Signal Quality Pricing

Decomposition for Spectrum Scheduling and System Configuration

Eric Anderson[†⋆] Caleb Phillips[⋆] Douglas Sicker[⋆]
Dirk Grunwald[⋆]

[†]Carnegie Mellon University
Electrical and Computer Engineering

[⋆]University of Colorado
Computer Science

DySPAN 2011
6 May 2011

Background: Separate Scheduling and Configuration

Scheduling

Which transmissions occur when?

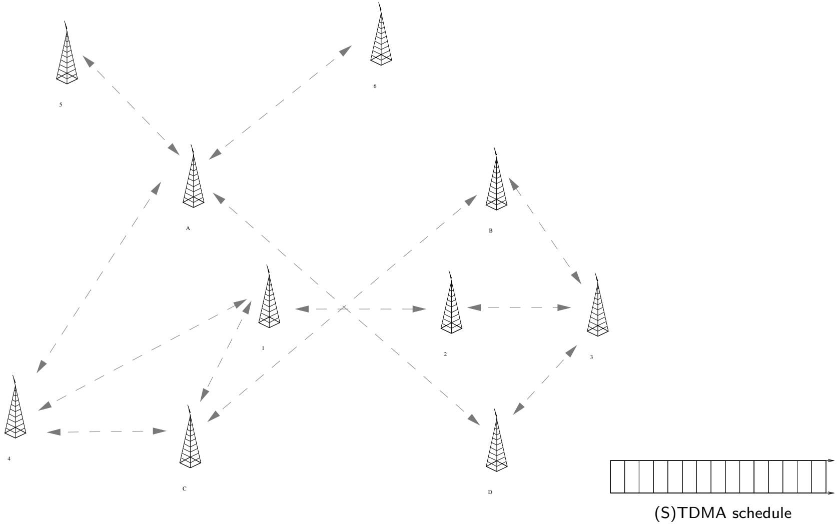
- Partition transmissions into *compatible* groups.
- Assign groups to times,
- Or frequencies.

Configuration

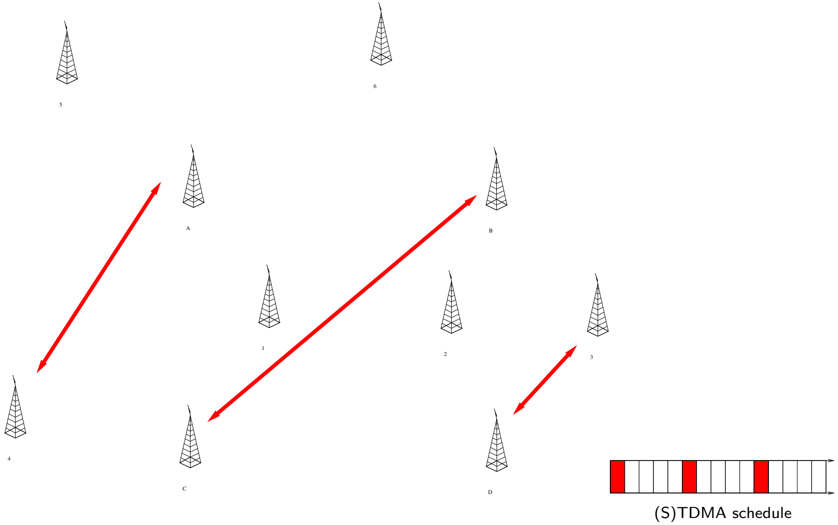
How does each transmission (and reception) occur?

- Transmission power,
- Modulation / rate,
- **Antenna steering / selection**,
- Frequency (sometimes).

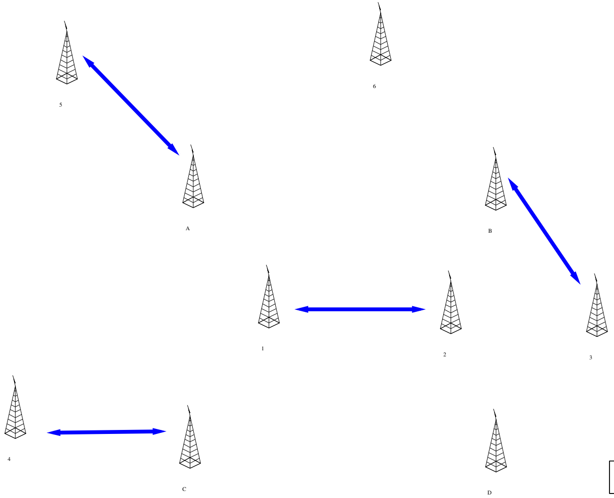
Scheduling Example

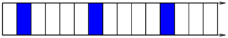


Scheduling Example



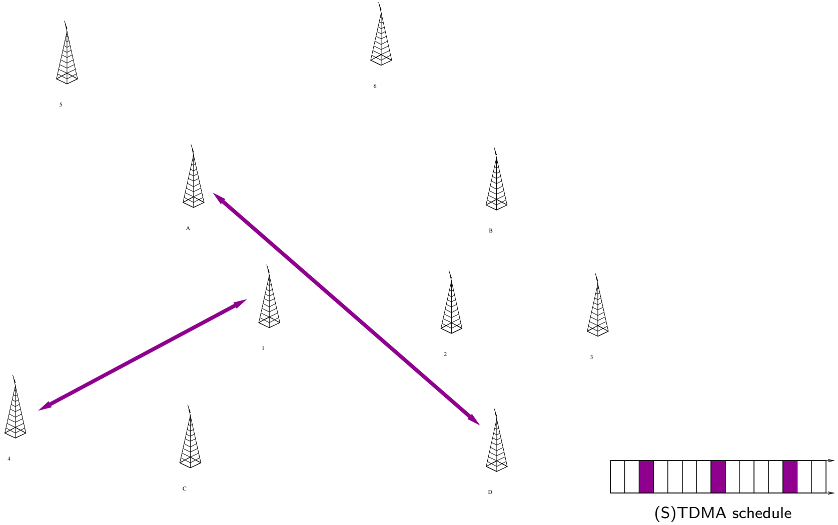
Scheduling Example



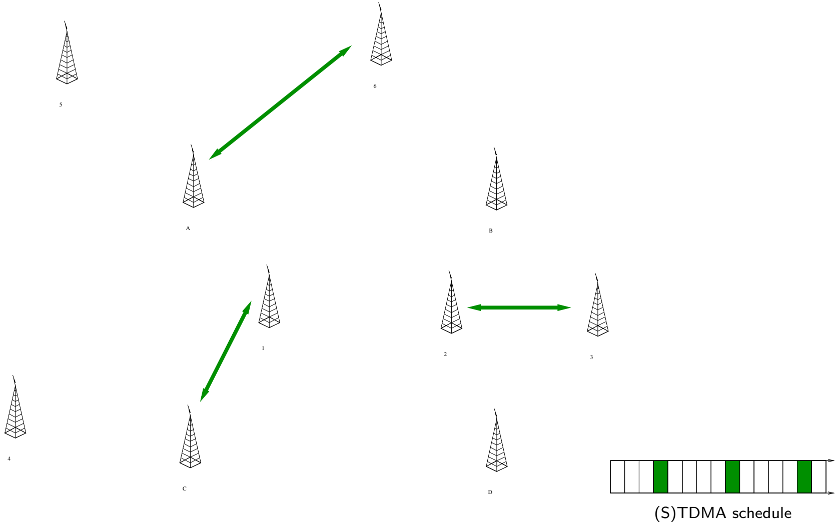


(S)TDMA schedule

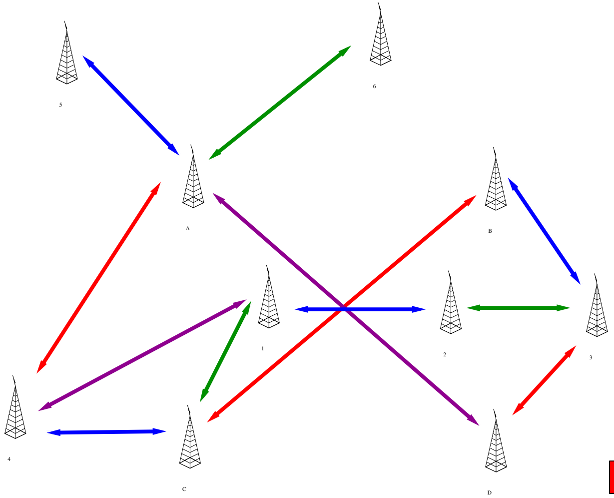
Scheduling Example



Scheduling Example



Scheduling Example

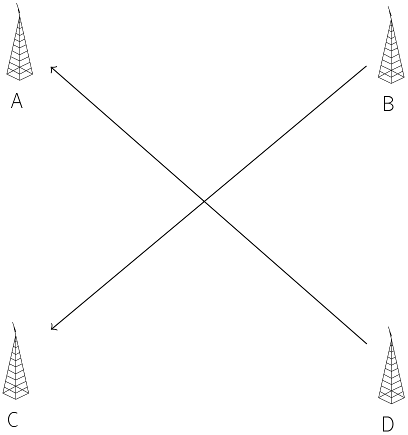


(S)TDMA schedule

"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

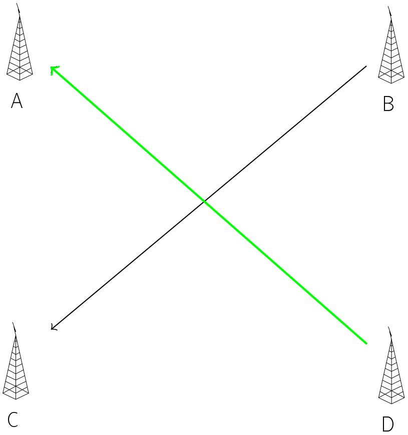
Constraints:
Link SINR,
Half-duplex,
...



"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

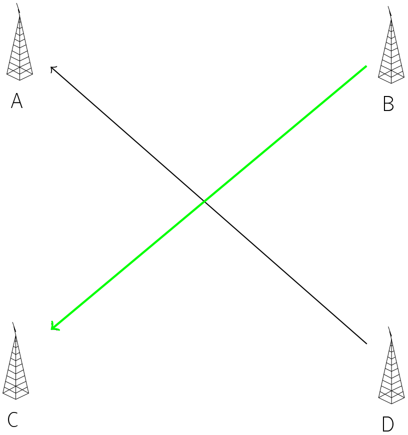
Intended signal:
D→A



"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Intended signal:
B→C

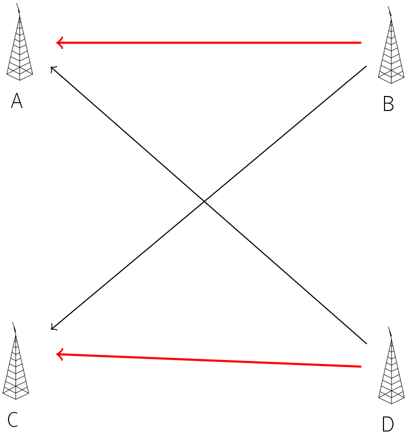


"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Interference:
B→A
D→C

(If both links were in use)

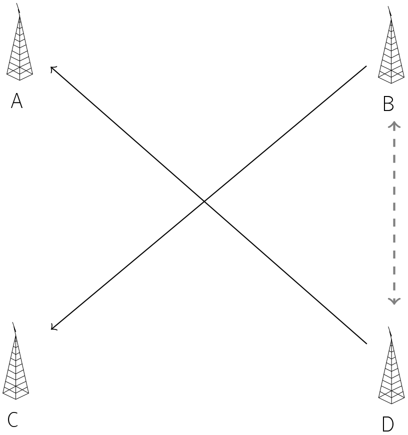


"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Signal between transmitters:
Not an issue

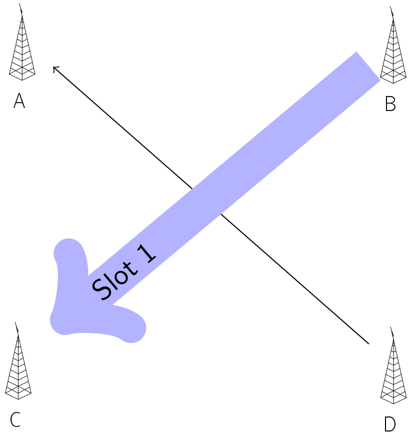
(Would matter for CSMA)



"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

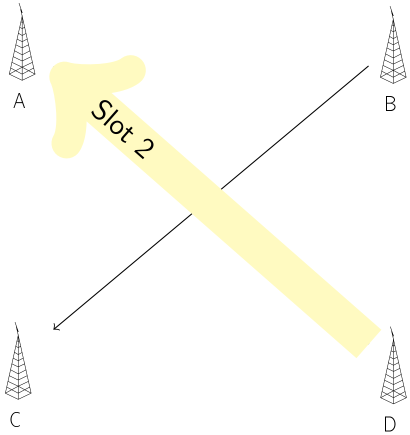
Trivial Schedule:
Each link gets one slot (TDMA).



"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

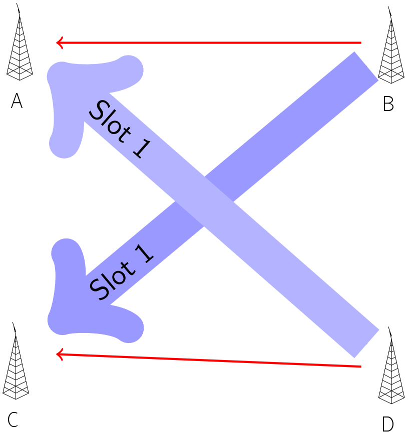
Trivial Schedule:
Each link gets one slot (TDMA).



"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

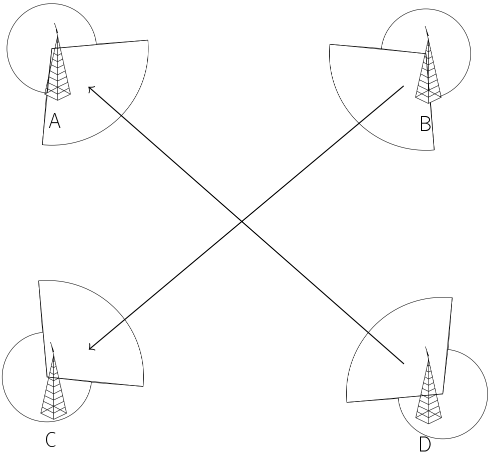
Faster Schedule:
Concurrent links,
Interference



"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

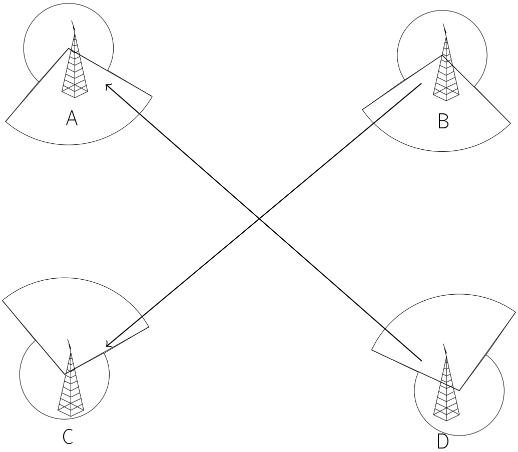
Per-link best (maximum SNR) antenna choices:
Boosts interfering signals, too.



"Chicken and Egg" Example

Link demand:
B→C: 1 slot
D→A: 1 slot

Scheduling-aware antenna selection:
Low gain for interference.



Integration and decomposition

Configuration and scheduling can be expressed as a combined problem
— but the state space is huge: $\Theta(n^2 2^m)$ *variables*

Key Idea

Transform problem into many coupled subproblems.

- Individually simple to solve
- Naturally parallel
- Iterate and update (not too many times)

Decomposition Process (Idealized)

Goal: Optimize complete schedule

Given:

Subject to: Complete constraints (PHY, MAC, Network, user, . . .)

Goal: *Marginally* improving concurrent group

Given: Current schedule

Subject to: Link (set) compatibility (PHY, MAC constraints)

Goal: Best set of active links

Given: *Estimated* configurations

Subj. to: PHY, MAC constraints

Goal: Best configurations

Given: *Estimated* active link set

Subj. to: PHY constraints

Decomposition Process (Idealized)

Goal: Optimize complete schedule

Given:

Subject to: Complete constraints (PHY, MAC, Network, user, . . .)

↓

Goal: *Marginally* improving concurrent group

Given: Current schedule

Subject to: Link (set) compatibility (PHY, MAC constraints)

Goal: Best set of active links

Given: *Estimated* configurations

Subj. to: PHY, MAC constraints

Goal: Best configurations

Given: *Estimated* active link set

Subj. to: PHY constraints

Decomposition Process (Idealized)

Goal: Optimize complete schedule

Given:

Subject to: Complete constraints (PHY, MAC, Network, user, . . .)

Goal: *Marginally* improving concurrent group

Given: Current schedule

Subject to: Link (set) compatibility (PHY, MAC constraints)

Goal: Best set of active links

Given: *Estimated* configurations

Subj. to: PHY, MAC constraints

Goal: Best configurations

Given: *Estimated* active link set

Subj. to: PHY constraints

Decomposition Process

. . .

Goal: *Marginally* improving concurrent group

Given: Current schedule

Subject to: Link (set) compatibility (PHY, MAC constraints)

Lagrangian dual problem: *Price* PHY & MAC constraints

e.g.

- Signal to Interference and Noise Ratio (SINR) threshold
- Half-duplex requirement

Decomposition Process

. . .

Goal: *Marginally* improving concurrent group

Given: Current schedule

Subject to: Link (set) compatibility (PHY, MAC constraints)

↓

Lagrangian dual problem: *Price* PHY & MAC constraints

e.g.

- Signal to Interference and Noise Ratio (SINR) threshold
- Half-duplex requirement

What do Constraint Prices Mean?

(Lagrangian relaxation in 60 seconds)

Original problem: Minimize objective subject to constraints.

$$\begin{aligned} \min_x \quad & & f(x) \\ \text{s.t.} \quad & g_i(x) & \leq c_i \end{aligned}$$

Lagrangian: Minimize (objective + penalty) w/o constraints.

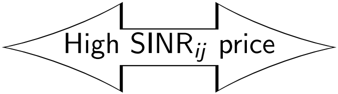
$$\min_x \quad f(x) + \lambda_i(g_i(x) - c_i)$$

Price (λ_i): For each constraint i, marginal cost per unit of violation.

Dual: Find the lowest prices such that the degree of violation ≈ 0.

Look up, this is important!

Scheduling
Avoid using link *ij*
or interfering with *ij*



Configuration
Increase gain for *ij*
attenuate interference

Subgradient / Relaxed Primal Problem method

Solution of dual problem:

Link Activation Problem

- Choose (estimate) link sets.
- Given:
  - Estimated antenna configuration
  - Estimated prices (dual multipliers)

Antenna Reconfiguration Problem

- Choose (estimate) antenna configuration.
- Given:
  - Estimated link selection
  - Estimated prices (dual multipliers)

Subgradient / Relaxed Primal Problem method

Solution of dual problem:

Link Activation Problem

- Choose (estimate) link sets.
- Given:
  - Estimated antenna configuration
  - Estimated prices (dual multipliers)

Antenna Reconfiguration Problem

- Choose (estimate) antenna configuration.
- Given:
  - Estimated link selection
  - Estimated prices (dual multipliers)

Subgradient / Relaxed Primal Problem method

Solution of dual problem:

Link Activation Problem

- Choose (estimate) link sets.
- Given:
  - Estimated antenna configuration
  - Estimated prices (dual multipliers)

Antenna Reconfiguration Problem

- Choose (estimate) antenna configuration.
- Given:
  - Estimated link selection
  - Estimated prices (dual multipliers)

Subgradient / Relaxed Primal Problem method

Solution of dual problem:

Link Activation Problem

- Choose (estimate) link sets.
- Given:
  - Estimated antenna configuration
  - Estimated prices (dual multipliers)

Antenna Reconfiguration Problem

- Choose (estimate) antenna configuration.
- Given:
  - Estimated link selection
  - Estimated prices (dual multipliers)

Combined estimates may not satisfy complicating constraints.

Subgradient / Relaxed Primal Problem method

Solution of dual problem:

Link Activation Problem

- Choose (estimate) link sets.
- Given:
  - Estimated antenna configuration
  - Estimated prices (dual multipliers)

Antenna Reconfiguration Problem

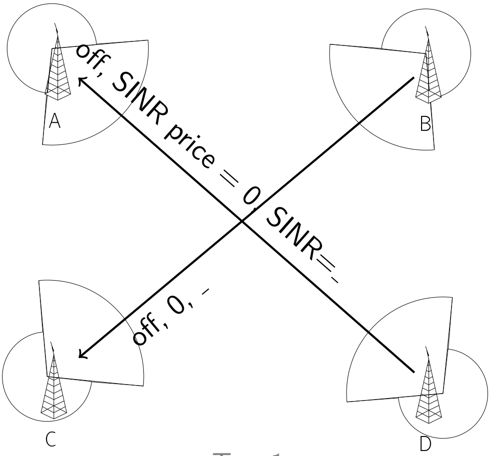
- Choose (estimate) antenna configuration.
- Given:
  - Estimated link selection
  - Estimated prices (dual multipliers)

Combined estimates may not satisfy complicating constraints.
Non-compliance determines subgradient. Update price estimates.

Example (simplified)

node	value
B	0
C	0
D	0

node	value
A	0
C	0
D	0



node	value
A	0
B	0
D	0

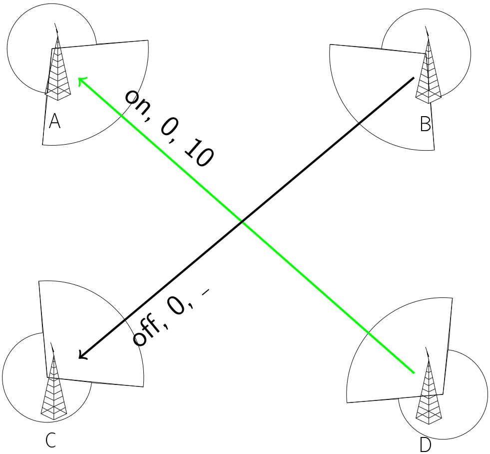
node	value
A	0
B	0
C	0

$T = 1$

Example (simplified)

node	value
B	0
C	0
D	0

node	value
A	0
C	0
D	0



node	value
A	0
B	0
D	0

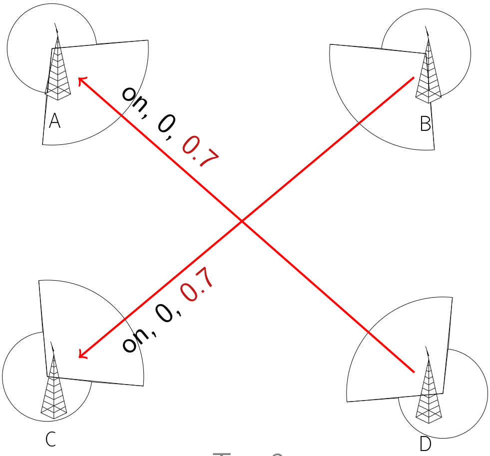
node	value
A	0
B	0
C	0

$T = 2$

Example (simplified)

node	value
B	0
C	0
D	0

node	value
A	0
C	0
D	0



node	value
A	0
B	0
D	0

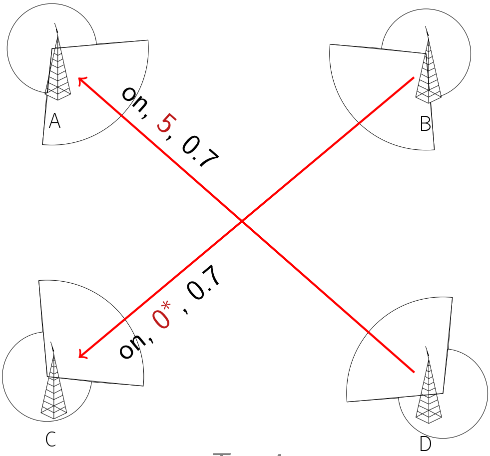
node	value
A	0
B	0
C	0

$T = 3$

Example (simplified)

node	value
B	0
C	0
D	5

node	value
A	-5
C	0
D	0



node	value
A	0
B	0
D	0

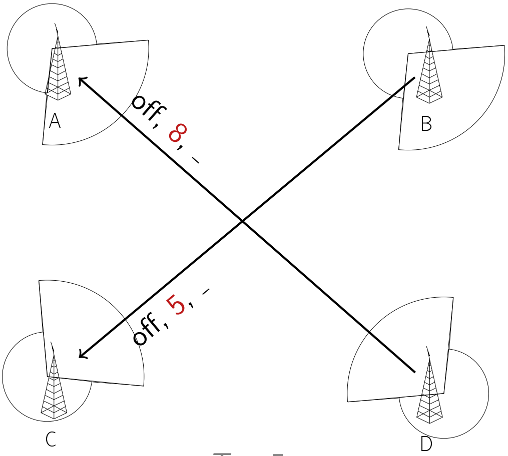
node	value
A	5
B	0
C	0

$T = 4$

Example (simplified)

node	value
B	0
C	0
D	8

node	value
A	-8
C	5
D	0



node	value
A	0
B	5
D	0

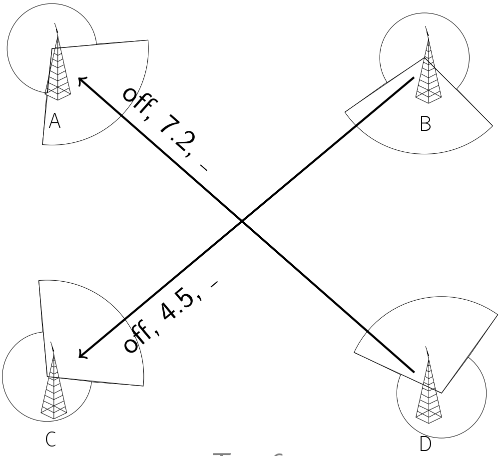
node	value
A	8
B	0
C	-5

$T = 5$

Example (simplified)

node	value
B	0
C	0
D	7.2

node	value
A	-7.2
C	4.5
D	0



node	value
A	0
B	4.5
D	0

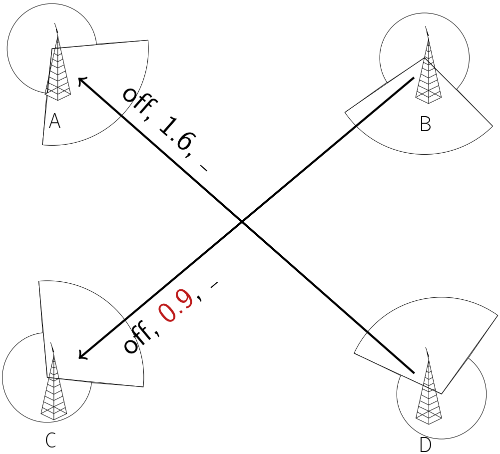
node	value
A	7.2
B	0
C	-4.5

$T = 6$

Example (simplified)

node	value
B	0
C	0
D	1.6

node	value
A	-1.6
C	0.9
D	0



node	value
A	0
B	0.9
D	0

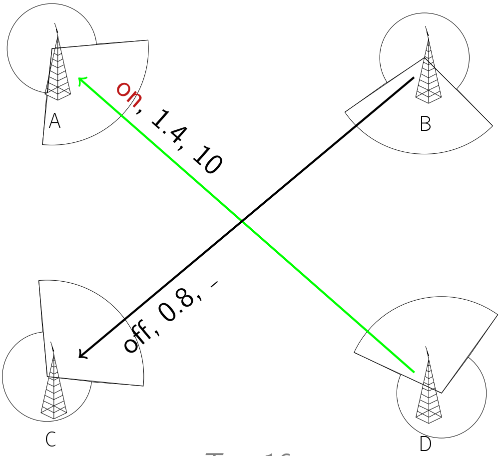
node	value
A	1.6
B	0
C	-0.9

$T = 15$

Example (simplified)

node	value
B	0
C	0
D	1.4

node	value
A	-1.4
C	0.8
D	0



node	value
A	0
B	0.8
D	-0.8

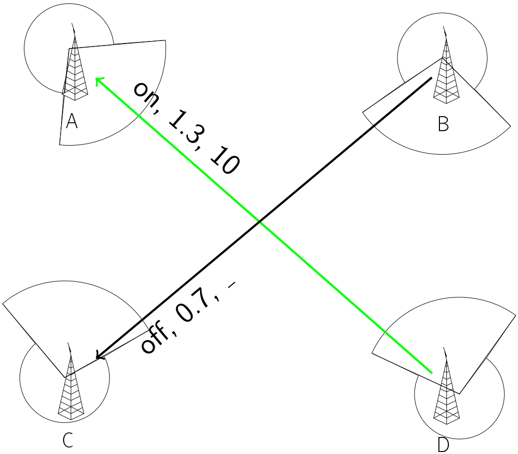
node	value
A	1.4
B	0
C	-0.8

$T = 16$

Example (simplified)

node	value
B	0
C	0
D	1.3

node	value
A	-1.3
C	0.7
D	0



node	value
A	0
B	0.7
D	-0.7

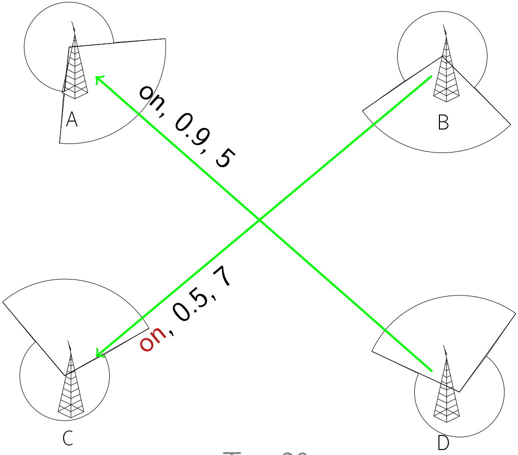
node	value
A	1.3
B	0
C	-0.7

$T = 17$

Example (simplified)

node	value
B	-0.9
C	0
D	0.9

node	value
A	-0.9
C	0.5
D	0



node	value
A	0
B	0.5
D	-0.5

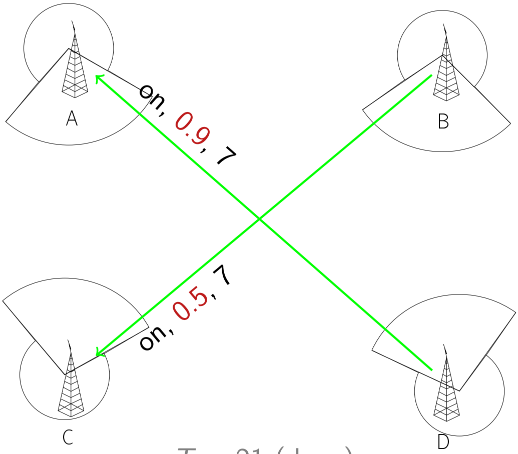
node	value
A	0.9
B	0
C	-0.5

$T = 20$

Example (simplified)

node	value
B	-0.8
C	0
D	0.8

node	value
A	-0.8
C	0.5
D	0



node	value
A	0
B	0.5
D	-0.5

node	value
A	0.8
B	0
C	-0.5

$T = 21$ (done)

Proof-of-Concept System

Dual problem: Node-local price and configuration estimates, distributed consensus algorithm.

- Asynchronous
- Delay- and loss-tolerant
- Eventually consistent

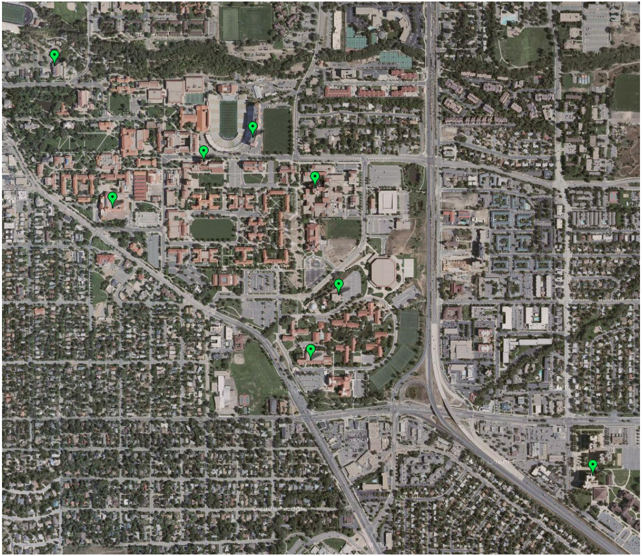
Global “restricted master” problem, flooding updates.

- Passively observes dual problem results.
- Recomputes (global) schedule when possible.
- Local computation, but requires global data.

Implemented on top of 802.11 PHY with STDMA MAC using switched-beam phased array antennas.

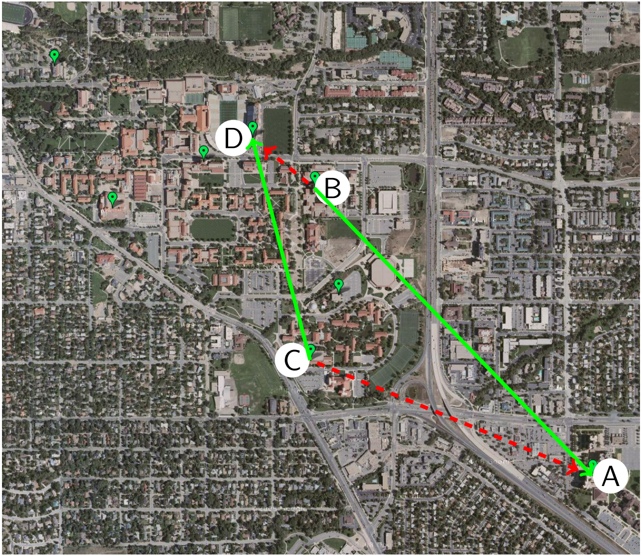
Experimental Test Bed

Phase array antennas installed around C.U. campus



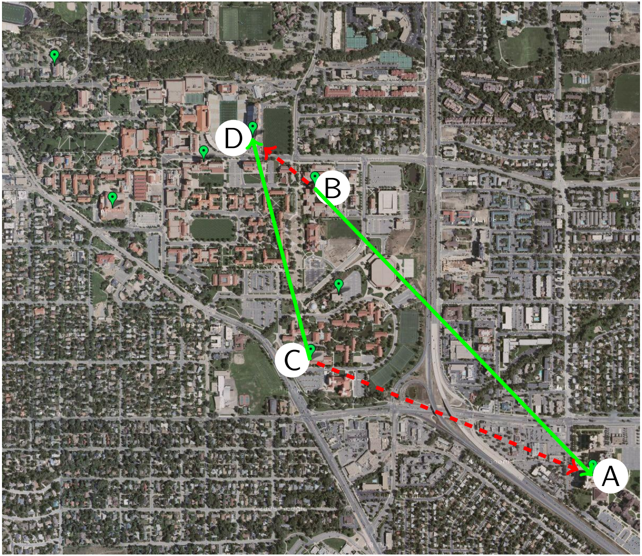
Test Load

- TDMA: 2 slots
- Best case: 1 slot
- Incompatible when using "obvious" antennas
- Algorithm achieves best case



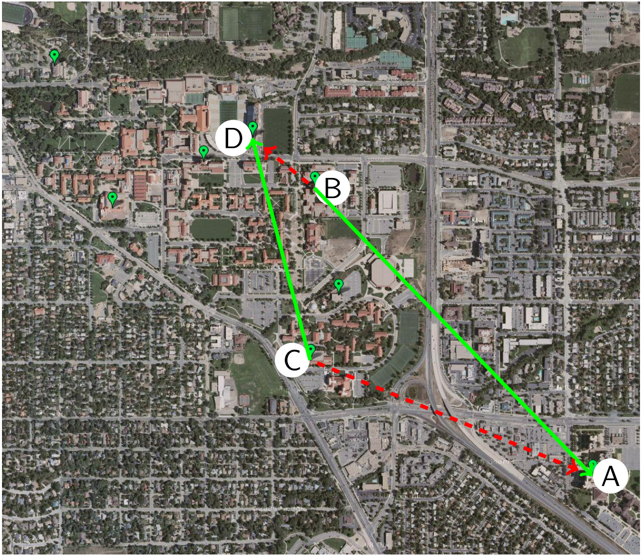
Test Load

- TDMA: 2 slots
- Best case: 1 slot
- Incompatible when using "obvious" antennas
- Algorithm achieves best case

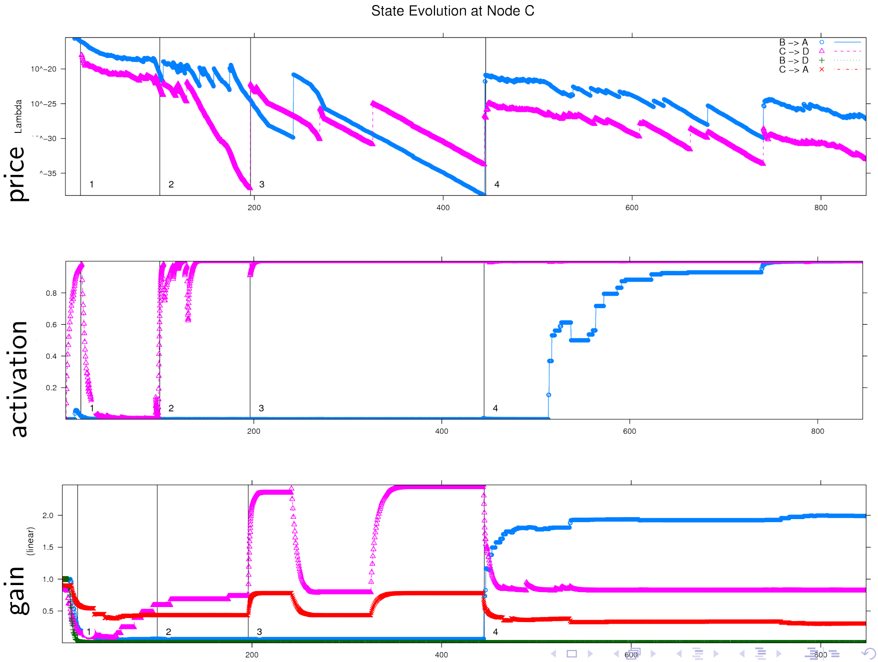


Test Load

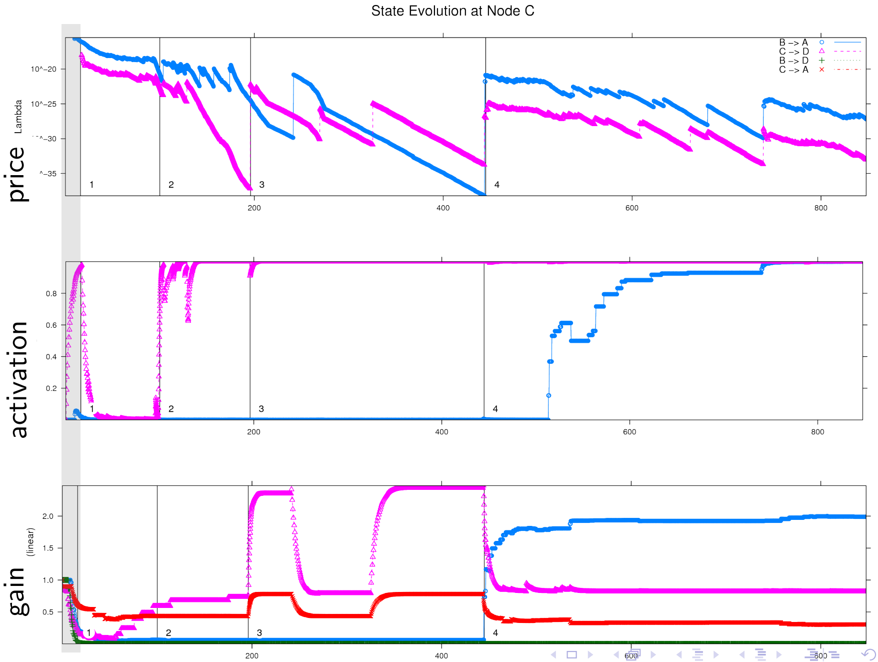
- TDMA: 2 slots
- Best case: 1 slot
- Incompatible when using "obvious" antennas
- Algorithm achieves best case



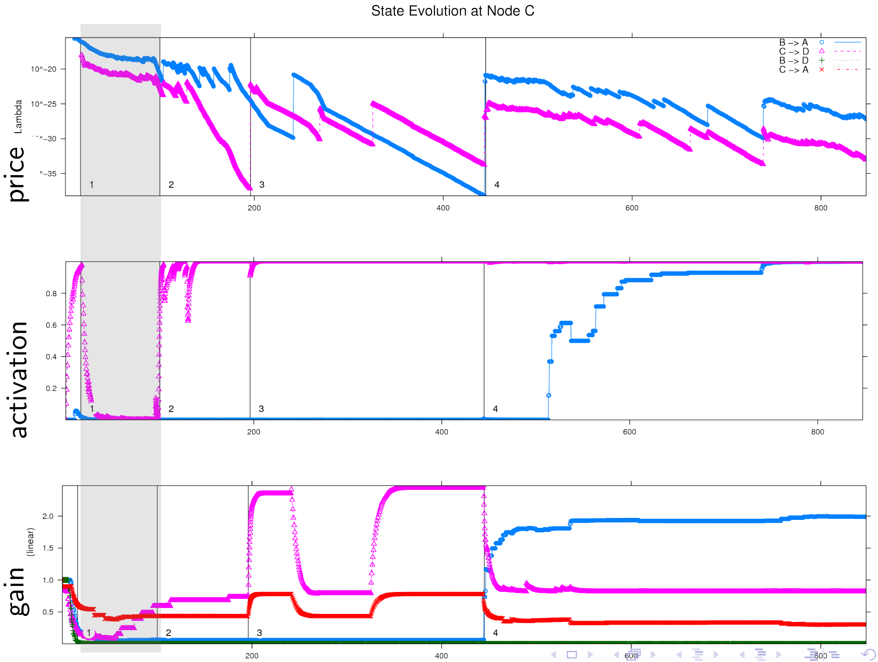
Execution Trace at Node C



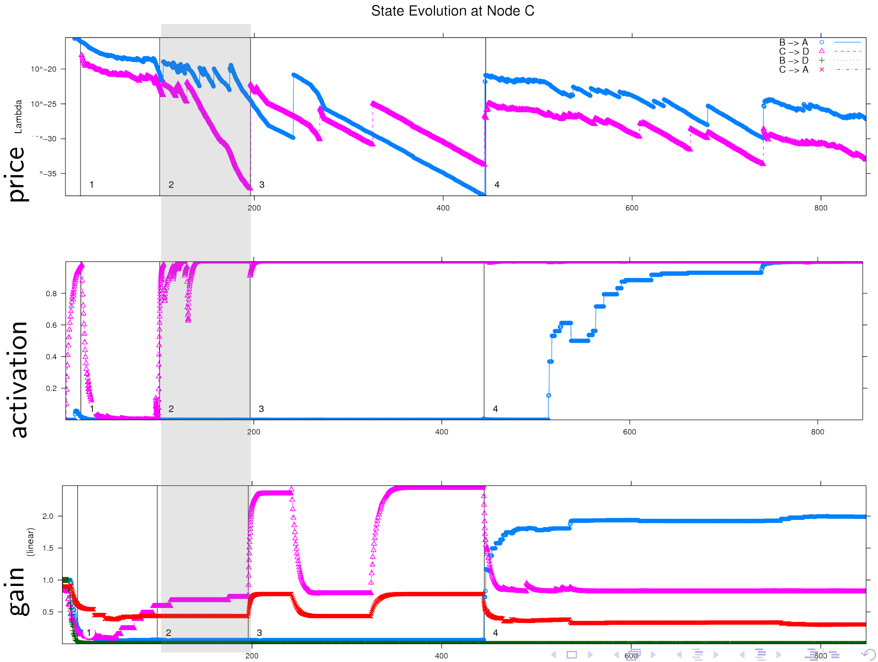
Execution Trace at Node C



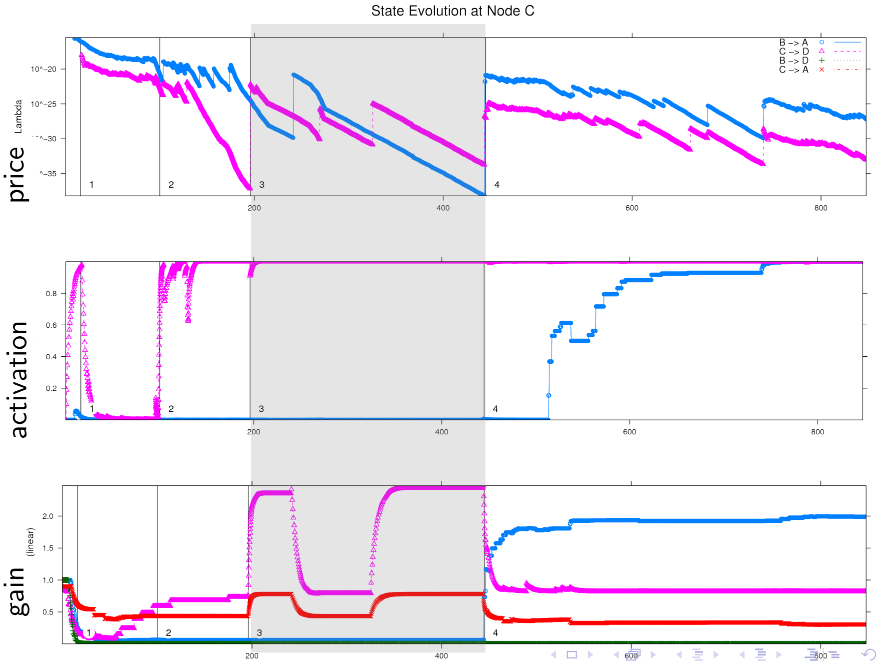
Execution Trace at Node C



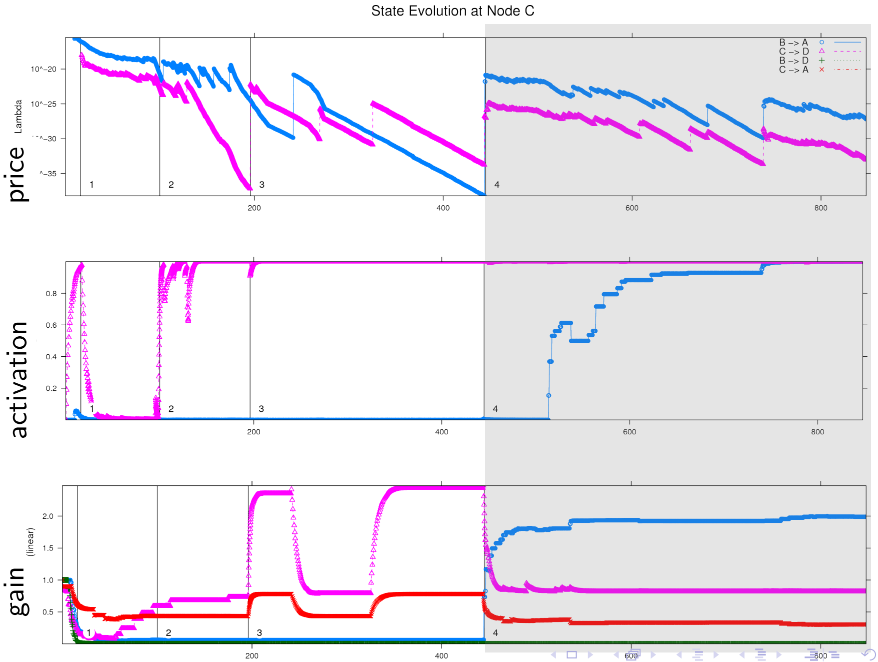
Execution Trace at Node C



Execution Trace at Node C



Execution Trace at Node C

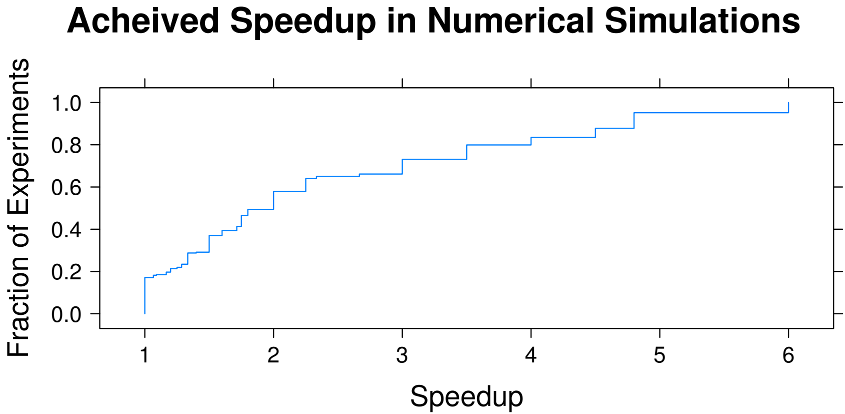


Numerical Results

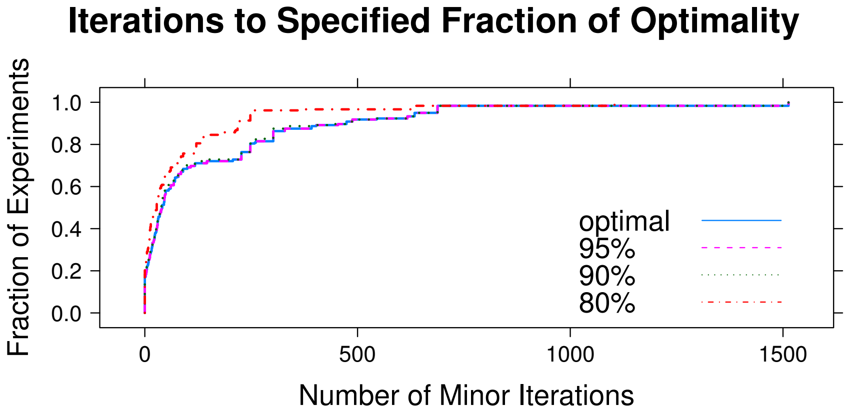
Numerical experiments: 1400 varying scenarios

- Number of nodes (2 - 48)
- Link density (1/2 - 3 per node)
- Size of simulated area (1 - 16 sq. km)
- Random seed

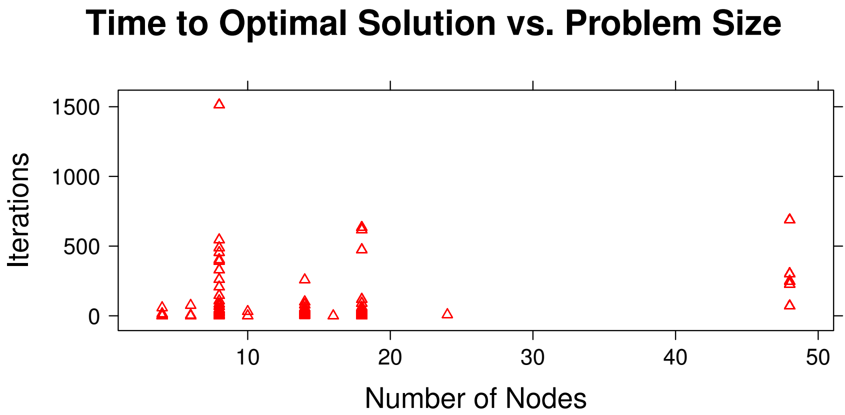
Improvement



Running Time 1



Running Time 2



Conclusions

- Tractable solution to optimal joint beam steering and scheduling
- Mean 234% speedup over simple TDMA
- Mean 150 iterations to optimality (90th %ile: 500)

- Dual-decomposition based scheduling works in practice
- More responsive on-line MAC in progress

Thank you!

Backup Slides

References I

Erdal Arikan.
Some Complexity Results about Packet Radio Networks.
IEEE Transactions on Information Theory, vol. 30, no. 4, pages 681 – 685, July 1984.

Patrick Björklund, Peter Varbrand & Di Yuan.
Resource optimization of spatial TDMA in ad hoc radio networks: a column generation approach.
In INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications Societies, volume 2, pages 818– 824. IEEE, March 2003.

Gurashish Brar, Douglas M. Blough & Paolo Santi.
Computationally efficient scheduling with the physical interference model for throughput improvement in wireless mesh networks.
In MobiCom '06: Proceedings of the 12th annual international conference on Mobile computing and networking, pages 2–13, New York, NY, USA, 2006. ACM.

J. Bibb Cain, Tom Billhartz, Larry Foore, Edwin Althouse & John Schlorff.
A link scheduling and ad hoc networking approach using directional antennas.
In Military Communications Conference, 2003, volume 1, pages 643– 648. IEEE, Oct 2003.

Lijun Chen, Steven H. Low, Mung Chiang & John C. Doyle.
Optimal Cross-layer Congestion Control, Routing and Scheduling Design in Ad Hoc Wireless Networks.
In Proceedings of IEEE INFOCOM 2006, 2006.

Imrich Chlamtac & Anat Lerner.
Fair Algorithms for Maximal Link Activation in Multihop Radio Networks.
Communications, IEEE Transactions on, vol. 35, no. 7, pages 739–746, Jul 1987.

References II

Ashish Deopura & Aura Ganz.
Provisioning link layer proportional service differentiation in wireless networks with smart antennas.
Wirel. Netw., vol. 13, no. 3, pages 371–378, 2007.

K. Dyberg, F. Farman L.and Eklof, J. Grönkvist, U. Sterner & J. Rantakokko.
On the performance of antenna arrays in spatial reuse TDMA ad hoc networks.
In MILCOM 2002, volume 1, pages 270– 275, Oct 2002.

Anthony Ephremides & Thuan V. Truong.
Scheduling broadcasts in multihop radio networks.
Communications, IEEE Transactions on, vol. 38, no. 4, pages 456–460, Apr 1990.

Jimmi Grönkvist.
Assignment methods for spatial reuse TDMA.
In MobiHoc '00: Proceedings of the 1st ACM international symposium on Mobile ad hoc networking & computing, pages 119–124, Piscataway, NJ, USA, 2000. IEEE Press.

Bruce Hajek & Galen Sasaki.
Link scheduling in polynomial time.
Information Theory, IEEE Transactions on, vol. 34, no. 5, pages 910–917, Sep 1988.

M. Johansson & L. Xiao.
Cross-layer optimization of wireless networks using nonlinear column generation.
Wireless Communications, IEEE Transactions on, vol. 5, no. 2, pages 435–445, 2006.

Tzu-Ming Lin & Juin-Jia Dai.
A Collision Free MAC Protocol using Smart Antenna in Ad Hoc Networks.
In CCNC 2004, Jan 2004.

References III

Xi Liu, Anmol Sheth, Michael Kaminski, Konstantina Papagiannaki, Srinivasan Seshan & Peter Steenkiste.
DIRC: Increasing Indoor Wireless Capacity Using Directional Antennas.
In Proc. SIGCOMM 2009, pages 171 – 182, Barcelona, Spain, August 2009. ACM.

Randolph Nelson & Leonard Kleinrock.
Spatial TDMA: A Collision-Free Multihop Channel Access Protocol.
Communications, IEEE Transactions on, vol. 33, no. 9, pages 934– 944, Sep 1985.

B. Radunović & J.-Y. Le Boudec.
Optimal power control, scheduling, and routing in UWB networks.
Selected Areas in Communications, IEEE Journal on, vol. 22, no. 7, pages 1252–1270, Sept. 2004.

Marvin Sánchez & Jens Zander.
Adaptive Antennas in Spatial TDMA Multihop Packet Radio Networks.
In RadioVetenskap och Kommunikation (RVK), Karlskrona, Sweden, June 1999.

Karthikeyan Sundaresan, Weizhao Wang & Stephan Eidenbenz.
Algorithmic aspects of communication in ad-hoc networks with smart antennas.
In MobiHoc '06: Proceedings of the 7th ACM international symposium on Mobile ad hoc networking and computing, pages 298–309, New York, NY, USA, 2006. ACM.

Karthikeyan Sundaresan & Raghupathy Sivakumar.
A unified MAC layer framework for ad-hoc networks with smart antennas.
IEEE/ACM Trans. Netw., vol. 15, no. 3, pages 546–559, 2007.

Master Problem (JBSS-MP)

Minimize total time

- Allocate sufficient time to each link
- Half-duplex unicast operation
- SINR on active links
- Antenna selection convexity
- Gain-antenna coupling

$$\min \sum_{l \in L_A} x_l$$

s.t.

$$\sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall_{i,j}$$

$$\sum_{j:(i,j) \in A} S_{ijl} + \sum_{j:(j,i) \in A} S_{jil} \leq 1 \quad \forall_{i,l}$$

$$\left.\begin{array}{r} \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j) N_r} S_{ijl} \; + \\ \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \; \geq \\ \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j) N_r} V_{kl} \right) \end{array}\right\} \quad \forall_{i,j,l}$$

$$S_{ijl} \leq V_{il} \quad \forall_{i,j,l}$$

$$\sum_{p \in P} B_{jpl} = 1 \quad \forall_{j,l}$$

$$D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall_{i,k,l}$$

$$x_l \geq 0 \quad \forall_{l \in L_A}$$

$$S_{ijl}, \; B_{jpl} \in \{0,1\}$$

Master Problem (JBSS-MP)

Minimize total time → $\min \sum_{l \in L_A} x_l$

- Allocate sufficient time to each link
- Half-duplex unicast operation
- SINR on active links
- Antenna selection convexity
- Gain-antenna coupling

s.t.

$$\sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall_{i,j}$$

$$\sum_{j:(i,j)\in A} S_{ijl} + \sum_{j:(j,i)\in A} S_{jil} \leq 1 \quad \forall_{i,l}$$

$$\left.\begin{array}{r} \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j) N_r} S_{ijl} \; + \\ \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \; \geq \\ \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j) N_r} V_{kl}\right) \end{array}\right\} \quad \forall_{i,j,l}$$

$$S_{ijl} \leq V_{il} \quad \forall_{i,j,l}$$

$$\sum_{p \in P} B_{jpl} = 1 \quad \forall_{j,l}$$

$$D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall_{i,k,l}$$

$$x_l \geq 0 \quad \forall_{l \in L_A}$$

$$S_{ijl},\; B_{jpl} \in \{0,1\}$$

Master Problem (JBSS-MP)

Minimize total time

- Allocate sufficient time to each link
- Half-duplex unicast operation
- SINR on active links
- Antenna selection convexity
- Gain-antenna coupling

$$\min \sum_{l \in L_A} x_l$$

s.t.

$$\sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall_{i,j}$$

$$\sum_{j:(i,j)\in A} S_{ijl} + \sum_{j:(j,i)\in A} S_{jil} \leq 1 \quad \forall_{i,l}$$

$$\left.\begin{array}{r} \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j)N_r} S_{ijl} \ + \\ \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \ \geq \\ \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j)N_r} V_{kl}\right) \end{array}\right\} \quad \forall_{i,j,l}$$

$$S_{ijl} \leq V_{il} \quad \forall_{i,j,l}$$

$$\sum_{p \in P} B_{jpl} = 1 \quad \forall_{j,l}$$

$$D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall_{i,k,l}$$

$$x_l \geq 0 \quad \forall_{l \in L_A}$$

$$S_{ijl},\ B_{jpl} \in \{0,1\}$$

Master Problem (JBSS-MP)

Minimize total time

- Allocate sufficient time to each link
- Half-duplex unicast operation
- SINR on active links
- Antenna selection convexity
- Gain-antenna coupling

$$
\begin{aligned}
\min \quad & \sum_{l \in L_A} x_l \\
\text{s.t.} \quad & \sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall_{i,j} \\
& \sum_{j:(i,j)\in A} S_{ijl} + \sum_{j:(j,i)\in A} S_{jil} \leq 1 \quad \forall_{i,l} \\
& \left.\begin{aligned} & \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j) N_r} S_{ijl} \; + \\ & \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \; \geq \\ & \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j) N_r} V_{kl} \right) \end{aligned}\right\} \quad \forall_{i,j,l} \\
& S_{ijl} \leq V_{il} \quad \forall_{i,j,l} \\
& \sum_{p \in P} B_{jpl} = 1 \quad \forall_{j,l} \\
& D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall_{i,k,l} \\
& x_l \geq 0 \quad \forall_{l \in L_A} \\
& S_{ijl},\; B_{jpl} \in \{0,1\}
\end{aligned}
$$

Master Problem (JBSS-MP)

Minimize total time

- Allocate sufficient time to each link
- Half-duplex unicast operation
- SINR on active links
- Antenna selection convexity
- Gain-antenna coupling

$$\min \sum_{l \in L_A} x_l$$

s.t.

$$\sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall_{i,j}$$

$$\sum_{j:(i,j)\in A} S_{ijl} + \sum_{j:(j,i)\in A} S_{jil} \leq 1 \quad \forall_{i,l}$$

$$\left.\begin{array}{r} \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j) N_r} S_{ijl} \;+ \\ \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \;\geq \\ \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j) N_r} V_{kl}\right) \end{array}\right\} \quad \forall_{i,j,l}$$

$$S_{ijl} \leq V_{il} \quad \forall_{i,j,l}$$

$$\sum_{p \in P} B_{jpl} = 1 \quad \forall_{j,l}$$

$$D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall_{i,k,l}$$

$$x_l \geq 0 \quad \forall_{l \in L_A}$$

$$S_{ijl},\; B_{jpl} \in \{0,1\}$$

Master Problem (JBSS-MP)

Minimize total time

- Allocate sufficient time to each link
- Half-duplex unicast operation
- SINR on active links
- Antenna selection convexity
- Gain-antenna coupling

$$\min \sum_{l \in L_A} x_l$$

s.t.

$$\sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall_{i,j}$$

$$\sum_{j:(i,j)\in A} S_{ijl} + \sum_{j:(j,i)\in A} S_{jil} \leq 1 \quad \forall_{i,l}$$

$$\left.\begin{array}{r} \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j)N_r} S_{ijl} \; + \\ \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \; \geq \\ \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j)N_r} V_{kl}\right) \end{array}\right\} \quad \forall_{i,j,l}$$

$$S_{ijl} \leq V_{il} \quad \forall_{i,j,l}$$

$$\sum_{p \in P} B_{jpl} = 1 \quad \forall_{j,l}$$

$$D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall_{i,k,l}$$

$$x_l \geq 0 \quad \forall_{l \in L_A}$$

$$S_{ijl},\; B_{jpl} \in \{0,1\}$$

Master Problem (JBSS-MP)

Minimize total time

- Allocate sufficient time to each link
- Half-duplex unicast operation
- SINR on active links
- Antenna selection convexity
- Gain-antenna coupling

$$\min \sum_{l \in L_A} x_l$$

s.t.

$$\sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall_{i,j}$$

$$\sum_{j:(i,j)\in A} S_{ijl} + \sum_{j:(j,i)\in A} S_{jil} \leq 1 \quad \forall_{i,l}$$

$$\left.\begin{array}{r} \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j) N_r} S_{ijl} \; + \\ \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \; \geq \\ \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j) N_r} V_{kl}\right) \end{array}\right\} \quad \forall_{i,j,l}$$

$$S_{ijl} \leq V_{il} \quad \forall_{i,j,l}$$

$$\sum_{p \in P} B_{jpl} = 1 \quad \forall_{j,l}$$

$$D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall_{i,k,l}$$

$$x_l \geq 0 \quad \forall_{l \in L_A}$$

$$S_{ijl}, \; B_{jpl} \in \{0,1\}$$

Master Problem (JBSS-MP)

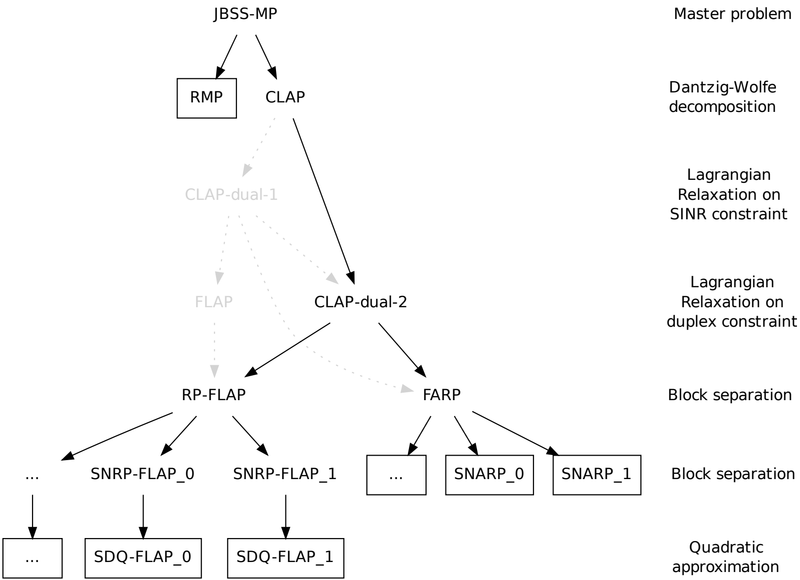
Symbol	Interpretation
S_{ij}	Activation of link ij
V_i	Node i is active (in current link set)
D_{ij}	Directivity of node i toward j
B_{jp}	Indicator: beam p used at node j

Table: Key Notation

$$
\begin{aligned}
\min \quad & \sum_{l \in L_A} x_l \\
\text{s.t.} \quad & \sum_{l \in L_A} S_{ijl} x_l \geq q_{ij} \quad \forall_{i,j} \\
& \sum_{j:(i,j)\in A} S_{ijl} + \sum_{j:(j,i)\in A} S_{jil} \leq 1 \quad \forall_{i,l} \\
& \left.\begin{array}{r} \frac{P_{il} D_{ijl} D_{jil}}{Lb(i,j) N_r} S_{ijl} \; + \\ \gamma_1 (1 + M_{ijl})(1 - S_{ijl}) \; \geq \\ \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_{kl} D_{kjl} D_{jkl}}{Lb(k,j) N_r} V_{kl}\right) \end{array}\right\} \quad \forall_{i,j,l} \\
& S_{ijl} \leq V_{il} \quad \forall_{i,j,l} \\
& \sum_{p \in P} B_{jpl} = 1 \quad \forall_{j,l} \\
& D_{ik} = \sum_{p \in P} G_{ikp} B_{ipl} \quad \forall_{i,k,l} \\
& x_l \geq 0 \quad \forall_{l \in L_A} \\
& S_{ijl}, \; B_{jpl} \in \{0,1\}
\end{aligned}
$$

▸ full notation

Decomposition Approach – Detailed



Dantzig-Wolfe Decomposition

Restricted Master Problem (RMP)

Given feasible link sets, allocates time to each.

Produces capacity constraint dual values ($\bar{\beta}$).

Subproblem

Given $\bar{\beta}$, finds improving link set.

Subproblem Complexity

	Functional	Degree	Variables S	V	D	B
Objective:	Reduced-Cost Column	1	✓			
Constraint:	Duplex	1	✓			
	Coupling	1	✓	✓		
	SINR	3	✓	✓	✓	
	Antenna coupling	1			✓	✓
	Antenna uniqueness	1				✓

Dantzig-Wolfe Decomposition

Restricted Master Problem (RMP)
Given feasible link sets, allocates time to each.
Produces capacity constraint dual values ($\bar{\beta}$).

Subproblem
Given $\bar{\beta}$, finds improving link set.

Subproblem Complexity

	Functional	Degree	Variables S	V	D	B
Objective:	Reduced-Cost Column	1	✓			
Constraint:	Duplex	1	✓			
	Coupling	1	✓	✓		
	SINR	3	✓	✓	✓	
	Antenna coupling	1			✓	✓
	Antenna uniqueness	1				✓

Dantzig-Wolfe Decomposition

Restricted Master Problem (RMP)
Given feasible link sets, allocates time to each.
Produces capacity constraint dual values ($\bar{\beta}$).

Subproblem
Given $\bar{\beta}$, finds improving link set.

Subproblem Complexity

	Functional	Degree	Variables S	V	D	B
Objective:	Reduced-Cost Column	1	✓			
Constraint:	Duplex	1	✓			
	Coupling	1	✓	✓		
	SINR	3	✓	✓	✓	
	Antenna coupling	1			✓	✓
	Antenna uniqueness	1				✓

Dantzig-Wolfe Decomposition

Restricted Master Problem (RMP)
Given feasible link sets, allocates time to each.
Produces capacity constraint dual values ($\bar{\beta}$).

Subproblem
Given $\bar{\beta}$, finds improving link set.

Subproblem Complexity

	Functional	Degree	Variables			
			S	V	D	B
Objective:	Reduced-Cost Column	1	✓			
Constraint:	Duplex	1	✓			
	Coupling	1	✓	✓		
	SINR	3	✓	✓	✓	
	Antenna coupling	1			✓	✓
	Antenna uniqueness	1				✓

Lagrangian Dual Problem

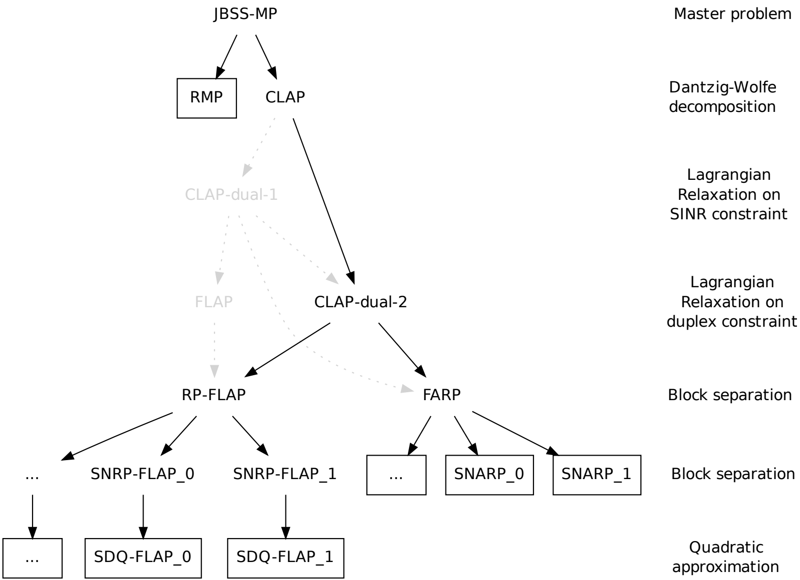
SINR and duplex constraints relaxed; multipliers are λ, μ. Constraint functionals are $d^s()$ and $d^d()$.

$$\mathcal{L}'(S, D, V, \lambda, \mu) = \bar{\beta}^T S - \lambda^T d^s(S, D, V) - \mu^T d^d(S)$$
$$\phi'(\lambda, \mu) = \max_{S,D,V} \mathcal{L}'(S, D, V, \lambda, \mu)$$

[CLAP-dual-2]

$$\begin{aligned} \min_{\lambda,\mu} \quad & \phi'(\lambda, \mu) & \\ \text{s.t.} \quad & S_{ij} \leq V_i & \forall_i \\ & D_{ik} = \sum_{p \in P} G_{ikp} B_{ip} & \forall_{i,k} \\ & \sum_{p \in P} B_{jp} = 1 & \forall_j \end{aligned}$$

Decomposition Approach – Detailed



Block Separation

$$\begin{aligned} \min_{x} \quad & f(x_1, x_2) \\ \text{s.t.} \quad & g_1(x) \leq c_1 \\ & g_2(x) \leq c_2 \end{aligned} \quad \leftrightarrow \quad \begin{aligned} \min_{x_1} \quad & f_1(x_1) \\ \text{s.t.} \quad & g_1(x_1) \leq c_1 \end{aligned} \qquad \begin{aligned} \min_{x_2} \quad & f_2(x_2) \\ \text{s.t.} \quad & g_2(x_2) \leq c_2 \end{aligned}$$

Relaxed Primal Fixed-antenna Link Assignment Problem (RP-FLAP)

[RP-FLAP]

$$
\begin{aligned}
\max_{S,V} \quad & \bar{\beta}^T S + \bar{\lambda}^T d'^s(S,V) - \bar{\mu}^T d^d(S) \\
\text{s.t.} \quad & S_{ij} \leq V_i \quad \forall_{ij}
\end{aligned}
$$

Fixed-Link Antenna Reconfiguration Problem (FARP)

[FARP]

$$\max_{D,B} \left\{ \begin{array}{r} \bar{\beta}^T \bar{S} - \sum_{ij} \bar{\lambda}_{ij} \left(\frac{P_i D_{ij} D_{ji}}{Lb(i,j) N_r} \bar{S}_{ij} + \right. \\ \gamma_1 (1 + M_{ij})(1 - \bar{S}_{ij}) \quad - \\ \left. \gamma_1 \left(1 + \sum_{k \in N \setminus \{i,j\}} \frac{P_k D_{kj} D_{kj}}{Lb(k,j) N_r} \bar{V}_k \right) \right) \end{array} \right.$$

$$\text{s.t.} \quad D_{ik} - \sum_{p \in P} G_{ikp} B_{ip} = 0 \quad \forall_{i,k}$$

$$\sum_{p \in P} B_{ip} = 1 \quad \forall_i$$

Single-Node Antenna Reconfiguration Problem (SNARP) I

Let x denote the vector of all antenna gains D. Now let i partition x as: $x_i = \cup_{k \neq i} D_{ik}$.

$$g_i(x) = \begin{cases} \sum_j \left(\frac{1}{2} \bar{\lambda}_{ij} \bar{S}_{ij} \frac{P_i}{Lb(i,j) N_r} D_{ij} \bar{D}_{ji} \right) + \frac{k}{|N|} & \text{if } i \text{ is a transmitter} \\ \sum_j \left(\frac{1}{2} \bar{\lambda}_{ji} \bar{S}_{ji} \frac{P_j}{Lb(j,i) N_r} \bar{D}_{ji} D_{ij} \right) + \frac{k}{|N|} & \text{if } i \text{ is a receiver} \end{cases}$$

$$h_i(x) = \begin{cases} \sum_j \left(\sum_{k,l \in N \setminus \{i,j\}} \left(\frac{1}{2} \gamma_1 \bar{S}_{ij} \bar{\lambda}_{kl} \frac{P_i}{Lb(i,l) N_r} D_{il} \bar{D}_{li} \right) \right) & \text{if } i \text{ is a transmitter} \\ \sum_j \left(\sum_{k,l \in N \setminus \{i,j\}} \left(\frac{1}{2} \gamma_1 \bar{S}_{ji} \bar{\lambda}_{ji} \frac{P_k}{Lb(k,i) N_r} \bar{D}_{ki} D_{ik} \right) \right) & \text{if } i \text{ is a receiver} \end{cases}$$

$$f_i(x) = g_i(x_i) - h_i(x)$$

$$f(x) = \sum_i f_i(x) \text{ given } \sum_j \bar{S}_{ij} \leq V_i \quad \forall_i$$

Single-Node Antenna Reconfiguration Problem (SNARP) II

[SNARP$_i$]

$$
\begin{aligned}
\max_{D,B} \quad & 1 - f_i(D) & & \\
\text{s.t.} \quad & D_{ik} - \sum_{p \in P} G_{ikp} B_{ip} &= 0 & \quad \forall_k \\
& \sum_{p \in P} B_{ip} &= 1 & \\
& B_{ip} &\leq 1 & \quad \forall_{p \in P} \\
& B_{ip} &\geq 0 & \quad \forall_{p \in P}
\end{aligned}
$$

Mathematical Components

Per-node:

- Link activation problem
- Antenna configuration problem
- Incremental subgradient calculation
- Primal estimate sequence

Inter-node exchange of:

- Primal and dual estimates

Distributed, asynchronous, incremental optimization process

Interference-Limited Wireless Networks

Shannon capacity of a narrowband Gaussian channel is given by:

$$C = B \log_2 (1 + \frac{P}{N}) \quad (1)$$

- B is a fixed resource.
- P has practical and regulatory limits.
- Your P may be someone else's N.

Interference-Limited Wireless Networks

Shannon capacity of a narrowband Gaussian channel is given by:

$$C = B \log_2 (1 + \frac{P}{N}) \tag{1}$$

- B is a fixed resource.
- P has practical and regulatory limits.
- Your P may be someone else's N.

Interference-Limited Wireless Networks

Shannon capacity of a narrowband Gaussian channel is given by:

$$C = B \log_2 (1 + \frac{P}{N}) \tag{1}$$

- B is a fixed resource.
- P has practical and regulatory limits.
- Your P may be someone else's N.

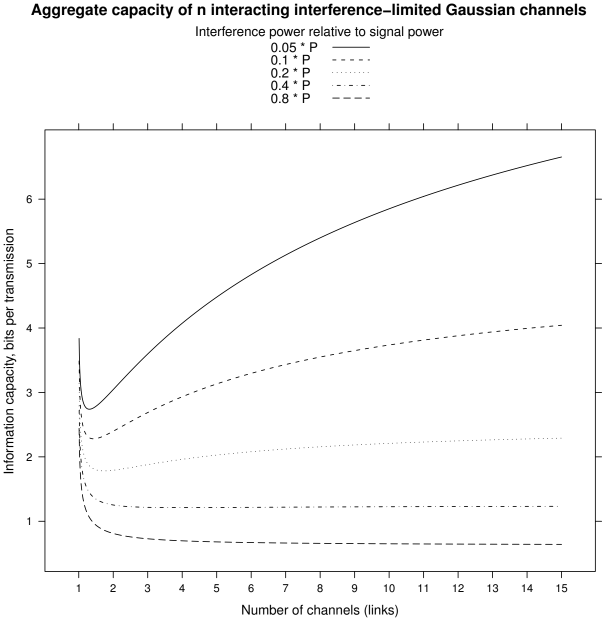
Interference-Limited Wireless Networks

Shannon capacity of a narrowband Gaussian channel is given by:

$$C = B \ \log_2 \ (1 + \frac{P}{N}) \tag{1}$$

- B is a fixed resource.
- P has practical and regulatory limits.
- Your P may be someone else's N.

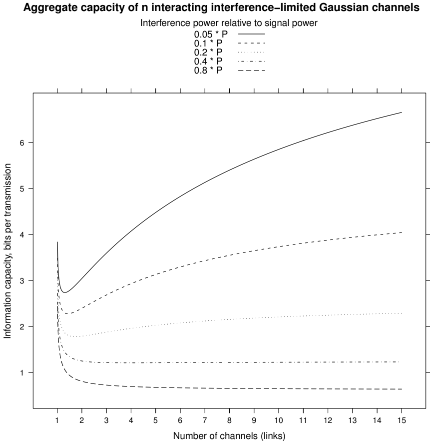
Interference-Limited Wireless Networks



Interference-Limited Wireless Networks

Absent some other bottleneck, Signal-to-Interference and Noise Ratio (SINR) limits throughput.

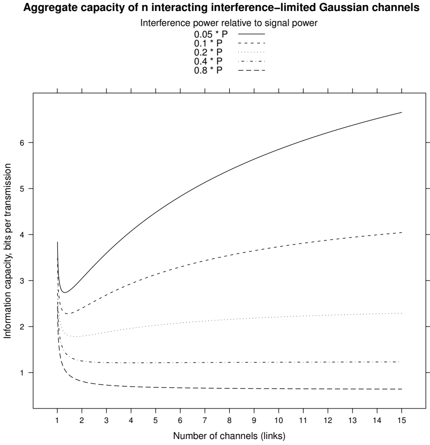
- Concurrent links increase total capacity,
- *If* the links don't unduly interfere with each other.
- *Identify* or *create* low mutual-interference link sets.



Interference-Limited Wireless Networks

Absent some other bottleneck, Signal-to-Interference and Noise Ratio (SINR) limits throughput.

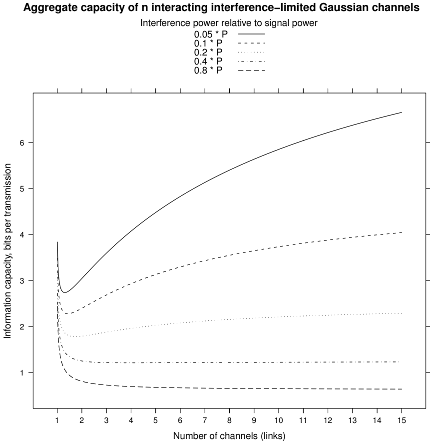
- Concurrent links increase total capacity,
- *If* the links don't unduly interfere with each other.
- *Identify* or *create* low mutual-interference link sets.



Interference-Limited Wireless Networks

Absent some other bottleneck, Signal-to-Interference and Noise Ratio (SINR) limits throughput.

- Concurrent links increase total capacity,
- *If* the links don't unduly interfere with each other.
- *Identify* or *create* low mutual-interference link sets.



Spatial-Reuse TDMA (STDMA)

Goal: Select *sets* of links or broadcasts such that spatial separation minimizes interference.

- Old idea: (goes back to [Nelson 85]).
- *Which* sets?
- *How much* time for each?
- *What* configuration?

Spatial-Reuse TDMA (STDMA)

Goal: Select *sets* of links or broadcasts such that spatial separation minimizes interference.

- Old idea: (goes back to [Nelson 85]).
- *Which* sets?
- *How much* time for each?
- *What* configuration?

Spatial-Reuse TDMA (STDMA)

Goal: Select *sets* of links or broadcasts such that spatial separation minimizes interference.

- Old idea: (goes back to [Nelson 85]).
- *Which* sets?
- *How much* time for each?
- *What* configuration?

Spatial-Reuse TDMA (STDMA)

Goal: Select *sets* of links or broadcasts such that spatial separation minimizes interference.

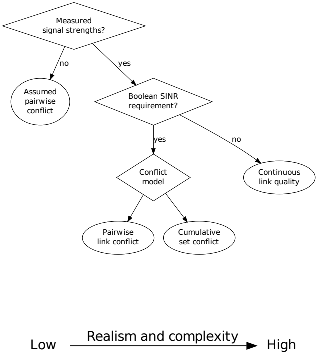
- Old idea: (goes back to [Nelson 85]).
- *Which* sets?
- *How much* time for each?
- *What* configuration?

STDMA Scheduling

Optimal scheduling is NP-Hard.

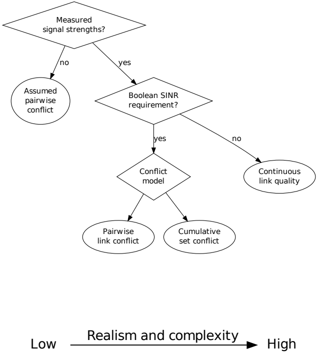
Responses:

- Relax objective ✓
- Relax constraints X
- Tighten constraints ✓



STDMA Scheduling

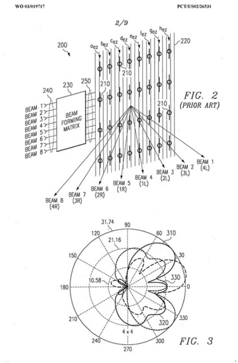
Optimal scheduling is NP-Hard.



Responses:

- Relax objective ✓
- Relax constraints X
- Tighten constraints ✓

Steerable, Switchable and Smart Antennas



Complication

If each node has p patterns, each set of m links has p^{2m} configurations.
Hairier than other adaptations:

- Power change affects signal and interference equally.
- Modulation change affects only the link in question.
- Antenna change affects everyone arbitrarily.

Complication

If each node has p patterns, each set of m links has p^{2m} configurations.
Hairier than other adaptations:

- Power change affects signal and interference equally.
- Modulation change affects only the link in question.
- Antenna change affects everyone arbitrarily.

Complication

If each node has p patterns, each set of m links has p^{2m} configurations.
Hairier than other adaptations:

- Power change affects signal and interference equally.
- Modulation change affects only the link in question.
- Antenna change affects everyone arbitrarily.

STDMA Scheduling

Goal: Partition links into concurrently-feasible sets to achieve desired throughput (delay, jitter, BER, etc.)

- **Ignoring RF interference**: Nodes can only participate in one link at a time. → Graph coloring-like algorithms (polynomial), *e.g.* [Hajek 88].
- **Pair-wise RF interference**: Link pairs are either compatible or not; any combination of links not including a forbidden pair is OK. → Polynomial graph algorithms, *e.g.* [Chlamtac 87, Ephremides 90, Chen 06, Liu 09].
- **Cumulative RF interference**: Combined interference from *all* other links must be acceptable for every link. Optimality is NP-hard [Arikan 84]. Greedy algorithms by, *e.g.* [Grönkvist 00, Brar 06]. Optimization algorithms by *e.g.* [Björklund 03, Johansson 06]. *
- **Continuous Interference Effect**: Link capacity as a function of SINR, not a threshold, *e.g.* [Radunović 04].

Antenna Capabilities

- Omnidirectional & Fixed Directional.
- Switched Beam
  - Sectorized antennas or arrays with pre-computed patterns.
  - Control consists of selecting among available patterns.
- Adaptive Array
  - Synthesizes beam patterns using on-line techniques.
  - Generally involves active measurement *e.g.* pilot tones.

- NO wedges, cones, pencil beams, etc.
- ... and the environment would distort them if there were.

Antenna Capabilities

- Omnidirectional & Fixed Directional.
- Switched Beam
  - Sectorized antennas or arrays with pre-computed patterns.
  - Control consists of selecting among available patterns.
- Adaptive Array
  - Synthesizes beam patterns using on-line techniques.
  - Generally involves active measurement *e.g.* pilot tones.
- NO wedges, cones, pencil beams, etc.
- … and the environment would distort them if there were.

Antenna Capabilities

- Omnidirectional & Fixed Directional.
- Switched Beam
  - Sectorized antennas or arrays with pre-computed patterns.
  - Control consists of selecting among available patterns.
- Adaptive Array
  - Synthesizes beam patterns using on-line techniques.
  - Generally involves active measurement *e.g.* pilot tones.
- NO wedges, cones, pencil beams, etc.
- ... and the environment would distort them if there were.

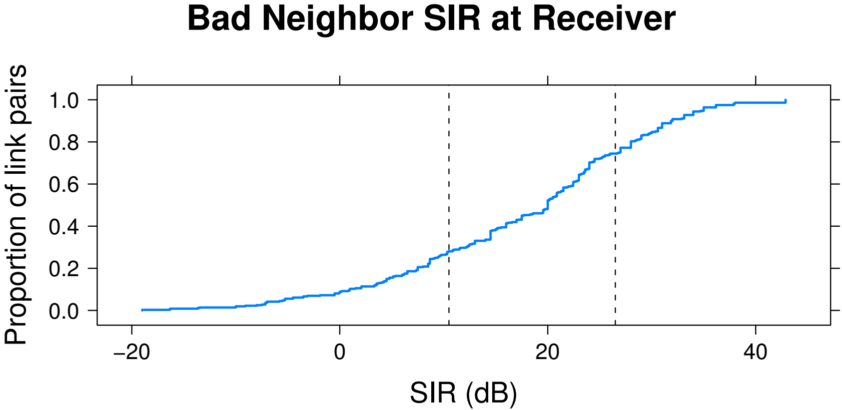
Controllable Antennas in Wireless Networks

- CSMA Protocols (not going to talk about)
  - "Deafness" problem, mixed directional/omni RTS-CTS, directional NAV, etc..
- Cellular (telephone or data)
  - One smart base station with many dumb clients.
  - $\approx$ No client-client interference.
  - Linear problem size, information & control all at BS.
  - (Some limited inter-cell interference mitigation exists.)
- STDMA

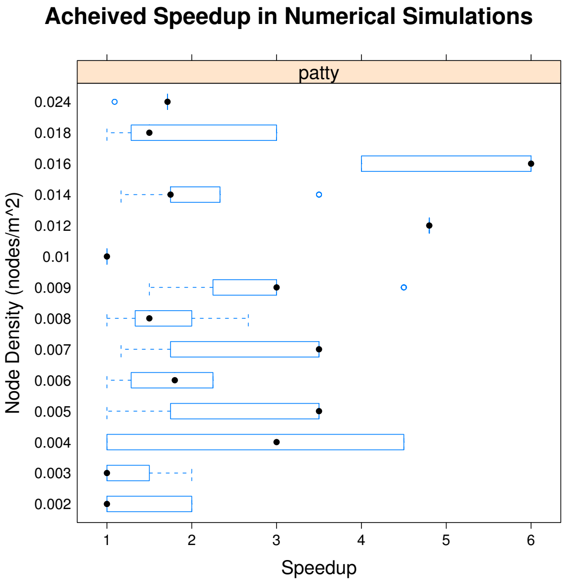
Controllable Antennas in STDMA

- Schedule then configure
  - [Lin 04]
- Configure then schedule
  - [Sánchez 99, Dyberg 02] and others. Special case: [Sundaresan 07]
- Schedule with assumed capabilities
  - Infinitesimal beam width [Cain 03]
  - Geometric rules *e.g.* significant signal propagates only in a wedge [Deopura 07].
  - Arbitrary k nulls [Sundaresan 06].
- Joint Scheduling and Configuration *
  - Pairwise configuration considered in scheduling [Sundaresan 07], *DIRC* [Liu 09].

“Bad Neighbor” links



Speedup by Node Density



Time to Algorithm Termination