Carnegie Mellon University
Department of Electrical and Computer Engineering

18-794 **Pattern Recognition Theory** **Spring 1999**

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Homework Assignment #5

Due: April 14, 1999

1. (20%) Suppose we are estimating $f(x)$, a Gaussian PDF (with mean m and variance σ^2) using the following Parzen density estimation procedure.

$$\hat{f}_n(x) = \frac{1}{nh_n} \sum_{j=1}^{n} W\left(\frac{x - x_j}{h_n}\right)$$

where $\{x_1, x_2, \ldots, x_n\}$ is the set of samples drawn independently from $f(x)$, $W(x))$ is a unit Gaussian window (with zero mean and unit variance) and h_n is the window width. Show that

$$\mathrm{Var}\{\hat{f}_n(x)\} \cong \frac{1}{2\sqrt{\pi}\, nh_n} f(x)$$

You may have to make some judicious assumptions to obtain the above approximation.

2. (20%) Consider a 2-class problem with equal a priori probabilities, i.e., $P(\omega_1) = P(\omega_2) = \frac{1}{2}$. The conditional PDFs $f(\mathbf{x}|\omega_1)$ and $f(\mathbf{x}|\omega_2)$ are uniformly distributed in hyperspheres of radius 1 unit whose origins are separated by 11 units. Show that the average P_e for K-NNM (assuming that K is odd) based on n independently drawn training samples is given by

$$P_e(n) = \frac{1}{2^n} \sum_{j=0}^{(K-1)/2} \binom{n}{j}$$

Determine $P_e(n)$ for $K = 1, 3, 5$ for $n = 10$. Do you see anything strange?

3. (20%) Consider a C class problem where all classes are equally likely and the class conditional PDF for the *i*-th class $(i = 1, 2, \ldots, C)$ is as follows.

$$f(x|\omega_1) = \begin{cases} 1 & \text{for } 0 \le x \le \dfrac{cr}{(c-1)} \\ 1 & \text{for } i \le x \le (i+1) - \dfrac{cr}{(c-1)} \\ 0 & \text{elsewhere} \end{cases}$$

Here $0 \le r \le (C-1)/C$. Determine $P_e{}^*$, the minimum (i.e., Bayesian) error probability as well as $P_{e,\,1-\mathrm{NNM}}$, the asymptotic error probability of 1-NNM.

4. (20%) Determine and sketch the decision boundaries resulting from a 1-NNM classifier using the following training data.

   class $\omega_1 = \{(1,0)^T, (0,0)^T, (0,1)^T\}$

   class $\omega_2 = \{(0,-1)^T, (1,-1)^T, (-1,0)^T\}$

5. (20%) In a 2-class problem using a single feature, following labeled samples are available. Sketch the decision boundaries obtained by using 3-NNM classifier with the following training data.

   class $\omega_1 = \{-4, -2, 0, 2, 4\}$

   class $\omega_2 = \{-3, -1, 1, 3\}$