Probabilistic a.c. load flow

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Abstract

The paper extends the presently available techniques for evaluating the load flow problem probabilistically. It presents two possible formulations of the problem that permit the probability-density curves of the angles, voltages, injected reactive powers and active and reactive power flows to be computed. Owing to the complexity of the problem, the load-flow equations were linearised. The errors introduced by this technique together with the results of a typical analysis are discussed.

List of symbols

B_{ik} = imaginary part of element ik of admittance matrix
$B_{i(sh)}$ = imaginary part of shunt admittance at node i
B'_{ik} = half of susceptance of line $i-k$
G_{ik} = real part of element ik of admittance matrix
n = number of nodes
n_1 = number of PQ nodes
n_2 = number of PV nodes including slack
P_i = injected active power at node i
P_{ik} = active power flow in line $i-k$
P_s = balance of power at slack node
Q_i = injected reactive power at node i
Q_{ik} = reactive power flow in line $i-k$
$Q_{i(sh)}$ = injected reactive power by shunt element at node i
t_{ik} = transformer tap ratio
V_i = voltage magnitude at node i
X_{ik} = reactance of line $i-k$
θ_i = angle at node i referred to slack node
θ_{ik} = difference in angles between nodes i and k
μ = expected value
σ = standard deviation

Subscripts and superscripts

g = generation quantity
i,j,k = node numbers
l = load quantity
sh = shunt element
$\hat{}$ = represents inverted matrix and its elements

1 Introduction

Recently a number of papers have been published[1-5] that have modelled the load-flow problem probabilistically instead of by the conventional deterministic method. The main reason for this is that the input data of any power-system load-flow study is known to vary statistically and the attempt has been to model these statistical variations in the input data. Some authors[4,5] have considered these variations to be normally distributed and hence the output information to be normally distributed also. These authors have produced an equivalent a.c. probabilistic load flow albeit by a linearisation process. However, it has been shown[2,6] that the assumption of normally distributed variables can be unrealistic and give very misleading results. For this reason the other papers[1-3] have restricted their initial formulation to an equivalent d.c. load flow but in the process to model the system variables by realistic probability-density curves. Using mathematical convolution techniques, the power flows can be defined by their probability-density curves without assuming them to be normally distributed.

In the present paper, the previous formulation[1-3] has been extended so that an a.c. load flow can be modelled such that angles, voltages, active and reactive power flows, reactive power losses and injected reactive power can be evaluated.

2 Formulation of the load-flow problem

In a deterministic load-flow study, the known quantities are the injected active powers (P_i) at all busbars (PQ and PV) except the slack, the injected reactive powers (Q_i) at all load (PQ) busbars and the voltage magnitude (V_i) at all generator (PV) busbars.

In general, the load-flow problem can be formulated as

$$P_i = g_i(\theta_1 \ldots \theta_n, V_1 \ldots V_n) \tag{1}$$

$$Q_i = h_i(\theta_1 \ldots \theta_n, V_1 \ldots V_n) \tag{2}$$

where $i = 1, \ldots, n$.

From these quantities and equations the system can be solved quickly, efficiently and precisely using one of a number of deterministic load-flow techniques.

The problem with these techniques is that, although very precise, they are as inaccurate as the input data. Any variation of this input data can cause very significant changes in the load-flow solution. To overcome this difficulty, the problem can be modelled probabilistically.[1-5]

The known quantities in a probabilistic study are the same as given above. However, the quantities P_i and Q_i are now defined by probability-density curves although the voltages V_i at PV busbars are still known exactly. A number of difficulties are encountered in solving this problem probabilistically since the quantities are no longer single values but are, instead, density functions. Briefly these difficulties are due to

(*a*) θ and V are not available in terms of P_i and Q_i
(*b*) the functions g and h are non-linear
(*c*) the random variables P, Q, θ and V are not necessarily independent.

To overcome these difficulties, the problem has been linearised and the random variables P and Q have been assumed to be independent. Convolution techniques can then be used to deduce the density functions of the unknown quantities. Presently, two linearisation formulations have been investigated. These are based on the previously published[1-3] techniques and form the basis of this paper.

3 Probabilistic formulations

3.1 Load-flow equations

Although well known, it is useful for explaining the probabilistic formulation and assumptions made, to first consider the more detailed form of the load-flow equations. These are

$$P_i = V_i \sum_{k=1}^{n} V_k(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) \tag{3}$$

$$Q_i = V_i \sum_{k=1}^{n} V_k(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) \tag{4}$$

(where $i = 1, \ldots, n$)

$$P_{ik} = -t_{ik}G_{ik}V_i^2 + V_iV_k(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) \tag{5}$$

$$Q_{ik} = t_{ik}B_{ik}V_i^2 - B'_{ik}V_i^2 + V_iV_k(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) \tag{6}$$

$$Q_{i(sh)} = V_i^2 B_{i(sh)} \tag{7}$$

3.2 Angles and active powers

In both of the present formulations, the angles and active power flows were deduced in the same way and with the same assumptions as published previously.[1-3] Briefly these assumptions are

$V_i = V_k = 1$ p.u.

$G_{ik} = 0$ because line resistance assumed zero

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Using these assumptions, it can be shown that (see Appendix 8.1)

$$\theta_i = \sum_{k=1}^{n-1} \hat{C}_{ik} P_k \quad (i = 1, \ldots, n-1) \tag{8}$$

$$P_{ik} = \frac{1}{X_{ik}} \sum_{j=1}^{n-1} (\hat{C}_{ij} - \hat{C}_{kj}) P_j \tag{9}$$

and

$$P_s = \sum_{i=1}^{n} P_i \tag{10}$$

In both formulations, as in previous publications, eqns. 8-10 were used to obtain the angles, active power flows and balance of power at the slack (reference) busbar.

3.3 Voltages and reactive powers

As discussed in Section 2, it is necessary to linearise the load-flow problem in order to compute the density functions of the voltage and reactive powers. One assumption in the present formulations was that the active and reactive powers could be considered as decoupled. In addition, linearisations similar to those assumed for the d.c. probabilistic load flow were made. Two such methods of linearisations were made which are best named as formulation 1 and formulation 2.

(a) *Formulation 1*

To deduce the voltages and injected reactive powers at generation nodes, eqn. 4 was linearised by assuming the voltage $V_i = 1$ p.u. Eqn. 4 therefore becomes

$$Q_i = \sum_{k=1}^{n} A_{ik} V_k \tag{11}$$

where

$$A_{ik} = G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik}$$

$$A_{ii} = -B_{ii}$$

where

$$G_{ik} = \frac{-R_{ik}}{R_{ik}^2 + X_{ik}^2} \text{ and } B_{ik} = \frac{X_{ik}}{R_{ik}^2 + X_{ik}^2}$$

As shown in Appendix 8, the voltages at all load busbars and the injected reactive powers at all generation nodes can then be deduced from eqn. 11.

To deduce the reactive power flows and the reactive power contributed by the shunt element, eqns. 6 and 7 were linearised by assuming

$$V_i^2 = V_i \text{ and } V_i V_k = V_k$$

Eqn. 6 then becomes

$$\left.\begin{aligned} Q_{ik} &= \alpha_{ik} V_i + A_{ik} V_k \\ &\text{where} \\ \alpha_{ik} &= t_{ik} B_{ik} - B'_{ik} \end{aligned}\right\} \tag{12}$$

and eqn. 7 becomes

$$Q_{i(sh)} = V_i B_{i(sh)} \tag{13}$$

As shown in Appendix 8, the reactive powers can be deduced from eqns. 12 and 13.

(b) *Formulation 2*

In this formulation, approximations were made from the knowledge that the voltages in a power system are close to unity. It was therefore assumed that

$$V_i = 1 + \delta V_i$$

$$V_k = 1 + \delta V_k$$

where δV_i and δV_k are small deviations in V_i and V_k from unity.

Therefore

$$V_i V_k = (1 + \delta V_i)(1 + \delta V_k)$$

$$\simeq 1 + \delta V_i + \delta V_k \text{ neglecting } \delta V_i \delta V_k$$

$$= V_i + V_k - 1 \tag{14}$$

$$V_i^2 = 2V_i - 1 \tag{15}$$

Using eqns. 14 and 15, eqns. 4, 6 and 7 can be linearised from which, as shown in Appendix 8.3, the voltages and reactive powers can be deduced.

Both of the above formulations transform the nonlinear equations into equations consisting of the sum or difference of independent random variables. Consequently, mathematical convolution techniques can be applied in a straightforward[3] manner to obtain the density functions of the unknown quantities.

4 Analysis of a typical system

4.1 Test system

The techniques described in the previous sections have been applied to a wide range of systems from a small 6 busbar test system to the 57 busbar IEEE test system. Brief details of these systems are shown in Table 1 which includes the size of the systems, the type of input distributions used and the executions times for formulation 1 using a CDC 7600. The executions times for formulation 2 were almost identical. It is evident from Table 1 that the execution time increases as the system size is increased and as the number of discontinuous (binomial and discrete) distributions is increased. This trend is that expected, since the number of convolutions required with these distributions is greater[3] than that required for normal distributions.

To illustrate the application and results, this paper considers in more detail the analysis of the 14-busbar, 20-line IEEE test system. Similar results were obtained for all the other cases though the details of the results differed depending on the type and number of input distributions. The data specified by the IEEE for this system is deterministic as shown in Table 2. However, the specified data for this system does not include an active power for the slack busbar. Therefore, this was assumed to consist of 10 units each rated at 25MW with a forced outage rate of 0·08. Consequently from the binomial distribution, the expected value

$$\mu = 10 \times (1 - 0{\cdot}08) \times 25$$

$$\doteq 230\text{MW}$$

To compare the probabilistic evaluations with those that can be computed conventionally, the 14-busbar system using the nodal data shown in Table 2 and the line data shown in Table 3 was analysed using a conventional deterministic a.c. load-flow program.

Since the data specified for the IEEE test system is deterministic, the probabilistic nature of the input quantities had to be devised. This data is shown in Table 4. In all cases, the deterministic values were assumed to be expected values. All the busbar loads (indicated by the negative values in Table 4), with the exception of that at busbar 9, were assumed to be normally distributed and a standard deviation was therefore associated with the expected values. For the load at busbar 9, a discrete load characteristic was assumed such that the expected value was equal to the deterministic value shown in Table 1. The only other source of generations (busbar 2), indicated by a positive quantity in Table 3, was represented by 2 units each having a rating of 22MW with a forced outage rate of 0·09; this giving an expected value equal to the deterministic value shown in Table 1.

4.2 Results

Using the techniques discussed in the previous sections, it was found that the expected value of the balance of power at the slack busbar 1 was 219·0 MW with a standard deviation of 16·27 MW. This compares with a value of 232·39 MW using conventional techniques. Also, the expected values and standard deviations of the busbar angles, voltages and injected reactive powers using formulations 1 and 2 are shown in Table 5 and some of the active power flows and reactive power flows are shown in Table 6. In some cases the expected values are compared with those obtained using a conventional deterministic method.

From Tables 5 and 6 it can be seen that the expected values of angles, voltages, active and reactive power flows and injected reactive powers are similar in magnitude to the values obtained from the deterministic analysis. The errors that exist are clearly expected however because of the linearisation process used in the probabilistic model of the problem. These errors can be considered tolerable however being in mind the significantly increased information that can be gained from a probabilistic analysis. Also the error can be minimised as will be discussed later.

System	Number of Busbars	PV Busbars	Lines	Shunts	Input distributions: Active power	Reactive power	Execution time on CDC 7600
							s
Test	6	2	7	3	binomials-1 normals-6	binomials-1 normals-3	1·60
IEEE	14	5	20	1	binomials-1 normals-11	normals-8	1·86
IEEE	14	5	20	1	binomials-1 discrete-1 normals-10	discrete-1 normals-7	3·96
Test	22	4	23	4	normals-21	normals-18	0·42
Test	27	6	28	0	normals-26	normals-21	0·53
IEEE	30	6	41	2	normals-29	normals-24	0·70
Test	38	4	40	0	normals-37	normals-34	0·76
IEEE	57	7	80	3	normals-56	normals-50	1·63

Table 2
DETERMINISTIC DATA FOR IEEE 14-BUSBAR SYSTEM

Busbar Number	Type	Voltage p.u.	Active powers: Generation	Load	load reactive powers MVARr
1	slack	1·0600	MW	MW	
2	PV	1·0450	40·04	21·74	12·70
3	PV	1·0100		94·20	19·00
4	PQ			47·80	−3·90
5	PQ			7·60	1·60
6	PV	1·0700		11·20	7·50
7	PQ				
8	PV	1·0900			
9	PQ			29·50	16·60
10	PQ			9·00	5·80
11	PQ			3·50	1·80
12	PQ			6·10	1·60
13	PQ			13·50	5·80
14	PQ			14·90	5·00

Table 3
LINE DATA FOR IEEE 14-BUSBAR SYSTEM

Busbar Sending	Receiving	Resistance *p.u.*	Reactance *p.u.*	Susceptance *p.u.*	Transformer tap
1	2	0·01938	0·05917	0·02640	%
1	5	0·05403	0·22304	0·02640	
2	3	0·04699	0·19797	0·02190	
2	4	0·05811	0·17632	0·01870	
2	5	0·05695	0·17388	0·01700	
3	4	0·06701	0·17103	0·01730	
4	5	0·01335	0·04211	0·00640	
4	7	–	0·20912	–	−2·2
4	9	–	0·55618	–	−3·1
5	6	–	0·25202	–	−6·8
6	11	0·09498	0·19890	–	
6	12	0·12291	0·25581	–	
6	13	0·06615	0·13027	–	
7	8	–	0·17615	–	
7	9	–	0·11001	–	
9	10	0·03181	0·08450	–	
9	14	0·12711	0·27038	–	
10	11	0·08205	0·19207	–	
12	13	0·22092	0·19988	–	
13	14	0·17093	0·34802	–	
9	9	–	−5·26000	–	

It is also seen from these two Tables that neither formulation 1 nor formulation 2 can be considered the better formulation, since more precise results are given in some instances by formulation 1 and in others by formulation 2.

Although expected values and standard deviations are quoted in Tables 5 and 6, this does not mean that the probability-density curves themselves are normally distributed. To illustrate this point, typical density curves for active power flow, voltage, reactive power flow and injected reactive power obtained using formulation 1 are shown in Figs. 1 – 4. It is clearly evident from these Figures that the computed quantities are not normally distributed. The amount by which the density curves deviate from a normal distribution depends on the system and the specified input nodal quantities; as the number of binomial and discrete powers are increased for a given system size, the computed density curves deviate more from a normal distribution. In the present example, most input quantities were defined by normal distributions, yet the computed results differ significantly from normal.

Probability density curves of almost identical shape to those shown in Figs. 1 – 4 were obtained using formulation 2 except that, because the expected values obtained from the two formulations were slightly

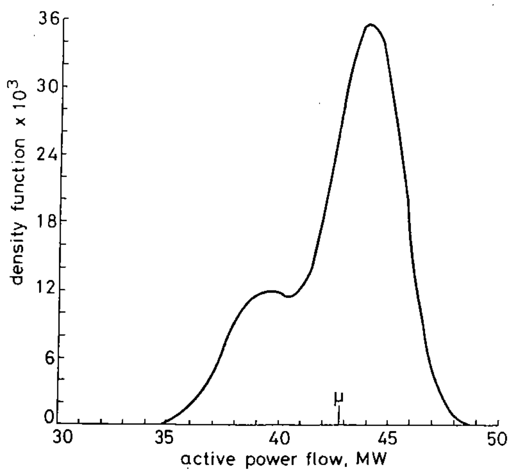

Fig. 1
Probability density curve for active power flow in line 5 - 6

Table 4
PROBABILISTIC DATA USED

1 Normal distributions

Busbar Number	Type	Voltage	Active power μ	Active power σ	Reactive power μ	Reactive power σ
		p.u.	MW	%	MVAR	%
2	PV	1·045	−21·74	9·00	−12·70	9·2
3	PV	1·010	−94·20	10·00	−19·00	10·5
4	PQ	–	−47·80	11·00	3·90	9·7
5	PQ	–	−7·60	5·00	−1·60	5·0
6	PV	1·070	−11·20	6·00	−7·50	6·3
7	PQ	–	0·00	0·00	0·00	0·0
8	PV	1·090	0·00	0·00	0·00	0·0
10	PQ	–	−9·00	10·00	−5·80	10·0
11	PQ	–	−3·50	9·50	−1·80	9·5
12	PQ	–	−6·10	7·60	−1·60	8·6
13	PQ	–	−13·50	10·50	−5·80	9·5
14	PQ	–	−14·90	8·60	−5·00	8·6

2 Binomial distributions

Busbar Number	Type	Voltage p.u.	Unit rating MW	Forced outage rate	Number of units
1	Slack	1·060	25·0	0·08	10
2	PV	1·045	22·0	0·09	2

3 Any discrete distribution

Busbar Number	Type	Voltage	Active power		Reactive power	
		p.u.	*MW*	*prob.*	*MVAR*	*prob.*
9	PQ	–	−13·4	0·10	−7·5	0·10
			−19·6	0·15	−11·0	0·15
			−30·2	0·30	−17·0	0·30
			−34·8	0·25	−19·6	0·25
			−37·3	0·20	−21·0	0·20

Table 5
ANGLES, VOLTAGES AND INJECTED REACTIVE POWERS

Busbar	Angles			Voltages				
	Determ.	μ	σ	Determ.	Formulation 1		Formulation 2	
					μ	σ	μ	σ
	deg	deg	deg	p.u.	p.u.	p.u.	p.u.	p.u.
4	−10·31	−10·58	0·66	1·0171	1·0156	0·0009	1·0154	0·0009
5	−8·76	−9·09	0·56	1·0187	1·0169	0·0005	1·0166	0·0005
7	−13·36	−13·91	0·96	1·0613	1·0593	0·0026	1·0603	0·0027
9	−14·93	−15·69	1·16	1·0557	1·0526	0·0051	1·0542	0·0052
10	−15·09	−15·97	1·10	1·0508	1·0469	0·0043	1·0480	0·0044
11	−14·79	−15·62	0·97	1·0568	1·0540	0·0023	1·0545	0·0023
12	−15·07	−15·97	0·88	1·0552	1·0526	0·0005	1·0523	0·0005
13	−15·15	−16·14	0·90	1·0503	1·0460	0·0010	1·0458	0·0010
14	−16·03	−17·19	1·06	1·0354	1·0287	0·0034	1·0292	0·0034
				Injected reactive power				
				Determ.	Formulation 1		Formulation 2	
					μ	σ	μ	σ
				MVAr	MVAr	MVAr	MVAr	MVAr
1	0·0	0·0	0·0	−13·31	−12·17	0·23	−15·10	0·23
2	−4·98	−5·01	0·41	36·10	35·24	0·75	35·93	0·75
3	−12·73	−12·95	0·92	7·29	8·43	0·44	8·78	0·44
6	−14·22	−14·85	0·84	5·68	7·63	1·81	5·06	1·84
8	−13·36	−13·91	0·96	17·78	17·41	1·47	16·85	1·52

Table 6
ACTIVE AND REACTIVE POWER FLOWS

Line	Active power flows			Reactive power flows				
	Determ.	μ	σ	Determ.	Formulation 1		Formulation 2	
					μ	σ	μ	σ
	MW	MW	MW	MVAr	MVAr	MVAr	MVAr	MVAr
1–2	154·78	147·84	12·16	−18·93	−18·05	*	−20·40	*
1–5	74·08	71·16	4·35	5·62	5·88	0·23	5·29	0·23
2–3	72·11	70·01	5·20	4·75	4·39	*	3·84	*
2–4	55·30	55·15	3·20	−0·40	−0·09	0·46	−0·67	0·46
2–5	41·07	40·97	2·33	2·61	2·74	0·29	2·39	0·28
4–3	23·44	24·19	4·62	−5·38	−5·92	0·44	−6·13	0·44
4–5	−61·34	−61·75	4·55	16·11	15·63	0·75	16·25	0·78
6–12	7·75	7·61	0·41	2·51	2·47	0·16	2·37	0·16
6–13	17·75	17·25	1·18	7·24	7·51	0·64	7·16	0·64
7–9	28·06	28·36	3·53	5·76	6·57	2·28	6·06	2·34
10–11	−3·80	−3·23	1·43	−1·66	−1·89	0·91	−1·55	0·94

* The standard deviations in these cases are zero since the voltage at both ends of the line was specified

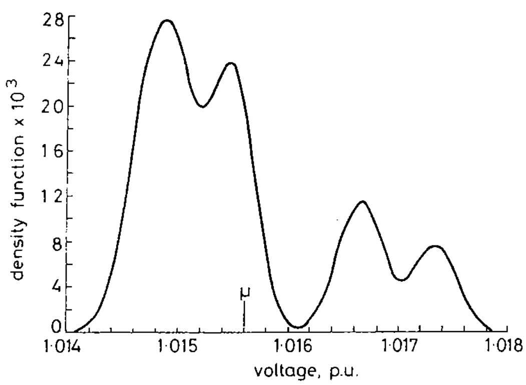

Fig. 2
Probability density curve for voltage at busbar 4

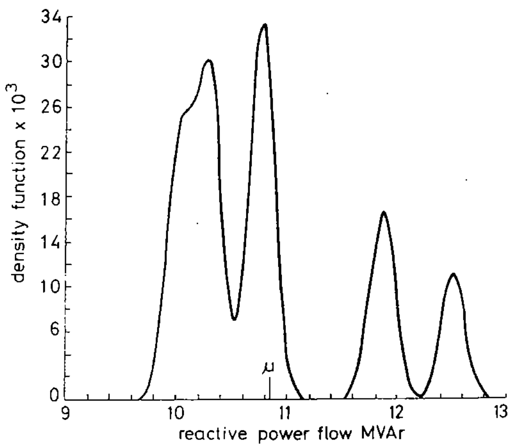

Fig. 3
Probability density curve for reactive power flow in line 7 - 4

different, the corresponding curves were shifted slightly relative to each other. This similarity in shape is confirmed by the results shown in Table 5 and 6. Comparing the standard deviations given by the two formulations shows that, in practically all cases, the results are virtually identical; in a few cases a very small difference exists.

Both of these aspects are interesting features since they indicate that the computed density curves and standard deviations are reasonably precise although small errors exist in the computed expected values. Because of this, the errors in expected values may be overcome by shifting the computed density curve, without changing its shape, until the expected value of it coincides with the value obtained from a conventional deterministic analysis. This newly positioned curve would then represent a reasonably precise distribution for the quantity being computed. Furthermore, the computational efficiency could be improved in such an exercise by using the expected values obtained from either formulation as the starting values for the conventional iterative analysis.

5 Conclusions

This paper has shown how the expected values, standard deviations and probability density curves of angles, voltages, active power flows, reactive power flows and injected reactive powers can be computed using linearised versions of the a.c. load-flow equations. This permits the previously published techniques to be extended such that the model more truly reflects the needs of a power-system load-flow study. The small errors that occur can be compensated for by simply shifting the computed density curve until its expected value coincides with the value obtained from a conventional deterministic analysis.

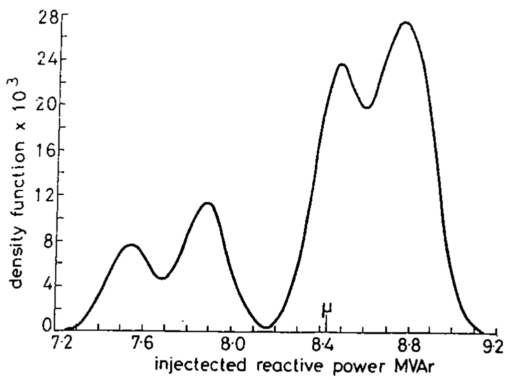

Fig. 4
Probability density curve for injected reactive power at busbar 3

From these density curves, it is a simple exercise to deduce confidence limits and the probability of any quantity being greater than or less than a certain predetermined value. The importance of this in power-system studies cannot be overstressed and forms a vital part in the analysis of data that is known to vary in practice or is in error due to forecasting problems.

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8 Appendixes

8.1 Angles and active powers

From the assumptions given in Section 3.2, eqn. 3 becomes

$$P_i = \sum_{k=1}^{n} \frac{1}{X_{ik}} \times \theta_{ik}$$

or in matrix form

$$P = C\boldsymbol{\theta}$$

where

$$C_{ik} = \frac{-1}{X_{ik}} \text{ and } C_{ii} = \sum_{k=1 \neq i}^{n} \frac{1}{X_{ik}}$$

in which the slack busbar row and column are deleted. Inverting gives

$$\boldsymbol{\theta} = C^{-1}P = \hat{C}P$$

or

$$\theta_i = \sum_{k=1}^{n-1} \hat{C}_{ik} P_k \quad (i = 1 \ldots n-1) \tag{16}$$

Similarly, from eqn. 5

$$P_{ik} = \frac{\theta_i - \theta_k}{X_{ik}}$$

from which after substituting eqn. 16 gives

$$P_{ik} = \frac{1}{X_{ik}} \sum_{j=1}^{n-1} (\hat{C}_{ij} - \hat{C}_{kj}) P_j \tag{17}$$

in which if node i is slack, $\hat{C}_{ij} = 0$

8.2 Formulation 1

Writing eqn. 11 in matrix form gives

$$Q = AV$$

partitioning into the load and generation quantities gives

$$\begin{vmatrix} Q_l \\ Q_g \end{vmatrix} = \begin{vmatrix} M & L \\ N & J \end{vmatrix} \begin{vmatrix} V_l \\ V_g \end{vmatrix} \tag{18}$$

where Q_l has n_1 elements and Q_g has n_2 elements. From eqn. 18

$$Q_l = M\,V_l + L\,V_g$$

i.e.

$$V_l = \hat{M}\,Q_l + \hat{M}\,H$$

where

$$H = -L\,V_g$$

writing explicitly gives

$$V_{i(l)} = \sum_{j=1}^{n_1} \hat{M}_{ij} Q_{j(l)} + \sum_{j=1}^{n_1} \hat{M}_{ij} H_j \quad (i = 1 \ldots n_1) \tag{19}$$

also from eqn. 18

$$Q_g = N\,V_l + J\,V_g$$

substituting for V_l gives

$$Q_g = D\,Q_l + E$$

where

$$D = N\hat{M} \text{ and } E = D\,H + J\,V_g$$

writing explicitly gives

$$Q_{i(g)} = \sum_{j=1}^{n_1} D_{ij} Q_{j(l)} + E_i \quad (i = 1, \ldots, n_2) \tag{20}$$

from eqns. 12 and 19

$$Q_{ik} = \sum_{j=1}^{n_1} (\alpha_{ik}\hat{M}_{ij} + A_{ik}\hat{M}_{kj}) Q_{j(l)} + \sum_{j=1}^{n_1} (\alpha_{ik}\hat{M}_{ij} + A_{ik}\hat{M}_{kj}) H_j$$

if i and k are load busbars

$$Q_{ik} = \sum_{j=1}^{n_1} A_{ik}\hat{M}_{kj} Q_{j(l)} + \sum_{j=1}^{n_1} A_{ik}\hat{M}_{kj} H_j + \alpha_{ik} V_i \tag{21}$$

if i is a generation busbar

$$Q_{ik} = \sum_{j=1}^{n_1} \alpha_{ik}\hat{M}_{ij} Q_{j(l)} + \sum_{j=1}^{n_1} \alpha_{ik}\hat{M}_{ij} H_j + A_{ik} V_k$$

if k is a generation busbar

Similar equations can be derived for Q_{ki} and the reactive power losses, $Q_{ik} + Q_{ki}$.

Finally, from eqns. 13 and 19

$$Q_{i(sh)} = B_{i(sh)} \sum_{j=1}^{n_1} \hat{M}_{ij} Q_{j(l)} + B_{i(sh)} \sum_{j=1}^{n_1} \hat{M}_{ij} H_j \tag{22}$$

if i is a load busbar

8.3 Formulation 2

Substituting eqn. 14 into eqn. 4 gives

$$Q_i = \sum_{k=1}^{n} (V_i + V_k - 1) A_{ik}$$

where A_{ik} is defined in Section 3.3

Thus

$$Q_i = \sum_{k=1}^{n} A'_{ik} V_k - W_i \tag{23}$$

where

$$A'_{ik} = A_{ik} \text{ for } k \neq i$$

$$A'_{ii} = A_{ii} + W_i$$

$$W_i = \sum_{k=1}^{n} A_{ik}$$

In matrix form:

$$Q = A'V - W$$

which when partitioned into load and generation quantities gives

$$\begin{vmatrix} Q_l \\ \hline Q_g \end{vmatrix} = \begin{vmatrix} M' & L' \\ \hline N' & J' \end{vmatrix} \begin{vmatrix} V_l \\ \hline V_g \end{vmatrix} - \begin{vmatrix} W_l \\ \hline W_g \end{vmatrix} \tag{24}$$

Using the same technique as described for formulation 1 gives

$$V_{i(l)} = \sum_{j=1}^{n_1} \hat{M}'_{ij} Q_{j(l)} + \sum_{j=1}^{n_1} \hat{M}'_{ij} H'_j \qquad i = 1, \ldots, n_1 \tag{25}$$

$$Q_{i(g)} = \sum_{j=1}^{n_1} D'_{ij} Q_{j(l)} + E'_i \qquad i = 1, \ldots, n_2 \tag{26}$$

where

$$H' = W_l - L'V_g$$

$$D' = N'\hat{M}'$$

$$E' = D'H' + J'V_g - W_g$$

Similarly, substituting eqns. 14 and 15 into 6 gives

$$Q_{ik} = \beta_{ik} V_i + A_{ik} V_k + \gamma_{ik} \tag{27}$$

where

$$\beta_{ik} = 2\alpha_{ik} + A_{ik}$$

$$\gamma_{ik} = -\alpha_{ik} - A_{ik}$$

Substituting eqn. 25 into eqn. 27 gives a similar set of equations for Q_{ik}, Q_{ki} and line losses as that given by eqn. 21 in formulation 1 above.

Finally, substituting eqn. 15 into eqn. 7 gives

$$Q_{i(sh)} = 2B_{i(sh)} V_i - B_{i(sh)} \tag{28}$$

from which an equation similar in form to eqn. 22 can be deduced.