HW2 Problem 8: Yet More On Metaproducts

▼ What you know
- Metaproducts are a way to represent SOP forms as BDDs
- Some BDD-like logic manipulations are “supposed to work…”
- …but results are very difficult to interpret
- HW2 problem 8: a 4-variable function is too big to see what’s going on.

▼ What you don’t know
- That metaproducts really represent sets of things (like we introduced in Lec05 on Formal Verification)
- How to really interpret ops like NOT( BDD in metaproduct form)

More On Metaproducts

▼ Suppose I have a Boolean function of 2 vars: $F(x,y)$
- If I want to consider writing an SOP equation for $F(x,y)$, how many possible product terms could there be?

▼ Can enumerate: there are $3^2 = 9$ terms:
- Every product has 2 “slots” for literals in it
- The first slot can be one of $\{\varepsilon, x, x'\}$ where “$\varepsilon$” means “empty”
- The second slot can be one of $\{\varepsilon, y, y'\}$ where “$\varepsilon$” also means “empty”
- Why 9 terms max? $|\{\varepsilon, x, x'\}| \times |\{\varepsilon, y, y'\}| = 3 \times 3 = 9$

▼ Examples
- Term $xy' == (1^{st} \text{ slot is } x)(2^{nd} \text{ slot is } y')$
- Term $x' == (1^{st} \text{ slot is } x')(2^{nd} \text{ slot is } \varepsilon -- \text{ empty})$
More on Metaproducts

Well, what are all 9 of these possible product terms?

<table>
<thead>
<tr>
<th>1st slot</th>
<th>2nd slot</th>
<th>Product Term Represented</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε</td>
<td>ε</td>
<td>ε = empty</td>
</tr>
<tr>
<td>x</td>
<td>ε</td>
<td>x</td>
</tr>
<tr>
<td>x'</td>
<td>ε</td>
<td>x'</td>
</tr>
<tr>
<td>ε</td>
<td>y</td>
<td>y</td>
</tr>
<tr>
<td>ε</td>
<td>y'</td>
<td>y'</td>
</tr>
<tr>
<td>x</td>
<td>y</td>
<td>xy</td>
</tr>
<tr>
<td>x</td>
<td>y'</td>
<td>xy'</td>
</tr>
<tr>
<td>x'</td>
<td>y</td>
<td>x'y</td>
</tr>
<tr>
<td>x'</td>
<td>y'</td>
<td>x'y'</td>
</tr>
</tbody>
</table>

OK, what does this have to do with metaproducts...?

A metaproduct is really a BDD that represents a set

The set it represents is some arbitrary set of product terms, chosen from the complete set of 9 (in this 2-variable case) on previous slide

Example: \( F(x, y) = x + y' \)
So, what really happens when you complement this BDD?

What is this?

It’s the BDD for the set of all the OTHER product terms NOT in the original BDD.

You get a new BDD that represents the 7 other products NOT in original set.

2 product terms in original set

7 other product terms that were NOT in original set.
More on Metaproducts

Subtle stuff

Interpreting what happens when you see missing variables

- **x** is here and negative, but no **y** occurrence var. Interpret as: all values of **y** are possible, including the empty “ε” **y** value. Result is: $x', xy, x'y$

- **x** is here and positive, **y** is here, but no **y** sign var. Interpret as: all “signed” values of **y** are possible, but not the empty “ε” **y** value. Result is: $xy, xy'$

- So both original metaproduct BDD and its complement are just sets of stuff. They represent subsets of these 9 terms

  - When you complement one of these, you don’t get $F'(\cdot)$. You get a set that represents all the other terms you didn’t represent originally
\section*{About HW2 Problem 8}

- The part about "..complement it and explain it" was aimed at this, but with $F(\ )$ = 4 variables, it's just way too complicated to see. (Sorry…)

- Do this \textit{instead} of the complicated 4-variable function:
  - Let $F(x,y) = x'y + xy'$
  - Draw the BDD for the metaproduct form for $F(\ )$
  - Draw the complement BDD for this metaproduct BDD
  - Like in these notes, show that the complement really does represent all of the other product terms not in the original BDD.