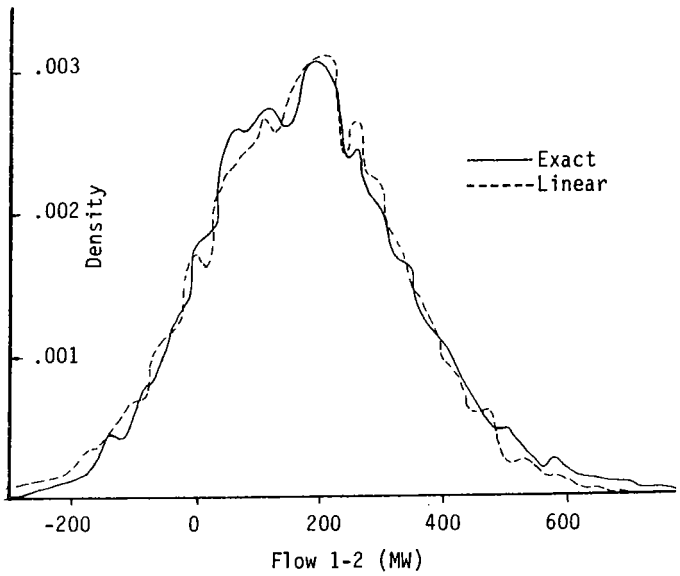

Figure 7 Comparison of Linear and Nonlinear (5000 trials) High Variance Case

CONCLUSIONS

The basic probabilistic load flow solution used in this paper was that proposed previously and is based on linearizing the load flow functions around the expected value and using convolution to evaluate the relevant density functions of the output variables. This paper has extended these techniques by replacing conventional convolution by Fast Fourier Transforms. It has been shown that the FFT method is a very significant improvement on the conventional method and gives fast, very precise results.

The problems associated with the Central Limit Theorem to justify a normal distribution for the output densities have been discussed and clarified.

The accuracy of the probabilistic load flow solution has been tested using Monte Carlo simulations. This has shown that the performance of the linear model is very good within a certain range of uncertainty of random variation of the input data. It has also been shown that the convolution technique used in the probabilistic load flow method to combine realistically an infinite range of solutions is a very powerful tool.

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APPENDIX 1

Convolution Using Fast Fourier Transforms

This appendix explains how to apply the FFT techniques to the PLF problem. Four steps are shown below:

Discretizing the Equivalent Normal

The basic concept of discretizing the normal is explained below with reference to Figure 8.

- The equivalent normal is truncated at a predetermined number of standard deviations to give the interval T_N. Generally the authors use ± 3 standard deviations. The required number of points will be discussed later.
- Each value α_K is divided by $(1-\varepsilon)$ where ε is the area neglected in the tail regions of the truncated normal. This ensures that the truncated normal has an area equal to unity and avoids subsequent numerical errors.

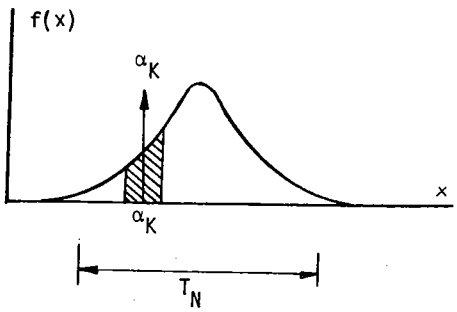

Fig. 8 Equivalent Normal Function

Intermediate Convolutions

Define the intermediate convolution of the density functions representing two random variables y_i and y_j:

$$f_{ij} = f(y_i') * f(y_j') \qquad (8)$$

where $y_i' = a_i(y_i - \bar{y}_i)$

This convolution process is illustrated in Figure 9 and continues as follows.

The interval T_{ij} in which the function f_{ij} will exist can be computed from:

$$T_{ij} = a_i T_i + a_j T_j$$

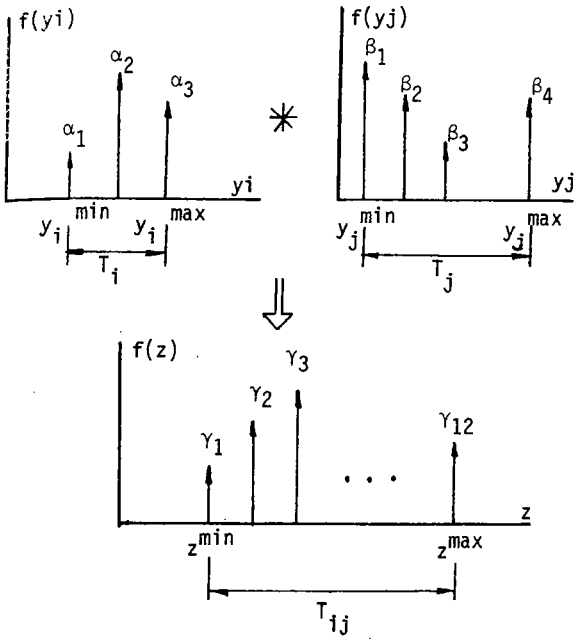

Figure 9 Example Convolution

The interval T_{ij} is then divided into N_{ij} points and each discrete function is represented in this interval. Each function to be convolved is shifted to the beginning of this interval for convenience in order to facilitate the addressing of the relative positions of the impulses. It should be noted that this shift only changes the position of the final function and not its shape. The final position can be evaluated in advance and repositioned after convolution.

If an impulse exists between two pre-determined points, it is shared between them using the weighted-average. This is illustrated in Figure 10. The representative impulses are now equally spaced which is fundamental for most FFT methods.

It should also be noted that the most common FFT algorithms require the number of points N_{ij} to satisfy the relation $N_{ij} = 2^M$, where M is an integer.

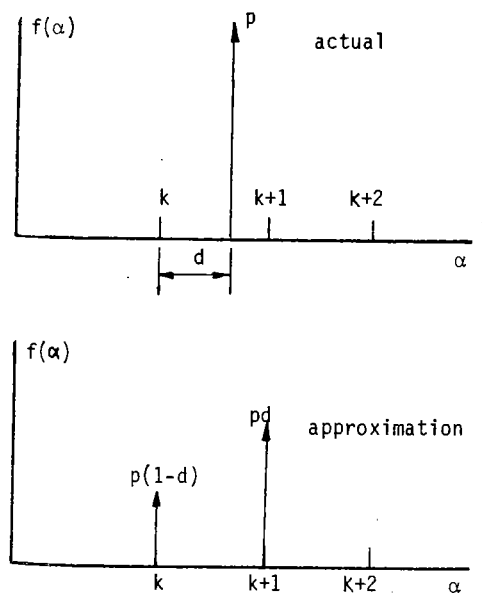

Figure 10 Impulse Sharing Process

Dynamic Optimal Ordering Approach

Let N functions be convolved, each having periods T_1, T_2, ... T_i, ... T_j, ... T_N. These periods represent the individual discrete functions and the equivalent discretized normal function. Assume that the functions have already been weighted by the appropriate sensitivity coefficients. The final period T will therefore be given by:

$$T = \sum_{i=1}^{N} T_i \quad .$$

Consider that 2^M points are required to represent the final function. The convolution process proceeds by convolving two functions at a time in a step-by-step fashion using the FFT method. Mathematically the functions can be convolved in any order. Consider therefore the convolution of functions f_i and f_j having periods T_i and T_j. This convolution will have a final period given by:

$$T_{ij} = T_i + T_j$$

and the number of points M_{ij} required to represent this interval given that 2^M points are required to represent the final function can be deduced as follows:

$$M'_{ij} = \log_2 \left(2^M \cdot \frac{T_{ij}}{T}\right)$$

$$M_{ij} = \text{integer}\,(M'_{ij}) + 1 \qquad (9)$$

The number of points required to represent the convolution of f_i and f_j is given by equation (9) a value which decreases as the periods T_i and T_j decreases. The efficiency increases and the execution time decreases as the number of points used in the FFT algorithm decreases. It is therefore much more efficient to convolve the functions in ascending order of period T_i and to use a dynamic process so that the number of points used to represent each step of the convolution process increases in accordance with equation (9).

Convolution Using FFT

Consider 2 discrete functions $f_i(k)$ and $f_j(k)$ with N impulses each. One of these functions may be the discretized normal equivalent evaluated conventionally. Using an FFT algorithm [10-12], these functions can be transformed into the frequency domain. Let these transforms be designated $S_i(n)$ and $S_j(n)$. In the frequency domain, the convolution process is simply one of term-by-term multiplications. The two transforms are therefore multiplied term-by-term to give the final function S(n) in the frequency domain. Finally an inverse - FFT algorithm is used to retransform S(n) into f(k), thus completing the convolution process. This technique is illustrated in Figure 11.

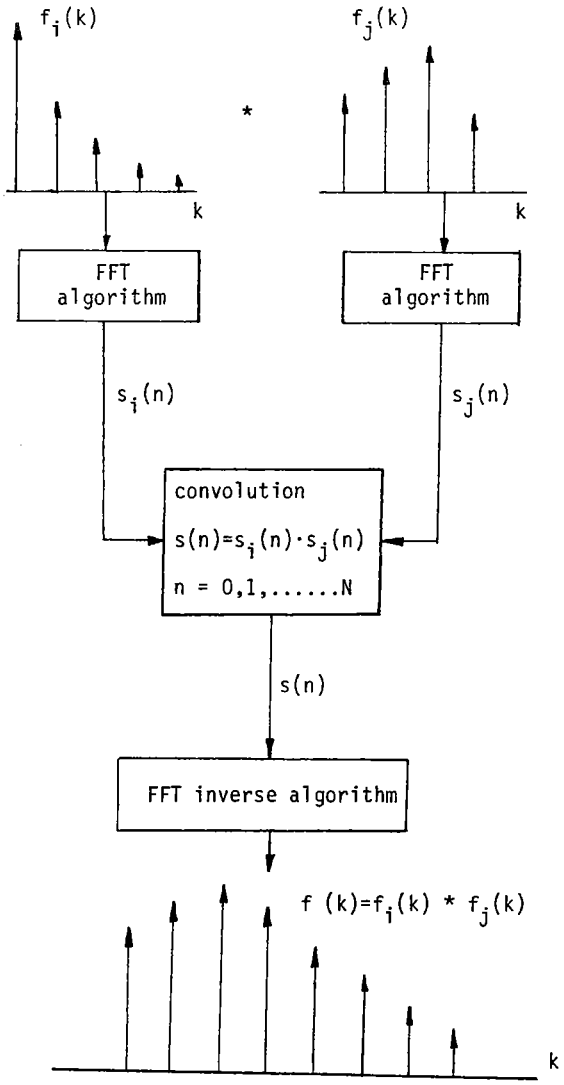

Fig. 11 FFT convolution process

APPENDIX 2

14 Bus System

The line and transformer data used in these studies are identical to that published in Reference [3].

Nodal Probabilistic Data

(i) Normal Distributions

Bus No.	Type	Voltage (p.u.)	Active Power μ(p.u.)	Active Power σ(%)	Reactive Power μ(p.u.)	Reactive Power σ(%)
1	Slack	1.060	2.3000	9.33		
2	Gen.	1.045	0.4004	9.00		
2	Gen.		-0.2174	9.00		
3	Gen.	1.010	-0.9420	10.00		
4	Load		-0.4780	11.00	0.0390	9.70
5	Load		-0.0760	5.00	-0.0160	5.00
6	Gen.	1.070	-0.1120	1.00		
7	Load		0.0	0.0		
8	Gen.	1.090	0.0	0.0		
9	Load		-0.2950	1.00	-0.1660	5.00
10	Load		-0.0900	10.00	-0.0580	10.00
11	Load		-0.0350	9.50	-0.0180	9.50
12	Load		-0.0610	1.00	-0.0160	8.60
13	Load		-0.1350	1.00	-0.0580	9.50
14	Load		0.0	0.0	-0.0500	8.60

σ expressed as a percentage of expected value, μ.

(ii) Discrete Distribution

Bus No.	Type	Active Power Power (p.u.)	Active Power Probability
14	Load	-0.1800	0.200
		-0.1500	0.450
		-0.1300	0.350