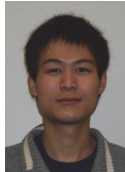


# Microelectromechanical Nonlinear Parametric Resonance



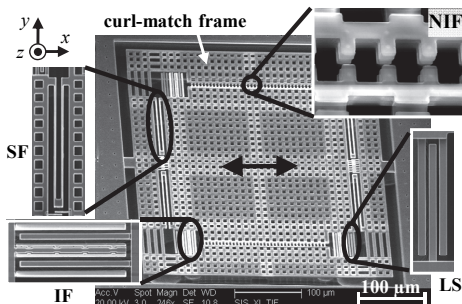
Congzhong Guo



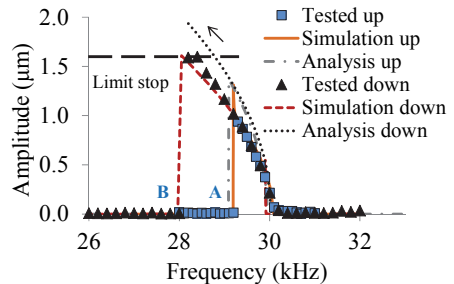
Gary Fedder

Modern MEMS design flow with high accuracy and fast modeling necessitates the emergence of CAD tools which capture the multiphysics and give rapid feedback to designers of complex MEMS systems. Parametric resonance in electrostatically actuated MEMS gives rise to complex nonlinear phenomena. Accurate predictive design of nonlinear systems featuring parametric resonance provides motivation to refine mixed MEMS/analog behavioral composable modeling and simulation methodologies. Toward this end, a parameterized schematic of a canonical parametric resonator tested is built with primitive beam, plate and gap models from a MEMS Verilog-AMS library. Behavioral modeling and the analytic perturbation solutions are validated by optical vibration measurements matching to 0.6% and 2.1%, respectively. This work has established a foundation for future studies of servoing at the system's bifurcation frequencies.

Shown in Fig. 1 is an SEM image of the fabricated CMOS-MEMS nonlinear plate-mass resonator. Four sets of serpentine flexures exhibiting a small cubic stiffening nonlinearity connect the curl-match frame to the perforated plate mass. Electrostatic force from the aligned non-interdigitated comb fingers (NIF) modulated by the pump drive voltage amplitude, parametrically excites the resonator. The NIF design makes use of the symmetric fringing field between non-interlocking fingers to generate a cubic force – displacement relationship. Fig. 2 is an overlaid comparison of the analysis, behavioral modeling and experimental frequency response for both sweep directions. The bi-state jump occurs on the boundaries of instability at points A and B. This sharp bifurcation makes it a good candidate for mass and stress sensing.



**Fig. 1:** SEM of a CMOS-MEMS nonlinear parametric resonator that moves in  $x$ , with zoom-in view of non-interdigitated comb fingers (NIF), interdigitated comb fingers (IF), serpentine flexure (SF), and mechanical limit stop fingers (LS).



**Fig. 2:** Analytic solution, behavioral simulation, and experimental results for bi-directional sweeps. The experimental results are obtained with  $V_{dc} = 40$  V and  $V_{ac} = 20$  V.